TEMPORAL LOGIC

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TEMPORAL LOGIC

From Ancient Ideas to Artificial Intelligence

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PREFACE

Time is ubiquitous. Look to such diverse fields as literature and computers, ethics and physics, logic and rhetoric, philosophy and natural science. If you are studying any of these subjects, professionally or *con amore*, you are very likely to come across temporality as a crucial factor to your studies.

For this reason, people are led into the study of time from a variety of highly different disciplines. For the same reason, the study of time is useful and enlightening, both for its own sake and for a large number of specific purposes. The rather ambitious goal of this book is to comprehend time in its diversity, and yet to do this in a focused manner.

Our study stretches from Antiquity to the present day, and spans the field from literature to computer science. It thus comprises a historical as well as a systematical dimension. We believe that such a comprehensive approach is necessary in order to achieve a fuller understanding of time. The cost of this approach is that not all aspects can be given a treatment quite as thorough as they deserve. Just for example, there is much more to say about such fields as program verification, trivalent and many-valued logic, and quantified temporal logic than what we have managed to cover here. There are also relevant topics which have been entirely left out: for instance, the current discussions on indexicals, and the logic of truth value gaps, to mention two of the most important omissions.

With these disclaimers we wish to make it clear that we are ourselves aware of some limitations of this book. But we also believe that it does contain an unusually comprehensive exposition of the study of time.

We must say a few words about the genesis of this book. Peter Øhrstrøm was the first of us to do research on the concept of time, leading to his 1988 dr. scient. thesis on this subject. This thesis put a special emphasis on the relation between the logic of time and the general history of natural science. Per Hasle began studies on the logic of time in 1988, adding to our incipient common project perspectives from linguistics and information

science. Differences in our backgrounds notwithstanding, the contribution contained in this volume is the result of essentially joint work. The book contains entirely new results as well as previously published material, which has been reworked and put into the wider context of our exposition here.

A particularly important source for our book has been several interviews with Dr. Mary Prior, who has graciously provided valuable and interesting information on the work of her late husband Arthur Norman Prior - the founder of modern temporal logic. Furthermore, Mary Prior has granted us access to A. N. Prior's papers kept at the Bodleian Library in Oxford, also a crucial source for some of the new findings presented here. A very special thanks must go to her.

We are indebted to many persons for advice, criticism and inspiring discussions. We especially thank Mogens Wegener and Marta Ujvari for carefully reading and constructively criticising our manuscript. We also want to thank Harmen van den Berg, Knud Capion, Jack Copeland, M. J. Cresswell, Dick Crouch, Sten Ebbesen, Milea Angela Simoes Froes, Claudine Engel-Tiercelin, Antony Galton, Richard Gaskin, Nils Klarlund, Inger Lytje, Claus Myltoft, Jakob Møller, Stig Andur Pedersen, Amir Pnueli, Anne Rasmussen, Stephen Read, Jan Schmidt, Peter Simons, and Jan Tapdrup. All these persons have in various ways been helpful and inspiring for our work.

Peter Øhrstrøm
Per Hasle
Aalborg, January 1995

INTRODUCTION: LOGIC AND THE STUDY OF TIME

What, then, is time? If no one asks me, I know: if I wish to explain it to one that asketh, I know not.
St. Augustine [Confessiones XI, c. XIV, xvii. / Gale p. 40]

Every concept of time arises in the context of some (no doubt useful) human purpose and bears, inevitably and essentially the stamp of that human intent.
N. Lawrence [1978, p. 24]

Philosophers have had much to say about the nature of Time. Mathematicians and Physicists add a lot from their perspective. More recently, linguists are also becoming interested in the temporal constructions of natural language. Can a logician add anything of value to all this wisdom? J.F.A.K. van Benthem [1983, xi]

According to the Ancients as well as most of the later European thinkers in philosophy and science, time is primarily to be understood as strongly related to movement. In addition it is assumed that time can be described by numbers. Time has consequently been thought of as a basic concept for natural science, first and foremost physics and astronomy. In many circles physics is still assumed to be the key science for anyone who wants to study the concept of time, so let us first say a few words about the contribution from physics in this respect. According to Newtonian mechanics time is viewed plainly as a co-ordinate. The bodies in the world are supposed to move according to the laws of dynamics. These movements can be fully predicted in principle. All past and future states are implicit in the present state. Predictions and retrodictions can be expressed by means of spatial and temporal co-ordinates. At this level there is no proper temporal asymmetry, since the laws of dynamics permit time reversal. A concept of entropy might, however, be defined at this level, and the probability of increasing entropy will be

high. In thermodynamics the law of ever increasing entropy has been used as an argument for the so-called 'arrow of time'. Things become even more complicated when quantum physics is taken into consideration, and relativity theory raises some special problems for the logical study of time.

The study of physical time is certainly very important and useful. In our opinion, however, it is even more important to realise the relevance of what N. Lawrence [1979] has pointed out in his study of various levels in the discourse about time: every concept of time bears the stamp of human intent. When humans are taken into consideration the concepts of activity and creativity become very important for the understanding of time. In this connection it is clearly also possible - in fact, necessary - to introduce the idea of the 'NOW' and the direction of time. This observation must have general consequences, if it is accepted that every concept, including the concept of time, has to be related to the human mind. Under this perspective it becomes more natural to describe time by means of tenses: past, present and future, than by means of instants (dates, clock-time, etc.). With tenses, we can express that the past is forever lost and the future is not yet here. Without these ideas we cannot hope to grasp the idea of the passing of time.

Phenomena such as memory, experience, observation, anticipation and hope are all essential for the way time is understood. Notions of past and future time, the interpretation of the past as well as expectations of the future are all interwoven in the human mind. In this qualitative sense the past has not ceased to exist when followed by the next time period.

There are many common expressions for the qualitative and quantitative aspects of time, for example 'the sight of time' - 'time will tell' - 'old time' - 'I don't have time' - 'to waste time' - 'to buy time' - 'long time' - 'short time'. Apparently, human beings experience a tension between time as a quantity and time as a quality. We can certainly see the numerical or quantitative aspect of time, witness the clock and the calendar. But we also highly value the qualitative aspects of time such as the 'nowness' of events, and the passing of time as expressed by the tenses [Lundmark 1991].

However, time should not be seen as an idea merely dependent upon the individual mind, but also as an intersubjective idea. An individual cannot understand time properly only from the viewpoint of his or her own mental life. It is a very important fact that the tenses past, present, and future are not private, but at least intersubjective, if not objective. A satisfactory understanding of time requires a careful study of temporal relations in human society. It must be admitted that this sociotemporality is very complex and that little has been done in order to reach a deeper understanding of it. But it is clear that language and communication are in general essential for an understanding of social time.

The above description of the various notions of time does not explicitly say which idea of temporality is the most fundamental. In our opinion, the answer to the question of fundamentality must be that the concepts of past, present and future are basic, but that they cannot be fully understood unless sociotemporal relations and especially the preconditions for communication are taken into consideration. Therefore a proper study of time must involve an analysis of the general means and features of communication. So the study of meaning and language is essential for the understanding of time.

Nobody has yet presented a satisfactory definition of time. Every attempt to tell what time is can be understood as an accentuation of some aspects of time at the expense of others. Plato's definition of time as the 'moving image of eternity' and Aristotle's suggestion that 'time is the number of motion with respect to earlier and later' are no exceptions (see e. g. [Whitrow 1972]). In our opinion the attempt to establish a conclusive definition of time ultimately leads to confusion. Time is not definable by any other concepts. Time, in its fullness, is unique and *sui generis*.

The Augustinean wisdom that time cannot be satisfactorily described using just one single formula, definition, or explanation, is now generally accepted among philosophers of time. In order to gain more knowledge about the temporal aspects of reality, time has to be studied within many different strands of science. If such studies are to lead to a deeper understanding of time it-

self, various disciplines have to be brought together in the hope that their findings may form a new synthesis, even though we should not expect any ultimate answer regarding the question of the nature of time! If a synthesis is to succeed, a common language for the discussion of time has to be established. We are convinced that temporal logic (or 'the logic of time') is a crucial part of such a language.

The use of numbers in the description of time has made it obvious to see a connection between time and mathematics. However, some people may be taken aback by the claim that the concept of time is a subject for the discipline of logic. This reaction is primarily caused by the idea that logic is essentially timeless. Nevertheless, we will here attempt to document that time has been relevant in the development of logic, and indeed, that its relevance has never been more acute than today. We shall argue that this relation between time and logic is two-ways: logical investigations into time are required for a deeper understanding of the concept, as well as for the development of a general language for the discussion of time. On the other hand, temporal notions are required for a richer logic, applicable to a wider scope of problems ranging from computer science to philosophy.

We intend to demonstrate that the concept of time can in fact be studied using temporal logic. According to St. Augustine we all have a tacit knowledge of what time is, even though we cannot define time as such. In a sense the endeavour of temporal logic is to study some manifestations of this tacit knowledge.

In the first part of this book the question will be discussed from the perspective of the history of logic. It will be documented that there is a rich tradition of temporal logic from the ancient and medieval periods. We shall take the liberty of presenting some of these old ideas utilising the explanatory power of symbolic logic. The application of symbolic logic to ancient and medieval logic is in fact disputed - some researchers claim that such a procedure is anachronistic and misleading. We shall not take up that methodological discussion, except in the form of 'arguing by doing' - showing how concrete examples do lend themselves to a discussion partly in terms of symbolic logic. The great Polish

School of Logic set the example for this approach, and indeed, temporal logic itself partly began from the conviction that some classical logical ideas could and should be so studied. In our opinion, some of the brilliant insights of those ancient and medieval logicians and philosophers can in fact only be fully understood and studied further by modern logicians when recast in a symbolic formalism.

The rediscovery in our century of the importance of time and tense is first and foremost due to the works of Arthur Norman Prior, who was deeply inspired by his studies in ancient and medieval logic. In the 1950s and 1960s Prior laid out the foundation of temporal logic and showed that this important discipline was intimately connected with modal logic. Prior revived the medieval attempt at formulating a temporal logic corresponding to natural language. In doing so, he also used his symbolic formalism for investigating the ideas put forward by these logicians. Prior argued that temporal logic is fundamental for understanding and describing the world in which we live. He regarded tense and modal logic as particularly relevant to a number of important theological as well as philosophical problems. Using his temporal logic Prior analysed the fundamental question of determinism versus freedom of choice. The second part of the book will describe this rediscovery of the logic of time, focusing on Prior's contribution for the reasons just given. But we shall also describe his most important forerunners in the field of temporal logic in the 19th century and the first decades of the 20th century.

In fact, Prior himself preferred the term 'tense logic', but it has since then become commonplace to call the general quest for a logic of time as well as the resulting systems 'temporal logic'. We shall adopt the modern usage in this respect; later, we shall clarify the special meaning of 'tense logic' within the general picture. The main parts of temporal logic have been developed using mathematical symbolism and calculi, but nevertheless it has first and foremost been a philosophical enterprise. During the last decades it has become clear that temporal logic also has a number of practical applications. In part three we intend to outline some modern issues of temporal logic.

1.1. THE SEA-FIGHT TOMORROW

Chapter IX of Aristotle's work, *On Interpretation,* is without doubt the philosophical text which has had the greatest impact on the debate about the relations between time, truth, and possibility. In this text we find the famous example of 'the sea-fight tomorrow'; the discussion of this example certainly bears witness to the fact that Ancient philosophy was highly conscious of tense-logical problems (see [Gaskin 1995]).

Central to the discussion is the question of how to interpret the following two statements:

'Tomorrow there will be a sea-fight'.
'Tomorrow there will not be a sea-fight'.

Aristotle makes the following observation:

> Let us take, for example, a sea-fight. It is requisite on our hypothesis that it should neither take place nor yet fail to take place on the morrow. These and other strange consequences follow, provided we assume in the case of a pair of contradictory opposites having universals for subjects and being themselves universal or having an individual subject, that one must be true, the other false, that contingency there can be none and that all things that are or take place come about in the world by necessity. [*On Interpretation*, 18 b 23 ff.]

It is natural to discuss this text with a special view to the tense-logical semantics of operators concerning the future. The two statements above can be symbolised by

$F(1)p$
$F(1){\sim}p$

where p stands for the statement 'there is a sea-fight', and $F(1)$ is read 'it will be the case in one time unit' - this is what we would today call a metrical tense operator, since it is combined with an

explicit measure of time. In the present context $F(1)$ simply means 'it will be the case tomorrow', p stands for 'there is a sea-fight going on', and $\sim p$ stands for the negation 'there is not a sea-fight going on'.

Can statements like $F(1)p$ and $F(1){\sim}p$ be said to be true (or false) already today? Alternatively, is the truth value of the statement undetermined, such that it cannot be said to have any actual truth value today? The answers to these questions in turn bear upon the interpretation of modality. For if we assume that $F(1)p$ is true today, is the statement then not also necessary today? And further, if it turns out that there is no sea-fight tomorrow, can $F(1)p$ then be possible today? Aristotle was clearly aware of these relations, and in the discussion of the example he as well as later thinkers also examined the related problems concerning the modal concepts of possibility and necessity.

On grounds of his basic assumption of indeterminism, Aristotle claimed that neither statement could be necessary today. However, the same does not apply to statements about the past or the present; they are either necessarily true or necessarily false. Aristotle is apparently a 'past-determinist' and a 'present-determinist', but a 'future-indeterminist'. The Aristotelian logic must therefore be assumed to allow the following proposition within its framework:

$$P(n)p \supset NP(n)p$$

$P(n)$ stands for 'it was the case n time units ago' and N stands for 'it is necessary that ...'. This implication must as a minimum be consistent with the general theory, that is, its negation must not be valid. But still more likely, it should in fact itself be a theorem of the theory. On the other hand, the theory must reject the validity of

$$NF(n)p \vee NF(n){\sim}p$$

i.e. it should accept that in some cases it holds that

$$MF(n)p \wedge MF(n){\sim}p$$

where M stands for 'it is possible that ...'.

The Polish logician Jan Łukasiewicz [1920] has argued that Aristotle in fact considered propositions about future contingents to be neither true nor false. Such an interpretation is not merely a modern construction. Richard of Lavenham (c.1380) said something very similar, when he formulated the Aristotelian position in the following way:

> The third opinion, which was Aristotle's opinion, opposes the Christian faith in so far as this opinion presupposes that God does not know more determinately that Antichrist will be than that Antichrist will not be; and that He does not know more determinately that the day of judgement will be than that the day of judgement will not be; and that He does not know more determinately that the resurrection of the dead will be than that the resurrection of the dead will not be. And the reason is that there is no determinate truth of any of the two propositions about contingent future events. But these propositions 'the day of judgement will be' and 'the resurrection of the dead will be' are contingent propositions about the future, therefore they are not determinated to truth, and in consequence not more determinated to truth than to falsity (and also not conversely). The consequence is clear, and the major premise is Aristotle's opinion in 'On Interpretation'. And this opinion presupposes that no contingent proposition about the future is true, and that no such proposition is false. This was Aristotle's intention as Ockham says in his book about 'On Interpretation'. [Øhrstrøm 1983]

Lavenham's version of Aristotle's statement clearly means that $F(n)p$ as well as $F(n)\sim p$ are neither true nor false. It is, however, unclear whether he had in mind a third true-value corresponding to 'indeterminate', or simply held that no truth-value is defined for such contingent future propositions. In any case, Lavenham regarded the Aristotelian view as contrary to the Christian faith, and he preferred a solution suggested by William of Ockham.

Łukasiewicz has argued that Aristotle's text in chapter IX of *On Interpretation* should be read as an argument for a three-valued logic. At least in the early 1950's A. N. Prior shared this view [1957a, p.86]. At this time he thought that this was the only way to construct an indeterministic tense-logic [1967, p.128]. Therefore he suggested a three-valued logic of tensed propositions [1953]. Later it became clear to him that indeterministic tense-logic with bivalence is possible in at least two important ways - known as 'the Ockhamist system' and 'the Peircean system'. These systems are Prior's formalisations of ideas by Ockham and Peirce; both systems will be examined in detail later on.

The interpretative problems regarding *On Interpretation* chapter IX are by no means simple. N. Rescher [1968] has shown in a word-by-word analysis of the critical passage of chapter IX how a realistic interpretation, which maintains the principle of bivalence, can be consistently defended, and in fact was defended by Scholastic and Moslem philosophers in the Middle Ages. As we shall see, this medieval interpretation and the tense logic pertaining to it provide an affirmative answer to the question of whether statements about the contingent future do have a truth-value (at the time of utterance). They also confirm that even if it turns out that there is no sea-fight tomorrow, $F(1)p$ can be regarded as possible today. In principle, past-determinism was also accepted, but it was observed that it only holds for statements which are properly about the past. What was at stake here was to rule out necessitation for sentences of the form $P(n)F(m)p$, whose grammatical form is in the past tense, but which are only spuriously about the past when $m > n$. Concerning the general discussion of the rôle of necessity, Rescher referred to Peter Abelard (c. 1079-1142), who stated:

> No proposition about the contingent future can be either determinately true or determinately false in the same sense, but this is not to say that no such proposition can be true or false. On the contrary, any such proposition is true if the outcome is to be true as it states though this is still unknown to us. What Aristotle wished to maintain in his De Inter-

> pretatione was that while a proposition is necessary when it is true, it is not therefore necessarily true simply and always. [Kneale p.214]

As we shall see later the Ockhamist system makes it possible that a proposition about the contingent future can be true now, even though its truth-value is still unknown to us. In this crucial sense Abelard's interpretation is in agreement with Prior's Ockhamist system.

Henning Boje Andersen and Jan Faye [1980] have, however, put forth a different interpretation of chapter IX. They claimed that Aristotle would probably reject the general validity of what could be called 'the law of excluded middle for statements in the future tense', i.e. for all p:

$$F(n)p \vee F(n){\sim}p$$

Given that this proposition is not valid, it must be accepted that

$${\sim}F(n)p \wedge {\sim}F(n){\sim}p$$

may indeed be true for some proposition p. In fact, according to this interpretation the latter formula is possible for any contingent statement about the future. On the other hand, it is also clear that $F(n)p$ and $F(n){\sim}p$ cannot both be true. Therefore

$${\sim}F(n)p \vee {\sim}F(n){\sim}p$$

must be a theorem in the Aristotelian system under this interpretation.

It is worth pointing out that this interpretation makes Aristotle's observations consistent with the aforementioned Peircean system. Thus, there is a line from the two basic interpretations of Aristotle's text presented here to Prior's two major indeterministic tense logical systems.

1.2. THE MASTER ARGUMENT OF DIODORUS CRONUS

Diodorus Cronus (ca. 340-280 B.C.) was a philosopher of the Megarian school [Sedley 1977]. He achieved wide fame as a logician and a formulator of philosophical paradoxes. The most well-known of these paradoxes is the so-called 'Master Argument' which in Antiquity was understood as an argument designed to prove the truth of fatalism. Unfortunately, only the premises and the conclusion of the argument are known. We know almost nothing about the way in which Diodorus used his premises in order to reach the conclusion. During the last few decades various philosophers and logicians have tried to reconstruct the argument as it might have been. The reconstruction of the Master Argument certainly constitutes a genuine problem within the history of logic. It should, however, be noted that the argument has been studied for reasons other than historical. First of all, the Master Argument has been read as an argument for determinism. Secondly, the Master Argument can be regarded as an attempt to clarify the conceptual relations between time and modality. When seen in this perspective any attempted reconstruction of the argument is important also from a systematic point of view, and this is obviously true for any version of the argument, even if it is historically incorrect.

Our approach in this chapter will in the first part be mainly historical. We shall comment on some of the reconstructions which have been suggested, and present an elaborated version of one of them. At the end of the chapter, we shall discuss some of the philosophical and conceptual problems related to the Master Argument.

The Master Argument is a trilemma. According to Epictetus, Diodorus argued that the following three propositions cannot all be true [Mates 1961, p.38] :

(Dl) Every proposition true about the past is necessary.
(D2) An impossible proposition cannot follow from (or after) a possible one.

(D3) There is a proposition which is possible, but which neither is nor will be true.

Diodorus used this incompatibility combined with the plausibility of (D1) and (D2) to justify that (D3) is false. Assuming (D1) and (D2) he went on to define possibility and necessity as follows:

(DM) The possible is that which either is or will be true.
(DN) The necessary is that which, being true, will not be false.

In order to reconstruct the Master Argument two fundamental questions must be answered:

(1) How should 'proposition' in (Dl-3) be understood?
(2) How should 'follow' in (D2) be understood?

For the sake of completeness it should be mentioned that for some reconstructions it is also relevant whether the structure of time is assumed to be discrete or continuous.
The first of the above questions can be answered in at least two ways :

(1.1) The propositions mentioned in (D1-3) are temporally definite statements.
(1.2) The Master Argument refers in fact to statements corresponding to propositional functions.

F.S. Michael [1976] has suggested a reconstruction of the Master Argument based on (1.1). According to Michael the truth or falsity of such statements is entirely unaffected by the time of assertion. In his version the first premise of the argument can be formulated in the following way:

(D1M) If the proposition p_0 is true at some time t' before t, then the truth of p_0 is necessary at t.
In symbols: $(T(t',p_0) \wedge t' < t) \supset N(t,p_0)$

Note that this can only be reasonable if the proposition p_0 in (D1M) itself takes the form $T(t'',r)$. Using (D1M) Michael could in fact construct an argument like the Master Argument without using (D2) directly. For his attempt at a reconstruction, however, Michael had to presuppose that a necessary proposition is true. This principle seems to be uncontroversial, but it is not implied by (D1-3) alone. His proof can be presented in the following way:

> According to (D3) it is assumed that there is a proposition q_0, which is possible, but false now and also at any future time. The proposition q_0 must in the argument itself be of the form $T(t'',r)$ by Michael's assumption of (1.1). This means that the following holds:
>
> $$M(n,q_0) \wedge T(n,\sim q_0) \wedge (\forall t: t>n \supset T(t,\sim q_0))$$
>
> Now, q_0 must be false also before n, since if for some t'
>
> $$T(t',q_0) \wedge t'<n,$$
>
> then (D1M) would give us $N(n,q_0)$ and therefore also $T(n,q_0)$, which would contradict the above assumption. - Hence it can be concluded that q_0 is false at any time, t, i.e.
>
> $$t < n \supset T(t,\sim q_0)$$
>
> for any t. It then follows from (D1M) that $N(n,\sim q_0)$. This means that $\sim M(n,q_0)$, which contradicts the above assumption about q_0 being possible at n.
> Q.E.D.

It follows from the argument as reconstructed by Michael that a true proposition is necessary and a false proposition is impossible. But then it can be said that 'possible', 'true', and 'necessary' are identical qualifications of propositions. Therefore, Michael proves too much, since (DM) and (DN) are obviously meant to carry different informative content - that is, they should not be made equivalent. So there is not sufficient reason for accepting Michael's assumption regarding the status of propositions in the Master Argument. And indeed, for other and independent rea-

sons it seems most probable that Diodorus thought of propositions as corresponding to what we today would call functions. His examples include statements like 'It is day', 'I am conversing', 'It is light'. As Mates [1961, p.36] has stated, these propositions 'are true at certain times and false at others', or equivalently, 'they become true and become false'. Furthermore, Mates could also conclude that Diodorean necessity would in most cases apply to such 'functional propositions', so generally speaking we should expect (1.2) to be the correct answer as regards the status or nature of propositions in the Master Argument. Nevertheless, Mates did not think that (D1) could make sense if 'proposition' is understood in this way [1961, p.39]. Therefore Mates' analysis apparently left us with an enigma: according to this analysis, (1.2) was the most probable answer, but Mates could not see how this assumption could be consistent with the context of the Master Argument.

However, as we shall see in the following, Prior has shown how a reading of (D1) consistent with (1.2) is in fact possible. But first we must examine the question regarding the understanding of (D2). This question can also be answered in at least two different ways:

(2.1) 'Follows' in (D2) refers to temporal order.
(2.2) 'Follows' in (D2) refers to logical implication.

Like the reconstructions of Zeller [1882] and of Copleston [1962], Rescher's reconstruction [1966] of the Master Argument is based on an assumption like (2.1), i.e. on a temporal version of (D2). Rescher assumes that the original formulation of this premise can be reformulated in the following way:

(D2x) The impossible does not follow after the possible.

(D2x) implies that what has been possible will always be possible. This 'principle of possibility-conservation' is obviously not very plausible. Even if some proposition p could once be regarded as possible, consistently with whatever else obtained at that time, some of the conditions for p may change permanently

at a later time such as to make it impossible always thereafter. Moreover, Mates observed that the word used by Epictetus in (D2), which Rescher translates into 'follow after', is the same word used by Diodorus for 'is a consequent of'. It should also be noted that Chrysippus, who rejected the Master Argument, understood its second premise as referring to logical consequence rather than temporal succession [Mates 1961, p.39]. Finally, a circumstantial but important piece of evidence that (D2) is concerned with logical consequence is the fact that Diodorus studied the nature of implication very carefully. The famous debate between Diodorus and Philo of Megara precisely concerned the relation between time and implication. Their views on implication were described in the following way by Sextus Empiricus:

> according to Philo such a conditional as 'If it is day, then I am conversing' is true when it is day and I am conversing, since in that case its antecedent, 'It is day' is true and its consequent, 'I am conversing', is true; but according to Diodorus it is false, for it is possible for its antecedent, 'It is day', to be true and its consequent 'I am conversing' to be false at some time, namely, after I have become quiet... [Adv.Math. VIII, 112ff; Mates, 1961, p. 98]

This conflict between Diodorus and Philo was obviously concerned with whether one could allow the truth values of the implication to vary with time or not. As Mates [p.46] has argued, a conditional was proved to be Diodorus-true by showing that it never has a true antecedent and a false consequent. That is, Diodorus favoured what we today could call temporally strict implication, whereas Philo argued for material implication. The quotation also bears on the status of propositions, for Diodorus' argument as referred by Sextus Empiricus presupposes that propositions are understood as functions.

It appears that Diodorus regarded logical implication as very important. Therefore, it is only natural to assume that it played an important rôle in his Master Argument. We believe that (2.1) should be rejected and that (2.2) should be accepted, and

also that it is natural to assume that the implication in question was the Diodorus-implication, which is true just in case it never has a true antecedent and a false consequent.

PRIOR'S RECONSTRUCTION

Prior's reconstruction [1967, p.32 ff.] of the Master Argument follows the line of the interpretations (1.2) and (2.2). Thus it basically adopts the same understanding of 'proposition' and consequence as we have been arguing for above. Prior uses tense- and modal operators in his reconstruction, and interprets the logical (Diodorean) consequence involved in (D2) as what is in modal logic usually called 'strict implication', symbolised by $\rightarrow$.

On these assumptions it is possible to restate the reconstruction problem. Using symbols, (D1-3) can be formulated in the following way:

(D1') $Pq \supset NPq$
(D2') $((p \rightarrow q) \wedge Mp) \supset Mq$
(D3') $(\exists r) (Mr \wedge \sim r \wedge \sim Fr)$

where F is read as 'it will be the case that...', P is read as 'it has been the case that ...'. , and $\rightarrow$ is the strict implication defined as

$$p \rightarrow q \equiv N(p \supset q)$$

We are now ready to reformulate Prior's reconstruction. In doing so, we shall at first leave aside some of the problematic points about it, in order to make the main thrust of the argument as clear as possible. We shall use the propositional function q: 'Dion is here' as an example. The reconstruction, then, runs as following way. Let us make the following two assumptions:

(P1) It is possible for Dion to be here.
In symbols: Mq

(P2) Dion is not here and he never will be here.
In symbols: $\sim q \wedge \sim Fq$

Obviously, (P1) and (P2) together make up an instance of (D3). Now intuitively speaking, if Dion is not here now and from now on never will be here, then in the 'immediate past' it was true simply that Dion never would be here. Thus, it follows from (P2) that

(P3) It has been the case that Dion never will be here. In symbols: $P{\sim}Fq$

By substitution into (D1') we have $(P{\sim}Fq \supset NP{\sim}Fq)$. Therefore, it follows from (P3) and (D1') that

(P4) It is necessary that it has been the case that Dion never will be here. In symbols: $NP{\sim}Fq$

For the sake of exposition, it is useful to subject (P4) to two transformations. First, since N is equivalent with ${\sim}M{\sim}$, we directly obtain

(P5) It is impossible that it has not been the case that Dion never will be here. In symbols: ${\sim}M{\sim}P{\sim}Fq$

We can now make use of the common tense-logical symbol H, which is an abbreviation of ${\sim}P{\sim}$, and which may be read 'it has always been the case that ...' Using H in (P5), we get

(P6) It is impossible that it has always been the case that Dion will be here. In symbols: ${\sim}MHFq$

If Dion is here now, then at any time in the past it has been true to say 'Dion will be here'. Hence, the following implication is true:

(P7) If Dion is here, then it has always been the case that Dion will be here. In symbols: $q \rightarrow HFq$

By conjoining (P1) and (P7) we obtain $((q \rightarrow HFq) \wedge Mq)$. Using (D2') we can then deduce $MHFq$.

We have now arrived at a contradiction, since on assuming (P1) and (P2) we have derived $\sim MHFq$ (P6) as well as $MHFq$. Therefore, the combined assumption of (P1) and (P2) must be rejected.

Unfortunately, it is clear that Prior is not able to reconstruct the argument only using (D1), (D2) and (D3). In addition to these, he needs two extra premises. In order to make sure that the argument from (P2) to (P3) is valid, he must assume that

$$(\sim q \wedge \sim Fq) \supset P\sim Fq$$

or, to put it in a general form, that

(D4) $(p \wedge Gp) \supset PGp$

where $G \equiv \sim F \sim$ ('it will always be the case that...'). Furthermore, he must assume that (P7) is in fact a valid strict implication such that

(D5) $N(p \supset HFp)$

is valid in general.

Prior's proof that the three Diodorean premises (D1', D2', D3') are inconsistent given (D4) and (D5) can be summarised as a reductio ad absurdum proof in the following way:

(1)	$Mr \wedge \sim r \wedge \sim Fr$	(from D3')
(2)	$M r$	(from 2)
(3)	$N(r \supset HFr)$	(from D5)
(4)	$MHFr$	(from D2, 2 & 3)
(5)	$\sim r \wedge G\sim r$	(from 1)
(6)	$PG\sim r$	(from 5 & D4)
(7)	$NPG\sim r$	(from 6 & D1)
(8)	$\sim MHFr$	(from 7; contradicts 4)

Q.E.D.

O. Becker [1960] has shown that the extra premises (D4) and (D5) can be found in the writings of Aristotle. For that reason Becker concludes that it seems reasonable to assume that the extra premises were generally accepted in antiquity.

However, Prior's addition of (D4) and (D5) is nevertheless problematic (even though the argument thus reconstructed is interesting in its own right). (D4) is in fact a rather complicated statement and not so innocuous as it may seem at first glance - observations which will indeed become clear when we are going to discuss the Ockhamist and Peircean systems. It is not very likely that Diodorus would involve such an argument without making it an explicit premise in the Master Argument. As regards (D5), we know that Diodorus used the Master Argument as a case for the definitions (DM) and (DN). That is, in the argument itself *M* (or *N*) should in a sense be regarded as primitive. It is hard to believe that Diodorus would involve a premise about *N* without stating it explicitly.

A New Reconstruction of the Master Argument

As we have argued, Mates in his excellent analysis gave all the essential information needed for a reconstruction of the Master Argument. On the basis of the considerations so far we shall suggest a very simple argument as a possible reconstruction. We shall see that the argument can be formulated without the use of complicated extra premises as it is the case in Prior's reconstruction. We shall assume that in the Master Argument certain notions regarding time and propositions are taken for granted:

(a) Time is discrete.

(b) Diodorean propositions are functions of time. Thus, propositions are functions from instants into truth values - and conversely, such functions are propositions. For the function application of a proposition p to an instant t we write $T(t,p)$.

(c) The Diodorean implication involved in (D2) can be defined in terms of present-day temporal logic as

$$(p \Rightarrow q) \text{ if and only if } (\forall t)\ (T(t,p) \supset T(t,q))$$

Ad (a): It is not possible to prove directly that Diodorus took time to be made up of temporal atoms, although there is evidence that Diodorus believed in indivisible places and bodies [Adv. Phys. II,142-143]. Richard Sorabji [p.19] has maintained that a certain passage in the works of Sextus Empiricus [M 10.86-90] indicates that Diodorus was a temporal atomist. But even if Sorabji is wrong and Diodorus was not a temporal atomist, we might still undertake a reconstruction along the lines which we have been suggesting, provided that Diodorus held something like

> (A) No proposition has a first instant of truth. If a proposition is true, it has already been true for some time.

Although we have no direct information indicating that Diodorus actually made this assumption, it is indeed very likely that he was aware of Aristotle's point of view:

> For a change can actually be completed, and there is such a thing as its end, because it is a limit. But with reference to the beginning the phrase has no meaning, for there is no beginning of a process of change, and no primary 'when' in which the change was first in progress. [Phys. 236a 12-14]

It is not unreasonable to surmise that Diodorus tried to elaborate this observation, and that this work led him to an assumption like (A). We shall, however, omit a detailed reconstruction of the master argument on the basis of (A).

Ad (b): Diodorus apparently thought of propositions as though they contained time-variables. These propositions are true at certain times and false at other times. On the other hand, Mates has maintained that "although Diodorus usually predicates

necessity of what are in effect propositional functions, it seems that in the first of his three incompatibles, necessity is predicated of a proposition" [1961, p. 39]. We shall demonstrate how an understanding of the Master Argument based on (1.2) as well as (2.2) is possible.

Ad (c): According to Mates [1961, p.45] "a conditional holds in the Diodorean sense if and only if it holds at all times in the Philonian sense". (The Philonian implication is simply the material implication). Mates has demonstrated that his conclusion is a clear consequence of a number of passages from the sources.

Note that the assumptions (a), (b), and (c) are all well documented in the known sources about Diodorus' logic. Moreover, they do not involve the modal concepts which are at stake in the argument. For these reasons (a)-(c) should not be regarded as extra premises like Prior's (D4) and (D5).

In (c), we use '$\Rightarrow$' instead of '$\rightarrow$' in order to emphasise that our definition is distinct from Prior's definition of Diodorean implication, which was

$(p \rightarrow q)$ if and only if $N(p \supset q)$

If we did not keep these two definitions apart, (c) might be seen as *defining* modality in terms of temporality. However, the Master Argument was thought to lead to such a definition, to wit, (DM) and (DN), not to presuppose it. On (c), (D2) may be rendered as

$$(p \Rightarrow q) \wedge Mp) \supset Mq$$

where the possibility-operator should be understood as a still unanalysed concept. We shall assume, however, that Diodorus accepted the usual interdefinability between necessity and possibility (as he indeed most likely did). In symbols, this means

$$M = {\sim}N{\sim},\ N = {\sim}M{\sim}.$$

Using the assumptions (a) - (c), it is possible to reconstruct the argument.

It should be noted that although (c) defines $(p \Rightarrow q)$ in terms of temporality, it is very different from the kind of temporal definition involved in Rescher's understanding of the Diodorean 'follows'. Our understanding of $(p \Rightarrow q)$ refers to a quantification over temporal instants rather than a temporal order.

Let us assume (D3) for some statement q, e.g. 'Dion is here'. In symbols:

$$\sim q \wedge \sim Fq \wedge Mq$$

Then the statement is false now and at every future time, although Dion's being here is possible. We intend to show that the assumption of (D3) contradicts the premises (D1) and (D2).

Let *r* be a statement true only at the time just before the present time. Although any arbitrary statement fulfilling the requirement would do, we may choose the more intuitively appealing

r: 'The prophet says: Dion will never be here.'

From the propositional function *r*, we can construct the propositional function *Pr*, which is obviously false at any past time, true now and always in the future. We can illustrate the situation by the following figure, where the instant 'now' is represented by the number 10:

~Pr	~Pr	~Pr	Pr	Pr	Pr
~r	~r	r	~r	~r	~r
7	8	9	10	11	12
?q	?q	?q	~q	~q	~q

Clearly r is false at any instant other than 9, the instant immediately preceding the now. $\sim Pr$ is true at any past time, i.e. any instant lesser than 10. On the other hand, Pr is true now, at 10, and always thereafter. Finally, by our assumption of (D3), q is false now and always in the future. However, q might be true or false at any past time.

Since Pr is true now, we can by (D1) obtain NPr, which is equivalent with $\sim M\sim Pr$. It is also evident that

$$(q \Rightarrow \sim Pr).$$

This Diodorean implication is valid since if q is true at time t, then t must be a past time; this follows from our assumption of (D3) as illustrated in the figure. Furthermore, $\sim Pr$ is true at any past time. Therefore the antecedent can never be true when the consequent is false. But the validity of this Diodorean implication contradicts (D2), since the impossible, $\sim Pr$, follows from the possible, q. Therefore the assumption of (D3) has to be rejected.

In this way the Master Argument can be reconstructed using discrete time and the Diodorean idea of implication. We think it very likely that this was the kind of reasoning actually used by Diodorus.

It is interesting that the above argument works even if it is assumed that the first premise (D1) of the Master Argument is concerned only with propositions which are genuinely about the past. An example of a proposition which is not genuinely about the past would be 'One day ago it was the case that in two days, Dion will be here'. Such propositions should not be necessitated by (D1), although they may be necessitated on other grounds. In Prior's reconstruction, statements which are only spuriously about the past are regarded as necessary. In this way the validity of implications like $PGq \supset NPGq$ can be derived. In our reconstruction, however, such a questionable use of (D1) is completely unnecessary.

The way (D2) is used in our reconstruction bears some resemblance to one of the paradoxes of implication, since we can without loss of generality assume that q is not only false in the

present and the future, but also in the past - that Dion has never been here, is not here and never will be here. In this case any proposition will follow from q in the Diodorean sense. Indeed, it is not required that there be any semantical relation between q and r in the argument. In general, if q is any proposition which is always false, then the Dioderean implication $(q \Rightarrow p)$ holds for any arbitrary proposition p; in this case, the implication obviously never has a true antecedent and a false consequent. But then we may choose any possible proposition q in order to show that p must be possible. Hence, any proposition which is always false must be possible on the assumption of (D2).

In this connection it should be noted that the ancients were aware of the paradoxes of implication. There can be no doubt that Diodorus, too, realised that any proposition which is always false, implies any other proposition.

LOGICAL DETERMINISM

It is very likely that the Master Argument was originally designed to prove fatalism or determinism. Because of the apparent plausibility of (D1) and (D2), the argument was understood as a rather strong case against (D3). The denial of (D3) is equivalent to the view that if a proposition is possible, then either it is true now or it will be true at some future time. So in a nutshell the argument is that an event which never will happen and is not happening now cannot be possible, and hence everything happening now or in the future is necessary. It should be clear, then, that the argument is interesting not only for historical reasons. Its systematical content is entirely relevant for a modern discussion of determinism, too. The present-day philosopher wanting to argue against fatalism and determinism must relate to all known versions of the Master Argument, directly or indirectly. If the fatalistic or deterministic conclusion of the Master Argument is to be avoided, at least one of the two premises (D1) and (D2) has to be denied - at any rate, that is the case as long as we accept the tacit assumption that time is a lin-

ear structure. Now for any version of the Master Argument based on that assumption we believe that it is in fact quite reasonable to deny at least one of (D1) and (D2). Let us consider the versions which have been discussed above.

As mentioned above, the second premise in Rescher's version of the Master Argument turns out to be equivalent to a 'principle of possibility-conservation'. It would certainly be reasonable to deny the validity of this principle. In Michael's version of the Master Argument the first premise, (D1M), should be denied, since it is not reasonable to view a true proposition about the future as necessary, just because it is formulated as a prophecy stated in the past. Such a proposition is about the past only in a spurious sense. Regarding (D1) in Prior's reconstruction we can make a similar observation. The statement

'It has been that Dion never will be here', (in symbols: $P{\sim}Fq$)

should not be counted as necessary even if it is true. Even if we accept ${\sim}q$, ${\sim}Fq$, and $P{\sim}Fq$, there is no a priori reason to exclude the conceptual possibility of Dion's being here at some future time, or his 'having always been going to be here', i.e. MFq and $MPGq$. Therefore, the way in which (D1) is used in Prior's version of the argument should certainly be questioned.

In our reconstruction, we do not have to assume any more than the necessity of propositions which are genuinely about the past. When (D1) is seen in this way, it appears reasonable, whereas (D2) should be rejected if time is linear. The reason is that if there is a propositional function q which is possible but never true, then our version of (D2) implies that any absurdity $(p \wedge {\sim}p)$ also becomes possible. Obviously, it is not acceptable to regard an absurdity as being possible. Given that time is linear it seems entirely reasonable to deny (D2).

Prior himself questioned the validity of (D5) i.e.

(D5) $N(p \supset HFp)$

If we understand 'will be' as 'determinately will be', then (D5) can certainly be denied, as in fact it is in the Peircean system, which Prior elaborated and to which he indeed preferred himself . We shall return to this system in part 2.

SOME CONCEPTUAL CONSIDERATIONS

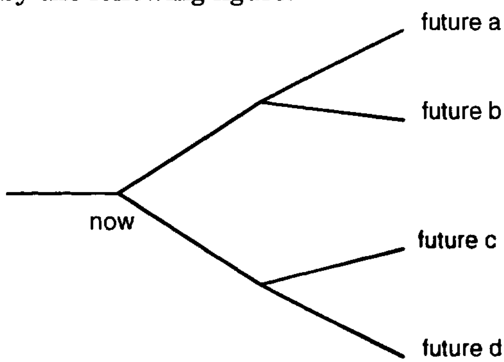
The Master Argument can also be read as an attempt to relate the modal concepts of possibility and necessity to the concept of time. The various versions of the argument emphasise the impact of temporal indices on the operators of possibility and necessity. For instance, what is possible now need not be possible in the future. And what is now not necessary but a mere possibility, can become necessary in the future. It is obvious that the notion of modality involved in such assumptions should be linked to the idea of time. A proposition is necessary if it is 'now-unpreventable', and a proposition is possible if its negation is 'now-preventable'. In formulating his argument Diodorus was aiming at a justification of his definitions of possibility and necessity, (DM) and (DN), which were:

(DM) The possible is that which either is or will be true.
(DN) The necessary is that which, being true, will not be false.

But if these definitions are accepted, and if time is understood as a linear structure, then we are led to some kind of fatalism or determinism.

As we have seen, we do not have to accept (DM) and (DN) on account of the argument itself, since at least one of the premises (D1) and (D2) should be rejected if time is implicitly or explicitly understood to be a linear structure. However, the picture is somewhat different if we avail ourselves of the modern notion of branching time: that is, if time is considered to be a branching structure, it is not representable as a subset of the real numbers, and both (D1) and (D2) as understood in our reconstruction become plausible. In part 2 we shall examine the notion of

branching time in detail. The basic idea can, however, easily be illustrated by the following figure:



The central idea is that for any given 'now' there are a number of possible and different futures - sometimes called the 'forking paths into the future'. Just one of these will become actualised in the course of time. In this kind of structure a propositional function cannot be represented by a series of truth-values. Rather, it must be represented as a complex structure of values. It should not be too hard to see that if the complex structures of branching time are discrete, then our new version of the Master Argument is still valid. The premises (D1) and (D2) as understood in our version can be accepted within all theories of branching time, in which case the conclusion of the Master Argument also has to be accepted within these theories. An adequate conception of the notion of 'possibility' can then be captured by the formula

$$Mr \equiv (r \vee Fr)$$

Obviously this means that the definitions (DM) and (DN) should also be adopted in theories of branching time. In fact, the very use of the idea of 'possible futures' can be understood as an acceptance of the conclusion of the Master Argument, since it is evident that if time is branching then any possibility must belong to some possible future. So when we investigate the Master Argument from the perspective of the historical development of the logical study of time, the argument turns out to be a demon-

stration of a fundamental relationship between time and modality rather than a case for fatalism or determinism.

The relation between time and modality and the attempt to define modality in terms of tense were very important to the founder of modern symbolic tense logic, A. N. Prior. As we shall see in part 2, Prior elaborated the formula above into a very complex and conceptually refined definition - his so-called fourth grade of tense-logical involvement, wherein the concept of modality becomes entirely absorbed by this tense logic. This fourth grade expressed Prior's own conception of time.

1.3. THE STUDY OF TENSES IN THE MIDDLE AGES

The Diodorean Master Argument can be seen as an example of that interest in the logic of statements involving time which is part of a tradition dating back to Aristotle and other Ancient philosophers. The Scholastic logicians in particular made a number of original contributions to tense-logic. We shall now devote a few chapters to a brief survey of the most important of these contributions.

Medieval logicians were engaged in an attempt to develop a logic of natural language. With this objective they had to take heed of the fact that some statements do not have fixed truth-values. A proposition like 'Socrates is alive' is true when Socrates is alive, and it is false when he is not alive. Therefore it is an integral part of medieval logic that the truth-value of a proposition can vary from time to time. For the same reasons it was natural, indeed inevitable, for them to analyse tensed statements in their logical studies. It was an important goal of theirs to be able to describe the logical content of propositions about past and future events.

The Scholastic logicians discussed the status of tensed statements with a view to theological problems. In the course of time the difference between statements such as 'Christ was born', 'Christ is born', and 'Christ will be born' had given rise to a theological and logical problem. On the one hand, a distinction between the three forms from a purely logical point of view was considered legitimate. On the other hand, some claimed that there was in principle no difference between what had been believed by the prophets (the third form), the contemporaries of Jesus (the second form), and what has been believed by Christians in all the succeeding centuries (the first form). The object of the faith is therefore the same one. But how can the unity of faith and its independence of time be maintained, when its main tenets are described by statements whose meanings seem to vary in time in the same manner as other tensed statements?

There were two different solutions in the Middle Ages, as pointed out by Nuchelman [1980, p.133]. Firstly, there was a

comparatively small group of thinkers, who defended one or another variant of the so-called 'res-theory'. These scholars wanted to consider the object of faith as an unchangeable entity. For instance, the object might be certain events which were believed to have actually occurred. Secondly, there was a larger group of Schoolmen, who adhered to the 'enuntiabile-theory'. They maintained that the three statements: 'Christ was born', 'Christ is born', and 'Christ will be born' are significantly different, although a hard core of meaning remains. It is quite conceivable that doctrines (e.g. the virgin birth) ought to be expressed without tense, but that a confession (e.g. 'I believe that Christ was born by a virgin') becomes tensed when the faith is to be expressed by an individual. The tense free doctrine is that which all the tensed creeds refer to.

Thomas Aquinas [Summa theologiae II 2. q. 1. art. 2 & De veritate q. 14, art. 12], among others, attempted to act as an intermediary between the two theories. He pointed out that one can consider the object of faith either from the point of view of the object itself, or from the point of view of the faith. This corresponds exactly to the difference between the 'res-theory' and the 'enuntiabile-theory'. The duality between these two perspectives is also evident when it comes to a discussion of the relation between God's and man's possibilities for having knowledge. According to Thomas, divine knowledge is primarily aimed at the object itself (res), while man can only know and believe in enuntiabile [Summa Theologia I q. 14. art. 15]. The tension between these two Scholastic theories, which take their starting points in enuntiabile and res, respectively, in a highly striking manner corresponds to the modern debate in tense-logic regarding A- and B-theories. We shall discuss those notions in part 2.

The debate about the semantic status of tense inflected statements can be regarded as an example of the scholastic emphasis on what we now call tense-logic. This subject matter was of general interest during the entire Middle Ages and covered a broad spectrum of theories, which also included work concerning questions bordering on tense-logic. These investigations sometimes went to the borderline of possible

language use, as in a discussion by Boethius de Dacia (c.1270) about the statements 'heri curram' ('I will run yesterday') and 'cras cucurri' ('I ran tomorrow') [Boethii Daci Opera, IV, I, p.203] - a discussion which was meant to be entirely serious.

Medieval logicians in general were also very much aware of the problems related to logical arguments involving tenses. In his famous *Logica Magna* - which is representative for a great deal of medieval logic - Paul of Venice (c. 1369-1429) dealt with a number of questions concerning reasoning about time and tenses. For instance, he considered the following argument [Part II Fasc. 8, p.271]:

(Arg. 1)
Socrates is in Rome at moment A;
You are in Rome at some moment;
therefore you are in Rome at moment A.

This argument is in fact based on two other arguments, which can be stated in the following way:

(Arg. 2)
Socrates is;
therefore Socrates is now (i.e. at the present moment).

(Arg. 3)
You are;
therefore you are now (i.e. at the present moment).

Paul obviously realised why the use of this kind of argumentation can be critised. We may reformulate the matter in terms of the 'res-theory' and the 'enuntiabile-theory'. According to the 'res-theory' (correspoding to the modern B-theory) (Arg. 2) and (Arg. 3) are invalid. According to the 'enuntiabile-theory' (corresponding to the modern tense logic) the arguments are all valid, but the premise 'Socrates is in Rome at moment A' is a contradiction unless the moment A is assumed to be the present moment. If A is not identical with the

present moment, one should according to the 'enuntiabile-theory' perhaps rather formulate the premise as

Socrates has been, is or will be in Rome at moment A;

but with that premise the argument is clearly invalid.

In his *Sophismata*, John Buridan (c. 1295-1358) dealt with the problem of self-reference in a setting, which related that problem to the subjects of time and tense. Medieval logicians used the term 'sophism' to describe propositions which were in a given situation considered true by some and false by others. That is, arguments could be made both for and against the truth of the proposition in question. Such was exactly the case in Buridan's discussion of this sophism: "You will throw me into the water" [Buridan 1966, p. 219]. For the discussion of that sophism Buridan imagined the following scenario: Socrates wants to cross a river and comes to a bridge guarded by Plato, who says: "Socrates, if in the first proposition which you utter, you speak the truth, I will permit you to cross. But surely if you speak falsely, I shall throw you into the water." Buridan assumed that Socrates then replies with the sophism in question. Obviously, it would be very hard for Plato to find out what he should do. He must admit that he cannot keep his promise. Buridan maintained that the sophism when uttered by Socrates has a truth value, i.e. it is either true or false. It is, however, "not determinately true or determinately false" [Buridan 1966, p. 220]. This means that we cannot determinately know whether it is true or false, until we have seen how Plato acts when Socrates is crossing. The two implications which can be found in Plato's statement are invalid, since in both cases the antecedent can be true and the consequence false. And since in this case what he promised is simply false, he cannot be under any obligation to keep his promise.

In order to deal with this sophism one also has to provide an answer to the very difficult question concerning the status of statements about the contingent future. Buridan's solution was that a statement about the contingent future is true or false,

although its truth value cannot be known by anybody now. That solution is just one of the possible answers - as we shall see.

The logicians of the Middle Ages in general took the Aristotelian view that a statement can change its truth value with time. A proposition such as 'Socrates runs' is not true at all times. The truth value depends on the actual state of affairs. The idea of 'the truth of a proposition at a given time' thus comes into the picture. Matters got more complicated when certain logicians introduced propositions such as 'Sortes fuit currens in a' ('Socrates was running at the time a') into the discussion. Assuming that Socrates actually ran at time a, such a proposition was regarded as false before and at time a, but true at all times after a [Nuchelman 1980, p.133].

The truth value of the proposition was thus regarded as relative to the time at which it was put forth - its 'moment of utterance'. Several factors are important in determining the truth value of a proposition: The 'present time' (understood as the moment of utterance), the time at which the event does or does not take place (tempus significantum), as well as the tense of the verb in the proposition (tempus consignificatum).

To maintain that the truth value of a proposition in any given case is to be determined relative to the 'present time' - the moment of utterance - is not so simple as it may seem. One can question the nature of that present time relative to which the proposition is to be evaluated. For we might very well consider the present as a duration rather than an atomic instant. John Buridan argued for this conception and noted that the 'present duration' which we have in mind may indeed be quite extended, "for we call this year present and this day present and this hour present " [Buridan 1966, p. 170]. Hence, Buridan argued that the truth value of a proposition should be understood as varying relative to the present regarded as a duration whose length is conventionally determined. The majority of logicians in the Middle Ages, however, took the view that the truth value should be discussed as 'truth value corresponding to a certain time'. They believed that what is needed in logic is 'durationless time', not determined as a part of a duration, but as a limit for the duration. Truth and time were considered as being closely

interconnected. Walter Burleigh (c. 1275-1345) examined the sophism "nothing is true unless at this instant" ("nihil est verum nisi in hoc instanti" [Burleigh p.173]). In his solution to that sophism he concluded that if a proposition is true it must be true now, that is to say at the present time. Even so, many medieval philosophers realised that the idea of the truth of a proposition at a durationless time was not without problems. We shall later return to these questions which mainly have to do with beginning and ending (incipit and desinit).

Buridan's answer to the question about the relation between time and truth represents an important alternative to the majority view. In his view the present, as well as the past and the future, were to be considered as having a certain span. Accordingly, one cannot give a definite answer to the question of the truth value of a proposition without knowing the presupposed convention of the duration of the present. The truth of a proposition thus depends on the choice of the duration which is considered to be the present. We shall analyse some details of Buridan's position in the next chapter.

In his article in *The Encyclopedia of Philosophy* [London 1967, vol.3, p.528], Ernest A. Moody identified the four most important components of Medieval logic to be the general theories of 1) suppositio terminorum (theory of terms) and 2) consequentia (theory of entailment), 3) modal concepts, and finally, 4) the general preoccupation with philosophical problems which were in the main related to logic and language. We shall in the following chapters consider some of the most characteristic features of the tense-logic of the Middle Ages. We shall see that the concept of time is relevant to all four areas mentioned by Moody.

It should be mentioned that many of the medieval texts still exist only in manuscript form. We have no guarantee that the texts which have been published are generally representative of the medieval logicians' views on the relation between time and logic. However, it may well turn out that temporal logic received even greater attention in the Middle Ages than it appears from the Medieval texts which have been published so far.

1.4. TEMPORAL AMPLIATION

The temporal reference of terms is one of the problem domains of tense-logic. The basic nature of the problem involved should be clear when considering sentences such as 'Young Socrates was going to argue', and 'The king of France was bald'. Obviously, an adequate logical analysis of these propositions requires an analysis of the temporal content of their subject terms. In the Middle Ages, this problem field was commonly called 'ampliatio', and great energy was invested into its solution. Indeed, the work of the Medieval logicians on 'ampliatio' is perhaps the clearest example of the great importance which they attributed to the logical study of temporal aspects of propositions. One can hardly think of a Scholastic author of a major logical work from the 14th century onwards, who would not also be concerned with the temporal reference of terms. However, this does not mean that all logicians called the problem domain 'ampliatio': to our knowledge Ockham did not use this particular term at all in his analysis, perhaps because he had his very own solution to this problem. Most Medieval logicians nevertheless did use the term 'ampliatio' when discussing how to determine the temporal reference of the subject. The three rules put forth by Walter Burleigh in his *De Puritate Artis Logicae* are probably typical:

> The first rule is that a common term standing (in a sentence) with a non-ampliating verb about the present stands only for present things. The second rule is that a common term standing (in a sentence) with a verb about the past is able to stand indifferently for present and past things. The third rule is that a common term standing (in a sentence) with a verb about the future is able to stand indifferently for present and future things. [Normore 1975 p.51]

One of the crucial problems motivating the work on 'ampliatio' was the problems regarding the naïve conception of tensed statements. According to that conception, a proposition of the type '*A* will be *B*' is equivalent to the claim of the existence of a

future in which '*A* is *B*', and similarly a proposition of the type '*A* has been *B*' is regarded as true if and only if the proposition '*A* is *B*' was true at some past time. But this naïve conception cannot be upheld in all cases. Consider for instance the statement

> 'The little boy will become a famous man'.

This proposition can certainly be true, even though the statement 'the little boy is a famous man' cannot be fulfilled at any time. The solution was to interpret the statement as being equivalent to:

> 'For a given person x, x is now a little boy and x will become a famous man'

'The little boy' thus refers to something in the present although the verb is referring to the future. But even this more refined treatment cannot encompass all cases, as we can see from the sentence 'Antichrist will be an orator'. Crucial to this example is the theological observation that Antichrist does not yet exist. The statement could consequently not be paraphrased in the same way as the statement about the little boy, but was understood as being equivalent to:

> 'For some person x: it is true that x will be Antichrist, and x will be an orator'.

In an analogous manner the proposition, 'Something white was black', might be true because the following statement is true: 'Something which has been white, was black', or it might be true because of the truth of, 'Something which is white was black'.

Actually, even this fairly innocuous formulation is slightly biased, for it implicitly favours Ockham's solution over the traditional one. Burleigh's rules for ampliatio quoted above are representative of the traditional solution, which treats sentences of the form '*A* has been *B*' as equivalent to 'For some x: x is or has been *A*, and x has been *B*', and analogously for future tense sen-

tences. It is possible that Ockham had Burleigh's logic at hand when he wrote his *Summa Logicae,* which states:

> For example if this proposition is true 'a white thing was Socrates', and if 'white' supposits for that which is white, it is not required that this will have sometime been true 'a white something is Socrates', but it is required that this will have been true 'this is Socrates' demonstrating that for which the subject stands in 'a white thing was Socrates'.' [Normore 1975 p.48]

With this explanation Ockham showed how the proposition 'a white thing was Socrates' can be true in the very first moment in which Socrates was white for the first time. We have indications that Ockham did not accept Burleigh's rules since Ockham thought that 'A was B' is either to be interpreted as - in modern terms - 'for some x: x is A and was B', or as 'for some x: x was A and was B', but not as the disjunction of the two possibilities. That is to say, Ockham considered such sentences as inherently ambiguous. Such an interpretation of Ockham has been defended by Graham Priest and Stephen Read [1981].

The difference between the two solutions is more significant than one might think at first glance. One of the most persuasive arguments in favour of the last kind of treatment - as opposed to the traditional 'ampliatio-theory' - emerges when we analyse propositions which require the use of a rule defining the past as well as a rule defining the future. Let us use an example of Buridan's [1966, p.150]: 'Young Socrates was going to argue'. In the case of this sentence, it does not seem acceptable to let the temporal reference of the subject term 'young Socrates' extend to the future just because an element of future occurs in the verb phrase. More specifically, the 'disjunctive treatment' forces upon us a reading equivalent to

> 'for some x : x was going to be young Socrates, and x was going to argue, or x was young Socrates, and x was going to argue'.

The first part of this reading does not seem plausible, and is not necessarily forced upon us by the 'ambiguity treatment'.

One of the most famous analyses of 'ampliatio' comes from Albert of Saxony, who was the rector of the University of Paris in 1353. Albert defined 'ampliatio' with a particular view to terms which do not refer to actually existing entities. That is, his rules accepted that statements can be made about thing which do not exist now or at any other time. He defined a number of rather precise rules for 'ampliatio' in a way similar to Burleigh's.

The study of 'ampliatio' was made a central part of logic during the later Scholastic period. The problem was studied as late as in the 17th century by the Portuguese logician John of St. Thomas (1589-1644), who was defending and still working within the tradition of Scholastic logic. He wrote two short passages directly concerning 'ampliatio', as well as a third passage, in which that phenomenon forms part of the problem. In the first of these passages he defined 'ampliatio', and in the second passage he presented four rules related to it. Two of these rules are similar to the above with respect to temporal and modal propositions, while the others are formulated in the following way:

> A term signifying a beginning amplifies all terms before and after to what is or what will be; a term signifying cessation, to what is or was... [John of St. Thomas, p.73]
> The term 'imaginatively' and the verb 'imagine' amplify all antecedent and subsequent terms to the imaginable... Similarly, signifying an interior act of the soul, as I wish, I understand, etc., can amplify to the imaginable the term on which it hits as its object. [John of St. Thomas, p.74]

The second of these two rules can naturally be looked upon as an extension of 'ampliatio'- namely a rule for modal propositions. The first rule demonstrates yet another connection between time and 'ampliatio'. Here the study of ampliatio is related to the extensive Scholastic debate on the logic of 'incipit' and 'desinit', to which we shall return.

1.5. THE DURATION OF THE PRESENT

The investigations in the last chapter clearly demonstrated that the naïve conception of tensed statements should be rejected as a general solution. On the other hand, it is also clear that Buridan defined tensed statements in a recursive way, reducing them to statements in the present tense. It turns out, however, that even the statements in the present are rather complicated. The reason for this is that Buridan regarded the present as a duration and not as a point in time. He explained the problems regarding the present in the following way:

> Also I say that it is not determined for us how much is the present time which we ought to use as the present. But we are allowed to use as much as we wish, for we call this year present and this day present and this hour present. [Buridan 1966 p. 170]

Obviously, Buridan's notion of the present was that of a duration. There was clearly an element of convention involved in this notion, since we are allowed to use as much of the present time as we wish as the present.

Buridan's concept of truth is relative to a choice of the present. That is, it only makes sense to talk about the truth of a contingent proposition if the present is specified. Buridan introduced his idea of the truth of a proposition in the following way:

> Thus, if in one part of the present time, Socrates stands or is white or is dead, it is simply true to say that he stands or is white or is dead. [Buridan 1966 p. 173]

According to Buridan's definition a proposition *p* is true during the present if and only if there exists a part of the present time during which the truth of *p* is given. The scope of that definition should perhaps be restricted by observing that Buridan's examples in this context are all concerned with what we would call 'stative propositions'. We may illustrate one of his examples by the following figure.

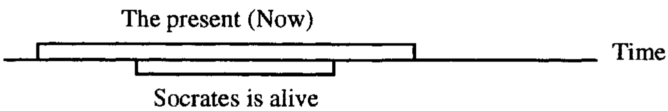


Figure 1

The truth of the proposition 'Socrates is alive' is given with respect to the interval I, which is a subinterval of the present assumed to be specified in the context. Thus, according to Buridan's ideas the proposition is to be regarded as true with respect to the entire present. Or in other words, if we have in mind the situation above as well as the present specified above, the present-tense proposition 'Socrates is alive' should be evaluated as true. This also means that there is a distinction between the general notion of being true with respect to an interval, and the notion of being *given* for as certain interval. The latter is the stronger notion of the two, and it reflects the intuition that the proposition in question is true throughout the interval.

A MODERN REPRESENTATION OF BURIDAN'S IDEAS

In order to give a symbolic representation of Buridan's ideas, we shall use the following conventions:

- variables p, q, ... stand for atomic propositions;
- variables I, I',... denote intervals (durations);
- I_n is understood to denote the present (as specified by some choice);
- *included(I,I')* means that the interval I is included in the interval I';
- the formula *T(I,p)* means 'p is true with respect to the interval I';
- the formula *given(I,p)* means 'the truth of p is given for the interval I'.

The relation 'included' is the usual inclusion relation among intervals, and it is thus both reflexive (I1) and transitive (I2):

(I1) $\forall I$: *included*(I,I)
(I2) $\forall I,I',I''$: (*included*(I,I') $\wedge$ *included*(I',I'')) $\supset$ *included*(I,I'')

The intuitions for '*given*' are not stated explicitly in Buridan's text, but it seems reasonable to assume that the following theses must hold:

(I3) *given*$(I,A) \supset \forall I'$. (*included*$(I',I) \supset$ *given*(I',A))
(I4) $\forall I$. (*given*$(I,A) \supset$ ~*given*$(I,\sim A)$)
(I5) ~*given*$(I_n,A) \supset (\exists$ I. *included*$(I,I_n) \wedge$ *given*$(I,\sim A)$)
(I6) (*given*(I_n,A) $\wedge$ *given*$(I_n,A \supset B)$) $\supset$ *given*(I_n,B)

We observed above that *given*(I,A) is a strong notion of truth, implying that the state of affairs denoted by A obtains throughout the interval I. This intuition is what is formalised by (I3)-(I5). Specifically, (I3) reflects the intuition that if A is given for some interval, then it is also given for any of its subintervals. (I4) captures the intuition that A and ~A cannot be given for the same interval. (I5) ensures that if given does not obtain for some predicate with respect to an interval, then the negated predicate is given for at least one of its subintervals. Finally, (I6) states that the given-relation is closed under a modus-ponens-like operation.

We suggest the following symbolic representation of Buridan's definition of truth with respect to an interval (understood to be 'the present'):

(B1) $T(I_n,A) \equiv_{def} \exists I$: *included*$(I,I_n) \wedge$ *given*(I,A)

A consequence of (B1) is the following one:

(B2) $T(I_n,A) \equiv \exists I$: *included*$(I,I_n) \wedge T(I,A)$

(B1-2) can of course be further generalised to cover all well-formed propositions, but we do not currently wish to state the rules for expressions like $T(I_n, p \wedge q)$.

Now let us consider a situation which gives rise to some intuitive problems. Suppose that it is given that in one part of the present time, Socrates is alive, and in another part of the present time, he is dead:

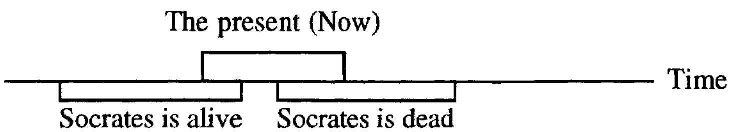


Figure 2

It follows from the definition (B1) that Buridan is obliged to accept the truth of the conjunction 'Socrates is alive and he is dead'. This seems to be a violation of the principle of contradiction, and so it would be if the following formula were valid:

$$(B3) \qquad T(I_n, p \wedge q) \equiv T(I_n, p) \wedge T(I_n, q)$$

Consequently, Buridan had to reject the principle embodied by (B3). But how could he establish a consistent framework such that (B3) would be invalid? In order to solve this problem Buridan had to make a distinction between, 'Socrates is alive and he is dead' and 'Socrates is alive and dead' . The latter can never be true, whereas the former can in fact in some cases be accepted as true.

In order to analyse this problem in further detail it was very important for Buridan to distinguish between affirmative and negative propositions. Affirmative propositions are statements of the form '*S* is *P*', which are not negated. Conjunctions like 'Socrates is alive and he is not alive' i.e.

$$T(I_n, p) \wedge \sim T(I_n, p)$$

can never be true. But if this is so, how can there be a duration for which the conjunction 'Socrates is alive and he is dead' is true? Buridan solves the problem by pointing out that while 'Socrates is dead' is an affirmative proposition, 'Socrates is not alive' is a negative proposition. According to Buridan, the conjunction

$$T(I_n,p) \wedge T(I_n,\sim p)$$

can in fact be true in some cases, namely in such situations where p is true in some part of the present duration I_n, and $\sim p$ is true in some other part of I_n (see figure 2). Obviously there are two kinds of negation involved in the temporal logic of Buridan:

(i) negation of predicates, e.g. 'non-alive' (='dead') is the negation of the predicate 'alive' ;
(ii) negation of propositions, e.g. 'Socrates is not alive' is the negation of the proposition 'Socrates is alive' .

We'll make use of the notation

(1) $T(I_n,A)$ for 'Socrates is alive',
(2) $T(I_n,\sim A)$ for 'Socrates is dead',
(3) $\sim T(I_n,A)$ for 'Socrates is not alive',
(4) $\sim T(I_n,\sim A)$ for 'Socrates is not dead'.

So the negation involved in (2) is a predicate negation, whereas (3) is the usual sentential negation. In (4) we see both kinds of negation occurring.

Before proceeding it should be noted that in our logical language, A and $\sim A$ are well-formed propositions in their own right. When they are not preceded by the T-operator, they are understood to refer to the present. Using (B2) we find the following truth conditions:

(B4) $T(I_n,\sim A) \equiv \exists I\text{: } included(I,I_n) \wedge T(I,\sim A)$
(B5) $\sim T(I_n,A) \equiv \forall I\text{: } included(I,I_n) \supset \sim T(I,A)$
(B6) $\sim T(I_n,\sim A) \equiv \forall I\text{: } included(I,I_n) \supset \sim T(I,\sim A)$

Intuitively, it is obvious that an analogue of the traditional rule for sentential double negation should also hold for predicate negation. Using (I1) - (I6) it can in fact be verified that

(B7) $\sim T(I_n, A \wedge B) \equiv \sim T(I_n, A) \wedge \sim T(I_n, B)$

Or equivalently,

(B8) $T(I_n, A \vee B) \equiv T(I_n, A) \vee T(I_n, B)$

From (I5) and (B4) we can now deduce that

(B9) $\sim given(I_n, A) \supset T(I_n, \sim A)$
(B10) $\sim given(I_n, \sim A) \supset T(I_n, A)$

By contraposition we find

(B11) $\sim T(I_n, \sim A) \supset given(I_n, A)$
(B12) $\sim T(I_n, A) \supset given(I_n, \sim A)$

It now follows from (B1) and (I1)-(I5) that

(B13) $\sim T(I_n, \sim A) \supset T(I_n, A)$
(B14) $\sim T(I_n, A) \supset T(I_n, \sim A)$

But obviously the opposite implications do not hold, for as we have seen it may very well be true that

$T(I_n, \sim A) \wedge T(I_n, A)$

is the case, as in figure 2 - in which case neither of $\sim T(I_n, A)$ and $\sim T(I_n, \sim A)$ are true! And in general, the consequences of (B12) and (B13) may well be true, without the respective antecedents being true.

One consequence of all these observations is of course that the natural language inference

'if Socrates is alive, then he is not dead'

is invalid on Buridan's account of the duration of the present. This last result seems to us to be in conflict with the logic of natural language. If that is so, then Buridan's ingenious investigations raise some problems of which any attempted interval semantics for natural language should take heed.

It may seem by now that this logic is suspiciously complicated. But we do not think that this observation by itself makes Buridan's ideas dubitable; interval semantics is in general a more complicated business than instant semantics. If we wish to study truth relative to durations, we must be prepared to accept a complicated framework.

TWO KINDS OF TENSES

As we have seen Buridan took it for granted that tense-distinctions are important to logical reflection. But he was also aware of the fact that a logic of tenses which pays due regard to a logic of durations is very complicated. For this reason, probably, he was content to sketch his ideas of tense logic. Buridan suggested two alternative ideas for the construction of the logic of tenses. The first one leads to the fairly natural kind of semantics, which we have discussed above. The tenses, past and future, are taken absolutely, in the sense that no part of the present time is said to be past or future.

Buridan made no attempt at formulating a detailed semantics for the tense operators, but he maintained that if the tenses are taken in an absolute sense, the Aristotelian proposition 'All which is moved was moved previously' cannot be valid [Buridan 1966, p. 177]. Generally speaking, the implication

$$moving(X) \supset P(moving(X))$$

is not a valid thesis in Buridan's temporal logic. In the same way he would also reject the validity of the implication

$$moving(X) \supset F(moving(X))$$

On the other hand, these two propositions become valid if the tenses are taken in the relative sense, which Buridan explained in the following way:

> But in another way, 'past' and 'future' are taken relatively, so that the earlier part of the present time is called past with respect to the later, and the later part is called future with respect to the earlier. This way of taking the terms is customary. [Buridan 1966, p. 175]

Buridan pointed out that if some thing is moving now, then there is a part of the present during which it is moving, and hence, it is moving in some part of the present which is earlier than some other part of the present. Therefore, if the thing is moving, then it was moving (if the past is taken in the relative sense). For this reason, the Aristotelian sophism must be conceded if the past is understood relatively.

It seems plausible to represent Buridan's idea of a relative past in the following way:

$$T(I_n, P_{rel}A) \equiv_{def} \exists I': included(I', I_n) \wedge \exists I''. before(I'', I') \wedge given(I'', A)$$

whereas the absolute past can be defined as

$$T(I_n, P_{abs}A) \equiv_{def} \exists I': before(I', I_n) \wedge given(I', A)$$

Let us assume that it is true for some 'now' I_n that some thing X is moving. According to the definition (B1) this means that there is an interval I' for which it holds that

$$included(I', \mathrm{I_n}) \wedge given(I', moving(X))$$

If I_1 is included in I' and I_2 is included in I_n such that $before(I_1, I_2)$, then by (I2) and (I3) we have

$$included(I_1,I_n) \wedge before(I_1,I_2) \wedge given(I_1,moving(X))$$

(Of course, our scenario here does not preclude that

$$given(I_2,moving(X))$$

may also hold - indeed, it might be true that X is moving throughout the present - but on the other hand, this is clearly not a necessary condition, i.e. it is not entailed.)

These observations fit nicely into the above definition of the relative past; it follows directly from our assumptions that

$$\exists I_2: included(I_2,I_n) \wedge \exists I_1:(before(I_1,I_2) \wedge given(I_1,moving(X))$$

which is the definition of $T(I_n,P_{rel}(moving(X)))$. Therefore, it follows that

$$T(I_n,moving(X)) \supset P_{rel}(moving(X))$$

Obviously, an analogous thesis cannot be proved for P_{abs} instead of P_{rel}. It turns out that the two intuitively different kinds of tenses are also very different from a formal point of view.

1.6. THE LOGIC OF BEGINNING AND ENDING

A very special chapter of Medieval logic was opened when philosophers of that time took up the analysis of the verbs 'incipit' (it begins) and 'desinit' (it ends). The starting point was found in Aristotle's *Physics*, books 6 and 8, so it was no coincidence that their deliberations proved to be relevant not only to logic but also to physics. The questions concerning beginning and ending naturally led to the consideration of temporal limits. The number of Medieval logicians who worked on these questions was very large [Kretzmann, 1976, pp.101ff]. As pointed out by William and Martha Kneale [1962, p.233-34], the very fact that so much attention was given to this type of problem constitutes an excellent proof of the formal character of Medieval logic. The general problem had to do with the correct understanding of 'incipit-statements' such as:

(1) 'Socrates begins to be white' ,
(2) 'Socrates begins to run',

and analogously for statements containing the verb 'desinit'. The task of the logician was to give clear semantic definitions of 'incipit' ('begins') and 'desinit' ('ends'). The most common definition given in order to clarify the meaning of the above examples was the following:

(1') 'Socrates is white and was not white immediately before'
(2') 'Socrates does not run, but will run immediately after'

This interpretation was for example defended by Peter of Spain (d. 1277). Obviously, the treatment offered by (1') and (2') does not fit into the same pattern, or paradigm; 'whiteness' and 'running' are treated differently. This difference - inspired by Aristotle's treatment in the Physics - originates in a distinction between permanent things or states (whose parts appear simultaneously), and successive things or states (whose parts appear one after another). Medieval logicians considered the

property of 'running' to be successive and 'whiteness' to be permanent. Hence the two kinds of predicates had to be treated differently. In addition to the two types of phenomena, the permanent and the successive, a third object type, the instantaneous, was observed and discussed by Burleigh and Thomas Bradwardine (c.1295-1349) [Nielsen, 1982, p.29].

The discussion of beginning and ending is in our opinion a striking example of the manner in which medieval logic is relevant even today for semantic discussions. When using symbolic language for the discussion, we shall use the following convention:

> p is a variable ranging over permanent state propositions, s is a variable ranging over successive state propositions, and q is a meta-variable ranging over both types of propositions.

The questions concerning 'incipit'/'desinit' were amongst the most discussed problems in the Middle Ages, as Simo Knuuttila [1985, p.165ff] has pointed out. The analysis of statements was central to the Medieval approach to scientific questions in general, and this particular problem was regarded as important in scientific and physical thinking as early as the 12th century (at which time 'the present' was regarded as a primitive concept). Thus, Scholastic natural philosophers who were interested in kinematics turned their attention to propositions regarding beginning and ending [Murdoch p.117ff].

In the medieval treatises on the question there is evidently a connection between the tense-logical analysis of the problems of 'incipit' and 'desinit', and the emerging awareness of the problem of continuity in connection with establishing the 'mathematical moment'. This is clearly the case with for instance Richard of Lavenham's analysis in his treatises *De Natura Instantium* and *De Primo Instanti* [Øhrstrøm 1985b].

The study of 'incipit' and 'desinit' is an extremely difficult matter. Two complicating factors ought to be mentioned. Firstly, a special challenge is constituted by statements making iterative use of 'incipit' and 'desinit'. Secondly, the use of 'immediately before' and 'immediately after' calls for very

specific tense-logical constructions, since it is not obvious how one is to precisely understand these expressions. It is the question of the continuity of time which is at stake here. Richard Kilvington (died 1361) amongst others analysed these problems thoroughly in his *Sophismata* [Kretzmann 1982, p.270ff].

Two characteristic features of Medieval logic was that it dealt with propositions whose truth-values could vary from time to time, and that it took tensed statements into serious consideration. On that basis medieval logicians put forth some very interesting ideas of temporal logic, also with respect to the problems of 'incipit'/'desinit'. In the following we will concentrate on some findings of William of Sherwood, which he formulated in his *Syncategoremata* [Kretzmann 1968].

According to Sherwood the terms 'incipit' ('begins') and 'desinit' ('ceases') can be used categorematically as well as syncategorematically. This distinction was well known in Medieval logic, as described by [Kretzmann et al.]:

> Medieval logicians regularly classified meaningful words into such as have meaning in their own right (*termini significativi* ...), and such as are meaningful only when joined to words of the first kind (*termini consignificativi*...). The former are also called categorematic terms..., the latter, syncategorematic terms... [p. 162]

Typical syncategorematic words are quantifiers such as 'every', 'all', 'some', etc. These remarks can be supplemented by the observation that in a syncategorematic *use* of an expression, the expression is considered to be 'incomplete'. For instance, the verb 'to begin' may be combined with an infinitive complement as in 'begin to run' in order to form a complete predicate (i.e. an intransitive verb phrase). In this use of 'begin', it is considered incomplete until adjoined with its complement.

A categorematic expression, on the other hand, is complete by itself - in the way a verb such as 'walk' may make up a full verb phrase by itself. Here, we will concentrate on the syncategorematic use of the verbs. In this syntactic role they indicate how things are qualified and how they are to be

interrelated, but they cannot by themselves be used as the predicate of a sentence.

In medieval logic some of the syncategorematic terms were classed as exponible terms, i.e. terms having an obscure sense which has to be explained or clarified. Sherwood stated that 'incipit' and 'desinit' are such exponible terms. He offered the following explication of 'desinit':

> Therefore, if I say 'he ceases to be sick, or unhealthy', then 'to cease' indicates that the thing is at the end of the time in which it was such and such, (in termino temporis in quo fuit talis). [Kretzmann 1968 p.109]

A MODERN REPRESENTATION OF THE IDEAS

Let the statement variable p stand for an arbitrary proposition, e.g. 'Socrates is alive', and let Cp represent the proposition stating that p ceases to be true, i.e. 'Socrates ceases to be alive'. Sherwood's description of the time at which 'he ceases to be sick' as 'terminus temporis in quo fuit talis', i.e. 'the end of the time during which the person was sick', gives rise to the following explication:

> (1) The proposition Cp is true at the time t only if t is a limit between times at which p is true and times at which p is false.

If Bp represents the proposition stating that p begins to be true, the following condition seems to be natural in addition to (1):

> (2) The proposition Bp is true at the time t only if t is a limit between times at which p is false and times at which p is true.

It must be noted that (1) and (2) are not complete definitions, since we have only stated some necessary conditions for Cp and *Bp* to be true at t. If time is continuous, and the distribution of the truth values of propositions corresponds to temporal intervals, the limits mentioned in (1) and (2) are well-defined. Then the definitions are in fact complete - so in that case, we may substitute 'only if' by 'if and only if'. If on the other hand time is discrete, the conditions in (1) and (2) are not sufficient. For let the truth-values (T: true, F: false) for *p* be as follows:

time	1	2	3	4	5	6	7	...
truth-value	F	T	T	T	T	F	F	...

- where the integers are used to indicate the succession of discrete instants (or discrete periods). Now there is a basic intuition according to which it is reasonable to say that *Cp* is true at t = 6 and that *Bp* is true at t = 2. But on the other hand, it also might be intuitively plausible to say that *Cp* is true at t = 5 and that *Bp* is true at t = 1. In fact, when operating on the basis of discrete time, (1) and (2) give rise to four possible combinations. It must be concluded that they are not yet full definitions. In fact, what the precise 'limits' of (1) and (2) should be determined to be also depends upon whether we are talking about successive or permanent states. So far, we have been using only the example of the permanent state p, but actually (1) and (2) in their general formulation are meant to apply to successive states also. A fully precise version of (1) and (2) will have to be differentiated according to the type of propositions in question. Sherwood realised all of this and did indeed arrive at a clear definition based on his concept of time.

In the notes to his translation of William of Sherwood's *Syncategoremata* Norman Kretzmann stated: "Sherwood's analysis is evidently based on a view of time as a sequence of discrete instants or periods..." [Kretzmann 1968, p.109]. We agree with Kretzmann's observation. Some of the phrases and expressions used by Sherwood obviously presuppose that it is possible to iden-

tify an immediate successor and an immediate predecessor for every instant in time. His use, for instance, of the expression 'in penultimo instanti vitae suae' ('in the next to the last instant of his life') clearly indicates that according to Sherwood there is an instant immediately preceding 'the last instant of his life'.

PERMANENTIA AND SUCCESSIVA

Since Sherwood used a discrete parameter of time, further explanations of 'incipit' and 'desinit' were required. In his expositions he was relying on the distinction between 'permanent states' (permanentia) and 'successive states' (successiva), which we discussed above. The parts of a permanent state are at one and the same time, whereas the parts of a successive state are not at one and the same time. The property of 'being white' represents a permanent state, and 'running' represents a successive state. As we also mentioned the distinction goes back to Aristotle. The philosophical starting point for Sherwood's discussion is to be sought in Aristotle's *Physics* book VI and book VIII. Aristotle stated that

> For a change can actually be completed, and there is such a thing as its end, because it is a limit. But with reference to the beginning the phrase has no meaning, for there is no beginning of a process of change, and no primary 'when' in which the change was first in progress. [Phys. 236a 12-14]

Obviously 'a process of change' is a successive state. Hence, according to Aristotle there is an end to, but no beginning of a successive state. The latter observation may seem counterintuitive, but what is meant is simply that a thing is not in the state when the thing begins to be in the state. Aristotle's view also means that a thing is in the state when the thing ceases to be in the state. In symbols:

(3.1) $(Bs \supset \sim s)$
(3.2) $(Cs \supset s)$

where *s* is an arbitrary proposition about a successive state. Sherwood maintained that (3) is valid for such propositions. Let the truth-values for s be as follows:

time	*1*	*2*	*3*	*4*	*5*	*6*	*7*	...
truth-value	*F*	*T*	*T*	*T*	*T*	*F*	*F*	...

We discussed above the same series of truth values for the permanent state proposition *p*. It was and observed that from an intuitive point of view, it could be argued that Bp was true at either t = 1 or t = 2, and analogously for *Cp* at t = 5 and t = 6. At a pre-theoretical level, a similar argument might be made about *Bs* and *Cs*. However, if we combine the insight of (3) with (1) and (2), we must arrive at the conclusion that *Bs* is true if and only if t = 1 and that *Cs* is true if and only if t = 5.

On the other hand, if *p* is a proposition corresponding to a permanent state with the above variation of truth-values, Sherwood's view can be formulated in the following way:

(4.1) $(Bp \supset p)$
(4.2) $(Cp \supset \sim p)$

That condition is also in good accordance with the observations by Aristotle in his Physics, book VIII:

> It is also evident that, when speaking of the subject of motion or change, unless we assign the instant that divides past and future time to the state into which it will be for the future rather than to that which it turns out of and in which it was in the past, we shall have to say that the same thing both exists and does not exist at the same instant, and when it has become something it is not that something which it has become. (263 b 10-14).

Aristotle examined the proposition: 'The object D is white'. He assumed that D is white before the instant *t*, and not white after *t*. Obviously, t is a limit: D ceases to be white at *t* and begins to be not-white at t. But since the change from white to not-white has

been completed at t, D must be not-white at t. Hence, according to Aristotle (4) is obviously valid for the proposition

p: 'The object D is not white'.

We can now generalise the results of the discussion into the following equivalences:

(5.1) $Bs \equiv (\sim s \wedge F(1)s)$
(5.2) $Cs \equiv (s \wedge F(1)\sim s)$
(6.1) $Bp \equiv (p \wedge P(1)\sim p)$
(6.2) $Cp \equiv (\sim p \wedge P(1)p)$

$P(n)$ is read 'n time units ago it was the case that ...' and $F(n)$ is read 'in n time units it will be the case that...'. In the following we intend to investigate Sherwood's solutions to some sophisms concerning 'incipit' and 'desinit'.

(A) WHAT BEGINS TO BE CEASES NOT TO BE

Using the same kind of symbolic language as above, we may render this sophism as:

(7) $Bq \supset C\sim q$

where q ranges over arbitrary propositions. Sherwood, of course, held this statement to be true. But since it is a sophism, there also exists an argument implying that the statement is false, namely this one:

'But what begins to be is, and what ceases not to be is not; therefore what is is not'. [Kretzmann 1968 p.113].
In symbols: $(Bq \supset q)$ and $(C\sim q \supset \sim q)$

But this combination of statements is neither in agreement with (5.1-2) nor with (6.1-2), i.e. q can neither be a permanent state nor a successive state. On the other hand, if we accept (5)

and (6), it turns out that (7) is a valid thesis, and hence a fortiori the sophism is true. Accordingly, the premise of the argument above should be rejected (and so should its conclusion).

Using (5) and (6) it can be verified that (7) holds in general. We shall show (7) for permanent propositions:

(i)	Bp	(assumption)
(ii)	$p \wedge P(1){\sim}p$	(by 5.1)
(iii)	${\sim}{\sim}p \wedge P(1){\sim}p$	(double negation)
(iv)	$C{\sim}p$	(by 5.1 used on ${\sim}p$)

Here it is assumed that if p is a permanent state, then ${\sim}p$ is also a permanent state. - The proof of $(Bs \supset C{\sim}s)$ is analogous, and combining the two we of course have (7). In fact, the implications in (7) can be substituted by equivalences, as should be obvious from the proof above.

(B) CEASING TO BE NOT CEASING

Sherwood formulates another sophism in the following way:

> 'Suppose that Socrates is in the next to the last instant of his life. Then Socrates ceases to be not ceasing to be'. [Kretzmann 1968 p.114]

In the following, let p represent the proposition 'Socrates is alive'. - According to Sherwood the sophism is not valid, if 'ceases to be' and 'not ceasing to be' are distributed as in 'Socrates ceases to be and Socrates doesn't cease to be'. This is obvious, for if the sophism is read in this way, its conclusion will be $Cp \wedge {\sim}Cp$, which clearly cannot be accepted. But if 'ceases to be' and 'not ceasing to be' are iterated as in $C{\sim}Cp$, the sophism should be regarded as true, indeed as a valid thesis. Let the truth-values for p be as follows:

time	... 3 4 5 6 7 ...
truth-value	... ? ? T F F ...

Before the validity of the sophism can be demonstrated, it must be clarified how 'in the next to the last instant of his life' ('in penultimo vitae suae') should be understood. Is it the instant immediately before the death of Socrates, i.e. t = 5, or is it the instant immediately preceding that instant, i.e. t = 4? It turns out that in the latter case, the sophism will not be valid. For this reason we shall assume that the instant in question is the instant immediately preceding the death of Socrates, t = 5. On the assumptions made so far the sophism can be symbolised in the following way:

(8) $F(1)Cp \supset C{\sim}Cp$

Note that the antecedent is true exactly at $t = 5$. For the proposition p corresponds to a permanent state, and consequently Cp has to be understood according to the exposition in (6). But it does not necessarily follow that its negation ${\sim}Cp$ is also a permanent state. To understand (8) fully we must know whether the proposition ${\sim}Cp$ corresponds to a permanent state or to a successive state. It can be shown that (8) is not valid if the exposition in (6.2) is used for both occurrences of C in $C{\sim}Cp$. Hence, it seems to be natural to assume that ${\sim}Cp$ corresponds to a successive state. In general, negating a predicate of one aspectual type may turn the predicate thus formed into another type (a fact which is also realised within modern linguistics).

On this observation, we can use the definition given in (5.2), and this makes (8) equivalent to

(9) $F(1)Cp \supset ({\sim}Cp \wedge F(1)Cp)$

which is valid if and only if the following is a valid thesis:

(10) $F(1)Cp \supset {\sim}Cp$

Since p is a permanent state proposition, we must now use (6.2), which makes (10) equivalent to

(11) $F(1)({\sim}p \wedge P(1)p) \supset (p \vee P(1){\sim}p)$

Let us take a semi-formal look at the antecedent of (11): on our assumptions, $F(1){\sim}p$ is clearly true ($\sim p$ is true at t = 6), so in this case the antecedent of (11) is true if and only if $F(1)P(1)p$ is true. As for the consequent, the truth value of $P(1){\sim}p$ cannot be determined by the assumptions legitimate in connection with the sophism in question. Nevertheless, if we can show that p must be true, the entire consequent is of course true, and we can neglect $P(1){\sim}p$. In short, if

(12) $F(1)P(1)p \supset p$

is a valid thesis, then (11) is also a valid thesis. Since $F(1)P(1)p \supset p$ is intuitively valid, it follows that the validity of (8) is a consequence of natural and obvious reasoning. We may add that in modern axiomatisations of metrical tense logic, (12) is also valid, that is, a theorem. In the context of the discussion of this sophism, Sherwood makes an interesting observation. He considers the proposition

(13) $Cp \supset C{\sim}Cp$ (where p is as above).

This proposition is a valid thesis if time is dense. To see this, assume that Cp is true at the time t and that time is dense. According to (1) it follows that t is a limit between times at which p is true and times at which p is false. Then t is also a limit between times at which $\sim Cp$ is true, and times - in this case only one time t - at which $\sim Cp$ is false. For this reason $C{\sim}Cp$ is true at t. Hence, (13) is a valid thesis. Obviously (13) is not valid if time is discrete, whereas (8) is not valid if time is dense. These results are rather remarkable. Sherwood did in fact demonstrate that the difference between accepting (8) as valid, or alternatively, accepting (13) as valid, corresponds to the distinction between discrete and dense time. For a modern logician, (8) and (13) will be natural candidates for an axiomatic description of this distinction.

(C) CEASING TO BE BEFORE SOMETHING

Sherwood presents another sophism pertaining to some of the same ideas in the following way:

> 'Suppose that Socrates is in the next to the last instant before Plato's death. Then Socrates ceases to be before Plato's death'. [Kretzmann 1968 p.114]

Let p represent the proposition 'Plato is alive'. As with the sophism in the above section we will assume the instant in question to be the instant immediately preceding Plato's death. Hence the antecedent in the sophism is true if and only if $F(1)Cp$ is true, whereas the consequent is true if and only if $CF(1)p$ is true. Therefore the whole sophism can be represented as follows

$$F(1)Cp \supset CF(1)p \tag{14}$$

According to Sherwood this sophism is valid, which is easily verified if $F(1)p$ is considered to correspond to a permanent state and time is discrete.

SOME FUTHER REMARKS

The above results should serve to demonstrate that Sherwood was committed to the view that the concept of time is needed within logical analysis. He considered the logical operators corresponding to 'beginning' and 'ceasing' to be interesting within a temporal logic, and formulated the semantics of these operators. He did so by giving some basic theses for each of them and arguing for the validity of those theses. By this work William of Sherwood provided a valuable contribution to the medieval study of temporal logic. It is remarkable that he was aware of the possibility of distinguishing between discrete and dense time by means of theses from temporal logic.

Another debate, which is to some extent related to the problems concerning 'incipit' and 'desinit', is the debate of the concepts of

'moment of change' and 'moment of creation', issues which bear on the ideas of 'creatio ex nihilo' and related questions. The analysis of 'creation', 'beginning', 'end', and similar concepts also played an important role after the Middle Ages. Moses Mendelssohn (1729-1786) was convinced that an analysis of time and logical relations in connection with change would constitute an argument for the immortality of the soul, an argument which Kant and Brentano resumed [Chisholm 1980].

It is also worth pointing out the distinction between successiva and permanentia clearly anticipate those syntactic and tense logical distinctions among types of verb phrases discussed in modern linguistics. In fact it may be easier to understand them when comparing them with present-day terminology. Zeno Vendler [1967] divided verb phrases into four major types:

(a) *states*, corresponding to permanent properties - such as 'is white',
(b) *achievements*, roughly corresponding to instantaneous events/properties, such as 'the deer was hit by an arrow',
(c) *accomplishments*, which may be described as non-instantaneous events, such as 'to draw a circle',
(d) *activities*, approximately the same as successive properties, such as 'to run'.

In modern formal semantics, it is realised that these different types of verb phrases call for different tense-logical treatments [Galton 1987, p. 13]. Also in Artificial Intelligence we find similar distinctions, as for instance in J. F. Allen's distinction between states, events, and processes [Allen 83, 84] - the last concept being comparable to that of successive states.

1.7. TIME AND CONSEQUENTIA

In the introduction, we emphasised the fact that the subject matter of logic has not been constant throughout the history of logic, and that the focus of interest has changed several times. Even so, the very notion of logical consequence is an almost definitional property of logical studies. The Middle Ages are no exception in this respect. The study of logical consequence, known as *consequentia*, constituted one of the most central fields within Medieval logic. Since we have been stressing the importance attributed to time and tense, we should now balance our account by observing that the concept of time was not of crucial importance to the formulation of most of theories on consequentia. On the other hand, in some cases time did in fact play a rôle in such theories.

Some medieval texts on consequentia appears to be about conditionals ('if *A* then *B*'), but in most of the texts on consequentia it seems that the author is in fact dealing with what we would now call inference. But as Alexander Broadie has pointed out [1987, p.51] it is plain that medieval logicians in general were aware of the difference between what we would call respectively conditionals and inferences, although they used the same term for the two relations. In this chapter we shall also use the term in this ambiguos way.

In medieval logic the study of syllogisms was considered to be one of the key parts. Sometimes other syllogisms that the ones of the classical figures were studied. Given the medieval awareness of the importance of temporal logic it is not surprising that they introduced a syllogistic tense logic. According to Broadie the first detailed discussion of the topic was given by William of Ockham. In his *Summa Logicae* he stated for instance:

> When both premisses are past-tensed in the second figure and the subject of each of these supposits for things which are, there always follows a present-tensed conclusion, and not a past-tensed conclusion. [*Summa Logicae* III, 1,18]

This means that although the syllogism

> No (present) A is B
> Some (present) C is B
> Ergo: Some C is not A

is in fact valid, the tensed syllogism,

> No (present) A was B
> Some (present) C was B
> Ergo: Some C was not A

has to be rejected in general, whereas the

> No (present) A was B
> Some (present) C was B
> Ergo: Some C is not A

should be accepted as valid syllogism. - In this way William of Ockham presented some rather precise components for the formulation of a formal (but of course not symbolic) tense logic.

Time was also involved in the medieval study of consequentia in another way. The conception of statements as units with temporally variable truth values led the medieval logicians to the notion of *consequentia ut nunc*, which was the medieval term for that form of logical consequence whose truth value varies with time. That is, this type of logical consequence is capable of being true at one time and false at another. Peter King, for instance, has described an ut nunc consequence as a statement with an antecedent and a consequent 'such that it is not the case that the antecedent obtains and the consequent fails to obtain', and he has stressed that we must take the tense of the verb in 'it is not the case' seriously [King p.62-63].

So in medieval logic we once again find a distinction between an implication which can be valid at one time (or some times, or some period of time), but invalid at other times, and an implication which must be valid at all times, if it is to be valid at all. That distinction had already been discussed in Ancient times

(Diodorus, Philo of Megara, the Stoics), an issue which we touched upon in our discussion of the Master Argument. There is an exact correspondence between the distinction underlying the Ancient discussion and the later Medieval distinction between 'as-of-now consequence' (*consequentia ut nunc*) and 'absolute consequence' (*consequentia simplex*) - as pointed out by L. N. Roberts [1967]. As we have seen, the famous debate between Diodorus and Philo of Megara was precisely concerned with the relation between time and implication. The question was whether to allow the truth values of the implication to vary with time or not. The Medieval logicians were aware of the problem, and solved it by allowing both kinds of implications, whilst duly distinguishing between them in any concrete case. Burleigh, for example, presented one implication of the same type as Diodorean implication, that is, it had to be valid at any time, and another - *consequentia ut nunc* - in the style of Philo, that is, it only had to be valid at one time.

Walter Burleigh put forth the following example of a *consequentia ut nunc*:

(1) 'Every man is running, so Socrates is running'

This is clearly a consequence, which is correct at certain times and not at others. It is obviously only valid during a period of time, in which Socrates is alive. On the other hand, a consequentia such as:

(2) 'All living beings are running, so all men are running'

is valid at all times, since the set of living beings must at any time include the set of human beings. However, the consequentia of (1) will prove to be false at times where no man by the name of Socrates exists.

While consequentia was in general a field of much interest for medieval logicians, they did not pay much attention to the specific *consequentia ut nunc*, nor did they develop any real theory about it. It appears that only a few leading medieval

logicians attempted to define rules for this kind of consequentia. Burleigh, for one, stated this rule:

> In an as-of-now consequence, however, the antecedent cannot be true without the consequent as of now. [Kretzmann 1988, p.285]

This means that at a time at which the consequence holds good, the antecedent cannot be true without the consequent.

John Buridan formulated another rule:

> From any false sentence any other sentence follows as a consequence *ut nunc*, and also any true sentence follows from any other as a consequence *ut nunc*. [King p. 196]

What this rule states is obviously close to what we would call the paradoxes of implication. That should come as no surprise, for it is evident that *consequentia ut nunc* is essentially the same as material implication - whereas *consequentia simplex* corresponds to strict implication.

> According to Buridan common people often use as-of-now-consequences [King, p.185] .

As an example he mentioned the consequence:

> 'Cardinal White has been elected Pope;
> therefore, a Master of Theology has been elected Pope.'

Clearly, this consequence can only be true if the proposition 'Cardinal White is a Master of Theology' is true.

In general, an ut-nunc-consequentia is true only when some tacit assumption related to the consequence is also true. As Alexander Broadie [1987, p.61] has pointed out a valid ut-nunc-consequentia can be transformed in to a valid inference (or conditional) by the addition of a relevant proposition, which is true now.

Buridan even claimed that the consequence:

> 'Socrates is running, Plato is running, Robert is running; therefore, every man is running.'

is true if it is perfected with the truth of the proposition 'Any man is Socrates, Plato or Robert'.

Buridan also pointed out that promissory consequences are as-of-now-consequences. He considered an example, where Plato says to Socrates:

> 'If you come to me, I shall give you a horse'

This consequence can be true as-of-now, if it is perfected with the following conditions, which must be assumed to be true [King p.186]:

> (a) 'Plato wills to give a horse to Socrates', and
> (b) 'whatever Plato wills to do in the future,
> (i) he will be able to do by holding to that volition (and holding in any circumstances to that what he wills); and
> (ii) when he is not prevented he does that thing when and how he wills'.

Attempts to involve past and future statements directly in the studies of the consequence-as-of-now were rare. One of the few examples is this one:

> If Antichrist will never be generated, Aristotle never existed. [King p. 196]

Buridan's accepted this consequence as being true as-of-now on the basis of his Christian belief that Antichrist is in fact going to be. Buridan did acknowledge that it is logically possible for the antecedent to be true, but he also asserted that its truth would be incompatible with other facts which should be accepted. Hence, the antecedent must be regarded as false, and therefore the consequence is to be accepted as true as-of-now. However, after

the generation of Antichrist it must be natural to reject the consequence.

Obviously, Buridan in this manner brought temporal considerations into the study of concrete examples of *consequentia ut nunc*. However, to do so in connection with the consequence-as-of-now was a fairly rare thing amongst medieval logicians, according to Bochenski [1961, p. 193]. On the other hand it was common-place in the late Middle Ages to base a distinction among different types of consequentia on criteria relating to the concept of time. But around the time of 1500 A.D., that kind of investigation was clearly fading away among logicians. *Consequentia ut nunc* was occasionally mentioned as *consequentiae vulgares*, since it was regarded as primarily concerned with everyday language rather than with scientific reasoning [Kneale p. 289]. This view was probably also the reason for the abandonment of *consequentia ut nunc*: as logic gradually distanced itself from everyday language, *consequentia ut nunc* came to be seen as uninteresting from a scientific point of view.

1.8. TEMPORALIS - THE LOGIC OF 'WHILE'

Medieval logicians generally accepted a distinction between atomic and molecular propositions. Molecular propositions are formed from atomic (or simple) propositions by means of propositional connectives. Propositions thus formed were also known as hypotheticals, a term which was applied not only to implicational statements, but also to the other kinds of molecular propositions such as conjunctions, disjunctions etc. In the Middle Ages, however, there was little agreement as to which connectives should be taken into consideration within logical studies. Thus the number of propositional connectives was not fixed in general. William of Ockham [EL, p.198] suggested that there were at least five: *conditionalis*, i.e. implication (in a broad sense), *copulativa*, i.e. conjunction, *disiunctiva*, i.e. disjunction, and *temporalis* and *causalis*. *Temporalis* will be discussed below. With respect to causalis, we may mention two of the many examples put forth by Paul of Venice (c. 1369-1429) in his *Logica Magna*:

> Because you are a man you are not a donkey [II, 3, 27e]
> Because the sun is this light is [II, 3,29e]

The use of causalis is not presently a concern of ours, and we shall leave it aside for now. It may be mentioned, though, that causalis roughly corresponds to 'because', and thus has to do with causation only in a very broad sense.

Ockham did not claim, though, that the corresponding molecular propositions are the only possible ones, or that they were mutually independent in a strong logical sense. He merely considered the molecular propositions listed above to be the most interesting and important ones. Walter Burleigh [1955, p.107] agreed with Ockham on the number of propositional connectives. He admitted that other types of molecular propositions might be thought of, but he also maintained that such further types would prove to be reducible into the five fundamental ones mentioned above. Ockham and Burleigh thus agreed on which molecular propositions were to be regarded as important or ba-

sic. Writing a few decades later, John Buridan listed six molecular propositions [MS Krakow BJ 662 f.7r.]. He accepted the five molecular propositions of Ockham and Burleigh, but in addition he considered one more type, which he called localis, to be a connective in its own right. Richard Lavenham [Spade 1973, p.57] later in the 14th century suggested the addition of even one more type of molecular proposition, called rationalis. Ockham and Burleigh, as well as Buridan, had probably considered rationalis to be nothing more than a yet another variant of conditionalis.

It appears that these medieval writers all agreed on the importance of 'temporal propositions' (*temporalis*), i.e. molecular propositions composed from two or more atomic propositions conjoined by some adverb of time. Temporal connectives such as 'cum', 'dum', and 'quando' appeared to these logicians to form an important class of logical constructions, just like conjunction, disjunction, and implication. This idea is in fact very old. The study of *temporalis* can be traced back to Boethius (c. 480-524), who discussed for instance the statement 'when a man is, an animal is' - 'when' being the temporal connective, of course. In the Middle Ages, one of the first philosophers to discuss temporal propositions was the Islamic logician Ibn Sina (980-1037), in Christian Europe known as Avicenna. In his logic he discussed such Arabic *temporalis* statements as can be seen in the following English counterparts:

> 'Whenever the sun is out, then it is day' ,
> 'It is never the case that if the sun is out, then it is night' ,
> 'It is never the case that either the sun is out or it is day' ,
> 'If, whenever the sun is out, it is day, then either the sun is out, or it is not day'

One cannot say that Avicenna developed a real theory of such temporal propositions, but he did try to work out the relationship between *temporalis* and the implication, as it can be seen from the following quotation:

> Take the word 'if'. You do not say 'If the day of resurrection comes, then people will be judged' because the consequent is

> not implied by the posited antecedent. For [the consequent] is not something necessary but depends on God's will. Rather you say 'When the day of resurrection comes, people will be judged'. Moreover, you do not say 'If man exists, then two is even' or 'the void is non-existent'. You say 'whenever man exists, then two is also even' or 'the void is non-existent'. It seems that the word 'if' is very strong in showing implication, while 'whenever' is weak in this respect and 'when' is in between. [Shehaby 1973, p.38]

According to Avicenna *temporalis* is an implication that is slightly weaker than the implication corresponding to 'if-then'. Here it must be observed that Avicenna had a sort of 'relevant implication' in mind. As he saw it, 'if p, then q' presupposes a strong semantical relation between p and q. Since Avicenna considered 'p while q' to be weaker than this kind of implication, he accepted that it could be true also in cases where the implication 'if q, then p' does not obtain. Following Avicenna, however, 'p whenever q' should be interpreted as the material implication, that is, p and q need not be semantically relevant to one another.

Most medieval logicians in the following centuries appear to have agreed that a necessary condition for the truth of 'p while q' is that both of p and q are true at some time in the past, the present or the future.

It is a fact that in ordinary language use *temporalis* is often conflated with implication. Thus, propositions such as 'When the sun shines, it is day', and 'Wood becomes warm, when fire is brought near to it', are often supposed to be equivalent to the corresponding conditionals 'If the sun shines, then it is day', and 'If fire is brought near to wood then it becomes warm'. Similar intuitions must have been at work in the attempts of various logicians, who tried to describe *temporalis* in terms of the conditional. Ockham was obviously acquainted with some of those attempts, but he nevertheless rejected their underlying idea of reducing *temporalis* into a conditional. Instead, he related the semantics of *temporalis* to the conjunction, as we shall see in detail below. According to Ockham, the following

propositions are all instances of *temporalis*:

(1) 'Socrates is running, while Plato is debating'.
(2) 'Socrates was running, before Plato was debating'.
(3) 'Socrates was running, after Plato was debating'.

However, on Buridan's definitions only (1) is a proper instance of *temporalis*, whereas (2) and (3) are not. Buridan seems to have assumed that molecular propositions constructed by means of 'after' and 'before' can be reduced into propositions with the connective 'while'. In the following we shall concentrate on temporal propositions formulated by means of 'while' (or some equivalent word); we shall use the connective w for 'while'.

Given Buridan's interest in durational logic, it would only seem natural if he had also tried to account for the logic of *temporalis* within a durational framework. To our knowledge, however, Buridan never formulated such an explication. Now what is the truth-condition for the temporal proposition $(p\ w\ q)$? Here all 14th century logicians gave almost the same answer: The truth-condition for the temporal proposition $(p\ w\ q)$, where both p and q are in the present tense, can be formulated in the following way:

(C) $(p\ \ w\ q)$ is true iff both p and q are true (now).

If (C) is accepted without any constraints, the following thesis will be valid:

(T1) $(p\ \ w\ q) \supset (p \wedge q)$

The same of course goes for the converse of (T1). Thus it would be a consequence of a completely general adoption of (C) that *temporalis* would simply be equivalent with the usual conjunction. However, according to Buridan (C) should not be accepted in general, but only for propositions in the present tense. Now let p and q represent atomic propositions (with the verbs in the present tense), and let Pp, Pq, Fp, and Fq stand for the corresponding propositions with verbs in the past and in the future tense,

respectively. With respect to the temporal propositions *(Pp w Pq)* and *(Fp w Fq)*,,the truth-conditions will be different from (C). As a precondition for giving the proper definitions, Dorp [1499, p.41] suggested that any adequate definition would have to comply with the following observations:

(B1) There is a temporal proposition *(p w q)*, which is false, although *p* and *q* are both true.
(B2) There is a temporal proposition *(p w q)*, which is true, although one of its parts is false.

In order to prove (B1) Dorp considered the temporal proposition

'My father was while Adam was'.

The constituents of this proposition, 'My father was' and 'Adam was', are both true, whereas the temporally combined proposition is false. This is indeed sufficient to prove (B1), and as a consequence of that principle the following proposition is not a valid thesis in Dorp's (and Buridan's) system:

(N1) $(Pp \wedge Pq) \supset (Pp \;\; w \; Pq)$

For this reason the *temporalis* and the conjunction are obviously not equivalent. (T1) can be valid only for propositions in the present tense. Thus restricted (T1) can on the other hand be strengthened into

(T1') $(p \;\; w \; q) \equiv (p \wedge q)$
- where *p* and *q* are atomic and present-tense propositions.

In order to prove (B2) Dorp fielded the following temporal proposition:

'My father was not while Adam was'
('Pater meus non fuit quando Adam fuit').

Let p stand for 'My father is', and q stand for 'Adam is'. It appears that the first part of this proposition, 'My father was not', is false, whereas the other part, 'Adam was', is true. It also appears that the combined temporal proposition is itself true - as already observed by Buridan. Then the proposition would seem to constitute a straightforward example of (B2). However, there obviously is a problem here. In general, the whole proposition is three-ways ambiguous, as signified by

(a) $\sim(Pp\ w\ Pq)$
(b) $(P{\sim}p\ w\ Pq)$
(c) $({\sim}Pp\ w\ Pq)$

The (a)-case is not interesting here, though, for clearly Pp and Pq are both true, and thus it could never be made into a case for (B2). In fact the (a)-case is another example of the principle (B1). So let us turn our attention to the (b)- and (c)-cases, which together reflect the fact that the proposition 'My father was not' is itself ambiguous. It can be understood as $P{\sim}p$ or as ${\sim}Pp$. Obviously it is ${\sim}Pp$ that is false, while $(P{\sim}p\ w\ Pq)$, $P{\sim}p$, and Pq are all true. It seems that Dorp mixed up his readings of 'My father was not'; in arguing that the constituent was false he adopted the reading ${\sim}Pp$, but in arguing that the whole *temporalis* was true he was referring to the reading $P{\sim}p$, i.e. the (b)-case. (It must be admitted, however, that the original Latin formulation of the example ('pater meus non fuit') takes a form that makes it tempting to understand it is as ${\sim}Pp$.)

We conclude that although it may be questioned whether Dorp managed to show (B2), but he did show that the following formula is not a thesis:

(N2) $(P{\sim}p\ w\ Pq) \supset {\sim}Pp$

What he did not manage to show was that this formula is not a thesis:

(N3) $(P{\sim}p\ w\ Pq) \supset P{\sim}p$

although he apparently thought he had done so. In fact, it may well be the case that (N3) is a thesis, and that (B2) is not correct. Ockham in his *Summa Logicae* actually denied the validity of (B2):

> It is also clear from what has been said that there is a valid consequence from a temporal proposition to one of its parts - but not conversely. Similarly, a conjunctive proposition follows from a temporal proposition - but not conversely. [Ockham 1980, p.192]

That is, Ockham claimed that

(T2) $(Pp\ w\ Pq) \supset (Pp \wedge Pq)$

is a valid thesis, whereas he (like Dorp) rejected (N1), i.e. he denied that there is a valid consequence from the conjunction of the parts of a temporal proposition to the temporal proposition itself. The validity of (T2) clearly implies the rejection of (B2).

Since (N1) is not a thesis, there is no straightforward reduction of temporal propositions into conjunctions, and (C) therefore has to be amplified in a manner, which will make it applicable also to tensed propositions such as $(Pp\ w\ Pq)$. It appears that the appropriate truth-condition according to both Buridan and Dorp can be formulated as follows:

(CP) $(Pp\ w\ Pq)$ is true if and only if there is some time in the past at which both p and q are true.

It is evident that this truth-condition is identical with the truth-condition for the proposition $P(p \wedge q)$. Therefore we have now arrived at the following equivalence, which must be adopted as a valid thesis within Buridan's and Dorp's logic for *temporalis*:

(T3) $(Pp\ w\ Pq) \equiv P(p \wedge q)$

It is reasonable to assume that the similar thesis for with respect to the future operator is also valid:

(T4) $(Fp\ w\ Fq) \equiv F(p \wedge q)$

Therefore the *temporalis* can in some sense be reduced to the conjunction, although it is no straight-forward reduction. The relation between the *temporalis* and the conjunction is explained in (C), (T3) and (T4). It is clear, however, that these theses do not apply to all possible syntactical constructions. For instance, one might ask for the truth-conditions of propositions like $(Fp\ w\ Pq)$. To the best of our knowledge no medieval logician took such propositions into serious consideration. The reason for this must have been the view that such hybrid propositions simply do not make sense. What meaning could be attributed to e.g. 'Socrates will be running while Plato was debating'? The acceptance of such propositions as meaningful presupposes the assumption that some future time is also past, that is, that the structure of time is circular (or possibly cyclical). We shall not rule out that such an assumption might be consistent at an ontological level, but it is not consistent with the semantics of Latin, or English, for that matter. For then consequences of the form

$Fp \supset Pp$

would have to be counted as intuitively valid - which they are not. It should be noted, however, that we do not rule out propositions like 'Socrates will be running while Plato will have been debating'. But the structure of such propositions is not $(Fp\ w\ Pq)$, where p and q are simple propositions with the verbs in the present tense. The structure of such propositions is rather to be represented as $(Fp\ w\ FPq)$.

The complete truth-condition for meaningful temporal propositions can be stated in the following way:

If p and q are atomic propositions in the present tense, then
$(p\ w\ q) \equiv (p \wedge q)$

If p and q are arbitrary propositions (atomic or molecular, in any tense), then

$$(Pp\ w\ Pq) \equiv P(p \wedge q) \text{ and } (Fp\ w\ Fq) \equiv F(p \wedge q).$$

This definition reflects the views of the logicians whose opinions have been discussed above, and on those views it covers all non-modal, temporal propositions which are considered to be meaningful.

In our opinion, this treatment does go a long way in getting us an appropriate semantics for *temporalis*, but as we suggested above there also are limitations to it. It seems to us that in certain cases *temporalis* should perhaps be related to implication rather than conjunction. Let us give just one example of this. Consider the sentence

(S) 'Socrates will be running, while Plato will be debating'.

Now the treatment offered above clearly implies that both of the events of S will occur. Let p represent 'Socrates is running', and q represent 'Plato is debating'. Then S will be represented as $(Fp\ w\ Fq)$, which is equivalent to $F(p \wedge q)$, which clearly entails the future 'occurrence' of p as well as q. But it seems that there is another reading which does not strictly foresee that either event will occur; it merely says that if p will ever be the case, then so will q.. Let G stand for 'it will always be the case that', i.e. $G = {\sim}F{\sim}$. Then the latter reading could tentatively be represented as $G(p \supset q)$. This reading is close to what we above called a 'generic reading'. However, it can be argued that this reading is the one that should be expressed as

(S') 'Whenever Plato will be debating, Socrates will be running',

and hence, the reading $G(p \supset q)$ for S might be viewed as merely a consequence of an imprecise use of language. But in fact, it seems that there is an even weaker reading of (S), which might be represented as

$Fq \supset F(p \wedge q)$

Loosely, this reading says that if Plato is ever going to debate at some future time, and possibly at several future times, then Socrates is going to be running at least one of those times. In other words, we here need to define 'while' in terms of implication rather than conjunction. We admit that this reading may be quite rare, but we believe that there is a systematical argument for it. The treatment of $(Fp\ w\ Fq)$ as equivalent with $F(p \wedge q)$ makes no difference between

(a) forming (S) by first forming p and q, then forming $(p\ w\ q)$, and finally putting the whole thing into the future tense; in this case (S) is seen to be equivalent with 'it will be the case that Socrates is running, while Plato is debating'. This is appropriately represented as $F(p\ w\ q)$, and adequately treated as equivalent to $F(p \wedge q)$;
(b) forming (S) by first forming two future proposition, Fp and Fq, and then conjoining them by means of the while-connective. This structure is immediately reflected by $(Fp\ w\ Fq)$. We see no prima facie evidence that this reading must also entail the actual occurrence of p and q.

Now we admit that no difference between the two structures can be seen in the syntactic surface structures of either Latin or English. But it is clear that (S) could be built in both ways suggested, and moreover, it is a fact that there has also been a persistent tradition of understanding *temporalis* in terms of the implication. From these observations we conclude that

(i) it is problematic to define *temporalis* exclusively in terms of conjunction (and tense operations) - implication should be granted a rôle, too;
(ii) *temporalis* in some cases invites a kind of generic reading, which can be pleasantly represented by the G-operator; it may be noted that this kind of generic reading may be relaxed somewhat if we would introduce intervals.

The logicians of the 14th century also considered temporal propositions involving modal operators. Walter Burleigh studied temporal propositions involving modal operators, and in this connection he also considered the Aristotelian principle "Omne quod est quando est necesse est esse" [Burleigh 1955, p. 130], which Aristotle proposed in *De Interpretatione IX*. According to Burleigh this principle can be understood in two different ways, which may be representable as follows (as usual, we shall let Mp and Np stand for 'it is possible that p', and 'it is necessary that p', respectively):

(N4) $N(p\ w\ p)$
(T5) $p \supset (Np\ \ w\, p)$

(N4) corresponds to the translation 'it is necessary, that everything is, when it is', where as (T5) can be read 'everything that is true, is necessary, when it is true'. Burleigh maintained that (T5) is a valid thesis, whereas (N4) is not.

But how does the notion of possibility more precisely attach to the *temporalis* operator? Burleigh pointed out that a precondition for the possibility of $(p\ \ w\ q)$, that is, for the truth of $M(p\ w\ q)$, is - rather of course - that both of p and q must be possible in their own right. So we have the following valid thesis :

(T6) $M(p\ \ w\ q) \supset (Mp \wedge Mq)$

However, the converse of (T6) is not valid:

(N5) $(Mp \wedge Mq) \supset M(p\ \ w\ q)$

In his *Summa Logicae* Ockham investigated several details regarding propositions containing both modal and *temporalis* operators. He first investigated which conditions would have to obtain in order for a temporal proposition to be necessary. Ockham claimed that

> ... in order for a temporal proposition to be necessary it is required that each part be necessary. [Ockham 1980, p.180]

This condition can be formulated by means of the following thesis:

(T7) $N(p \;\; w \; q) \supset (Np \wedge Nq)$

However, the converse of (T7) is not valid, that is, no temporal proposition $(p \; w \; q)$ is necessitated simply because each of its component propositions is necessary by itself. In fact, even temporal propositions such as

'a man is a bachelor, while he is not married',
'Socrates exists, while he exists', and
'Socrates is moving, while he is running',

are not necessary according to Ockham. This is so, in spite of the fact that these temporal propositions are composed of atomic propositions which could form necessary conditionals: 'Necessarily, if a man is not married, then he is a bachelor', and similarly for the other two sentences.

The fact that p necessarily implies q does not, however, imply that the temporal proposition of p and q is necessary, so according to Ockham's ideas

(N6) $N(p \supset q) \supset N(p \;\; w \; q)$

is not a valid thesis. Not even

(N7) $N(p \supset p) \supset N(p \;\; w \; p)$

is a valid thesis in Ockham's logic. The reason is of course that temporal propositions according to Ockham are modified conjunctions and not modified conditionals. Hence, $(p \;\; w \; p)$ will be equivalent to p, and consequently

(T8) $N(p \;\; w \; p) \equiv Np$

is a valid thesis in Ockham's logic. In his own words:

> Hence, the proposition 'Socrates exists while he exists' or 'Socrates is moving while he is running' is not necessary, but can be false. [Ockham 1980, p.192]

There is another question regarding modal temporal propositions to which Ockham gave an answer; that question is concerned with impossibility. He made it clear that a temporal proposition can be impossible, even though none of its parts is impossible, so we do not have the following formula as a thesis:

(N8) $\sim M(p \;\; w\; q) \supset (\sim Mp \vee \sim Mq)$

In fact, Ockham did not state a necessary condition for the impossibility of $(p\; w\; q)$, but he did formulate a sufficient condition:

> ... for a temporal proposition to be impossible it is not required that some part be impossible. Rather, it is sufficient that the parts be incompossible. Thus, this is impossible: God creates while he does not create. [Ockham 1980, p.192]

Since p and $\sim p$ are incompossible, the statement implies that

(T9) $\sim M(p \;\; w \sim p)$

is a thesis. It should be mentioned that (T9) can be shown as a consequence of two basic principles of modal logic: (i) the fact that the contradiction is impossible, i.e. the thesis

(T10) $\sim M(p \wedge \sim p)$,

and (ii), the well-known modal principle that no impossible proposition follows with necessity from a possible one, i.e. the thesis

(T11) $N(p \supset q) \supset (\sim Mq \supset \sim Mp)$

Because of (T1'), (T3), (T4), and standard modal logic

(T12) $N((p \; w \; {\sim}p) \supset (p \wedge {\sim}p))$

is also a thesis. It is easy to verify that the validity of (T9) follows from (T10-12).

In the 15th century *temporalis* became more and more neglected. The medieval discussion regarding the number of basic kinds of molecular proposition has been summed up by Paul of Venice in his *Logica Magna*:

> Some posit five kinds of molecular proposition, some six, others seven, others ten, others fourteen, and so on. But putting all these opinions to one side, I say that of kinds of molecular proposition which are not identical in their signification there are three and no more ... [Logica Magna, II, Fasc. 3, 2e].

According to Paul himself, there are only three species of molecular proposition: conditionalis, copulativa and disiunctiva. Nevertheless, he made some very careful studies of the logic temporal propositions such as

> 'when I was awake I did not sleep' [13e]
> 'while I shall not be Antichrist will not be' [15e]
> 'when every man disputed every man was white' [15e]
> 'when one single man will die every man will die' [18e]

and other temporal propositions like

> 'you will be a priest before you will be a bishop' [11e]
> 'you will begin to be after A will be' [16e]

However, although Paul of Venice considered such temporal propositions to be relevant for the logical reflection, he did not accept *temporalis* as one of the fundamental kinds of molecular propositions. He simply did not accept 'dum', 'ubi', 'quia' etc. as proposition-forming functors. The opinion expressed here by

Paul of Venice came to be the usual one in logical studies during the Renaissance.

In post-medieval logic *temporalis* eventually disappeared and can only exceptionally be found in 16th century logic. Ashworth mentions the Portuguese logician Petrus de Fonseca (1528-1599) as one of the exceptions.

But how can the fact that the *temporalis* sank into oblivion be explained? One factor seems to have been the growing humanistic criticism of scholastic logic. According to the humanists the language used by the logicians of the scholastic tradition was perverted. It is indeed likely that the humanists saw the medieval discussion of the *temporalis* as a clear example of scholastic linguistic perversions. On that basis, the *temporalis* should of course be rejected as an important element of logic.

It must, however, be admitted that this can only be a part of the explanation. For the very idea that temporal propositions formed a class of basic molecular propositions was rejected before the general downfall of scholastic logic - in fact, it was rejected within the scholastic tradition itself. As we have mentioned, Paul of Venice in his very popular *Logica Magna* claimed that there are only three species of molecular propositions. Obviously the majority of logicians of the late scholastic period agreed with Paul. They devoted significantly more interest to the conjunction, the disjunction, and the conditional, than to the other putative molecular propositions. One reason for this preference may have been the simple fact that it is relatively easy to formulate the truth-conditions for these three molecular propositions in terms of truth, falsity, and modality, whereas the truth-conditions for the other molecular propositions are more complicated. As the above discussion should have shown, the truth-value of a temporal proposition $(p \;\; w \; q)$ is not a simple function of the truth-values of its components. On the contrary, it comes out as a rather complicated combination of conjunctions and tense-operators. These properties do not seem to be adequate for a fundamental notion.

One further partial explanation should be mentioned. The rejection of the *temporalis* might be seen in the light of the general features of medieval logic. We believe that medieval logic can in

a general sense be characterised as a temporal logic. That is, it was a logic of propositions whose truth-values can vary from time to time, and a logic in which temporal expressions were considered to be important. It is only natural, then, that the logicians of the Middle Ages, who were working within that framework, and who took the *temporalis* into serious consideration, were also to put forth some of clearest statements ever of the basic assumptions of temporal logic. Late-scholastic and humanist logicians paid less attention to the temporal structures of logic than their predecessors, and so it seems understandable that the *temporalis* became increasingly neglected, and was ignored as an important propositional connective. Even so, the reasons for the rejection of the *temporalis* which we have mentioned here do not seem sufficient for fully explaining this development. There is certainly still much to be done with respect to reconstructing the medieval use of the *temporalis* as well as the final rejection of it as a connective. But we hope to have argued convincingly that the *temporalis* is an interesting construction of medieval logic, and that it deserves further study.

1.9. HUMAN FREEDOM AND DIVINE FOREKNOWLEDGE

During the Middle Ages logicians as a matter of course related their science to theology. Clearly they felt that they had something important to offer with regard to solving fundamental logical questions in theology. The most important question of that kind was the problem of the contingent future. This problem has since come to be regarded as one of the most central problems in the logic of time, together with the concomitant question of the relation between time and modality. In our day, it is not primarily seen as a theological problem, but intellectuals of the Middle Ages saw the problem as intimately connected with the relation between two fundamental Christian dogmas. These are the dogmas of human freedom and God's omniscience, respectively. God's omniscience is assumed to also comprise knowledge of the future choices to be made by men. But then the latter dogma apparently gives rise to a straightforward argument from divine foreknowledge to necessity of the future: if God already now knows the decision I will make tomorrow, then an inevitable truth about my choice tomorrow is already given now! Hence, there seems to be no basis for the claim that I have a free choice, a conclusion which violates the dogma of human freedom. To sum it up, the argument proceeds in two phases: first from divine foreknowledge to necessity of the future, and from that argument to the subsequent conclusion that there can be no real human freedom of choice. Among many others the great Danish 12th century philosopher, Boethius de Dacia [Sajo, vol.V, p.241] tried to solve this difficult problem. According to him the main question is whether the status of the contingent future is compatible with the certainty of divine knowledge, that is, the belief that God has certain knowledge of arbitrary contingent events in the future. Boethius in his analysis insisted that God fully knows future events, which among other things means that he knows events, which in a number of cases are not necessary but contingent.

The approach to the problem was to regard it as a consistency problem, which had to be solved within logic. It was primarily studied in connection with Aristotle's text from *De Interpretatione IX* (the sea-battle tomorrow etc.). Another piece of classical text which was occasionally taken into consideration was Cicero's *De Fato*, which among other matters describes the Diodorean Master Argument. The problem obviously bears on the theological task of clarifying questions such as 'In which way can God know the future?' or 'What is to be understood by 'free-will' and 'freedom of choice'?'

Extensive literature about this subject, primary as well as secondary, can be found, and any attempt to produce a detailed exposition of this subject seems hopeless at the outset. On the other hand, it is possible to get a systematical overview of basic approaches to the problem. We shall accordingly restrict ourselves to an exposition of the four possible solutions to the apparent conflict between the two dogmas, which Richard Lavenham (c.1380) enumerated in his treatise *De eventu futurorum*. Lavenham's central idea is quite clear: If two dogmas are seemingly contradictory, then one can solve the problem by denying one of the dogmas, or by showing that the apparent contradiction is not real.

Denial of the dogma of human freedom leads to fatalism (1st possibility). Denial of the dogma of God's foreknowledge can either be based on the claim that God does not know the truth about the future (2nd possibility), or the assumption that no truth about the contingent future has yet been decided (3rd possibility).

One can alternatively formulate a system, which shows that the two dogmas, rightly understood, can be united in a consistent way (4th possibility). Lavenham himself preferred the last approach which he called 'opinio modernorum', and which can justly be called the typical 'medieval solution' to the problem regarding human freedom and divine foreknowledge. The central feature of that solution was its use of the notion of a 'true future' among a number of possible futures. It was originally formulated by William of Ockham (d. 1349), although some of its elements can already be found in Anselm of Canterbury

(d.1109). It is also interesting that Leibniz (1646-1711) much later worked with a similar system as a part of his metaphysical considerations. In the following we shall follow this line from Anselm to Leibniz.

It seems that Lavenham, like Ockham, regarded the Aristotelian approach to propositions concerning the contingent future as being equivalent with the 3rd possibility. However, this interpretation of Aristotle is, as shown by Nicholas Rescher, by no means the only one. There is also a medieval interpretation of Aristotle, according to which his solution was thought to be identical with what we have called the 'medieval solution', i.e. the 4th possibility. On the other hand, Boehner [1945] has clearly demonstrated that a number of Ockham's contemporaries favoured the 3rd possibility. Peter Aureole (c.1280-1322), for instance, claimed that neither the statement 'Antichrist will come' nor the statement 'Antichrist will not come' is true, whereas the disjunction of the two statements is actually true. From that point of view, one can naturally claim that the dogma of God's omniscience is still tenable, even if God does not know if Antichrist will come or not. God knows all the truths given, and cannot know if Antichrist will come due to the simple reason that no truth value for the statement 'Antichrist will come' yet exists. It nevertheless appears quite sensible that Lavenham rejected the 3rd possibility as contrary to the Christian faith, since the understanding of the dogma of God's foreknowledge does seem somewhat clobbered.

The most characteristic feature of Lavenham's and Ockham's theory is its theoretical concept of 'the true future'. The Christian faith says that God' possesses certain knowledge not only of the necessary future, but also of the contingent future. This means that among the possible contingent futures there must be one which has a special status, simply because it corresponds to the actual course of events in the future. We have ventured to call this line of thinking 'the medieval solution', even though other approaches existed as described in the foregoing. The justification for this is partly that the notion of 'the true future' is the specifically medieval contribution to this problem, and partly that leading medieval logicians

regarded this solution as the best one ('opinio modernorum'). We shall now try follow the development of the medieval solution from Anselm to Leibniz.

SCT. ANSELM

Anselm treated the problem concerning divine foreknowledge and human freedom in his work *De Concordia Praescientiae et Praedestinationationis et Gratiae Dei cum Libero Arbitrio* [Hopkins 1967]. In this work Anselm undertook to answer three questions, of which the first one directly concerns the problem of divine foreknowledge and human freedom.

The central idea in Anselm's solution to the problem is his distinction between two kinds of modality. In chapter III of *De Concordia* he considers two propositions:

'There will be a revolution tomorrow', and
'The sun will rise tomorrow'.

The first of these sentences can be regarded as a contingent sentence, whereas the second one can be regarded as necessary. Using the day as the time unit these propositions can be symbolised as $F(1)p$ and $F(1)q$ respectively. If $F(1)p$ and $F(1)q$ are true, they are necessary on the basis of what Anselm calls subsequent necessity (necessitas sequens) - in symbols:

$$(1) \qquad F(1)p \supset N_s F(1)p$$

(and similarly for q). But according to Anselm there is another kind of necessity. He calls it antecedent necessity (necessitas praecedens). In terms of antecedent necessity the proposition $F(1)p$ is not necessary, so we have

$$(2) \qquad F(1)p \wedge \sim N_p F(1)p$$

or equivalently

(3) $F(1)p \wedge M_p{\sim}F(1)p$ (where $M_p \equiv {\sim} N_p{\sim}$)

according to which it is possible that there will not be any revolution tomorrow even though it is in fact true that there will be a revolution tomorrow. On the other hand the proposition $F(1)q$ is necessary on the basis of antecedent necessity. That is: $N_pF(1)q$.

But what is the difference between the two kinds of necessity? According to Anselm subsequent necessity follows from true propositions about the state of affairs, while the antecedent necessity of a proposition means that it is compelled to be true. Obviously subsequent necessity is 'factual necessity' - that is to say, it is necessity in terms of simply being true. Following Anselm, a proposition is necessary on the basis of subsequent necessity if and only if a contradiction follows from a conjunction of its negation and a number of true propositions.

Now in the argument from divine foreknowledge to necessity of the future one may interpret 'necessity' (N) as 'subsequent necessity' (N_s). Then the argument and its conclusion are fully acceptable to Anselm. Likewise, in terms of subsequent necessity Anselm did not have any misgivings about the thesis: 'What will be, necessarily will be', that is

(4) $\forall x: F(x)p \supset N_sF(x)p$

Anselm formulated his view as follows:

> For when I say, 'If a thing will be, then necessarily it will be', this necessity follows, rather than precedes, the presumed existence of the thing. [Hopkins, p.51]

This acceptance, however, does not imply any reduction of human freedom. To Anselm, the necessity involved is only verbal and factual, but it does not cause anything to be true concerning the future.

Antecedent necessity is stronger than subsequent necessity. If the occurrence of a certain event is necessary in terms of antecedent necessity, then the necessity causes the event to occur. Antecedent necessity can be described as a causal necessity.

This distinction between two kinds of necessity is originally Aristotelian. In his *Prior Analytics* Aristotle clearly drew a distinction between absolute and relative necessity:

> Further, it can be shown by taking examples of terms that the conclusion is necessary, not absolutely, but given certain conditions. [30b 32]

In *De Interpretatione* the distinction between the two kinds of necessity is also expressed. It is very likely that Anselm knew the Aristotelian distinction. In fact a Latin version of Aristotle's *De Interpretatione* along with Boethius' commentaries was certainly at his disposal.

Let us again consider the argument from divine foreknowledge to necessity of the future, and the subsequent conclusion that there can be no human freedom. Now, what is the Anselmian reaction to that argument, when N_p is used as N in the argument?

It is obvious that Anselm rejects the conclusion of the argument. According to him there is no insoluble conflict between the doctrines of divine foreknowledge and human freedom. He says:

> It is clear from these considerations that there is no inconsistency in maintaining both that God foreknows all things and that there are many things which, though having before they occur the possibility of never occurring, do actually occur through free will. [Hopkins p.55]

Therefore, according to Anselm there exists true propositions about the future such that their negations are also possible. The proposition $F(1)p$ about tomorrow's revolution is such a proposition , as expressed in (2). It is clear that if there will be a

revolution tomorrow, it cannot be possible - on the basis of subsequent necessity - that there is no revolution tomorrow. If it is possible that there is no revolution tomorrow, it has to be on the basis of antecedent necessity, as we can see in (2).

The acceptance of (2) clearly indicates that Anselm rejected the classical argument from divine foreknowledge to necessity of the future. This being so Anselm had to reject at least is one of its premises. It seems clear that he in fact denied that any true statement about the past is antecedently necessary. In *Cur Deus Homo* II.1 Anselm was discussing the Virgin's belief that Christ was going to die of his own will:

> It is in accordance with this consequent and non-creative necessity that since the belief or prophecy concerning Christ, and according to which he was to die voluntarily, and not from necessity, was true it was necessary that these things should be. [Henry p.176]

Here Anselm admits the truth of the proposition 'It was true to say: God knows that Christ is going to die voluntarily' According to Anselm, however, this proposition is necessary on the basis of subsequent necessity, but not on the basis of antecedent necessity. Let us clarify this position by using symbolic language. Let p stand for the proposition 'Christ dies voluntarily', D for the operator 'God knows that', and let x and y be suitable time units (numbers signifying for instance days or years). Consider now the statement

$$P(y)DF(x+y)p$$

which can be read 'y years ago God knew that Christ was going to die voluntarily $x+y$ years later'. In this case Anselm rejected $N_pP(y)DF(x+y)\ p$.

It should be noted that this position implies the rejection of the first of the premises in the so-called Master Argument of

Diodorus, wherein necessity (N) is understood in a completely general sense:

(Dl) $P(x)A \supset NP(x)A$,
where A is an arbitrary proposition.

Anselm obviously would reject (Dl), if N is interpreted as N_p and A is the proposition $DF(x+y)p$. Nevertheless, it seems that Anselm was willing to accept (Dl) in some limited sense. In *De Concordia* he said:

> Now, the past event has a characteristic which neither the present nor the future event has. For what is past can never become not-past as what is present can become not-present and as what is going to occur without necessity can be not going to occur. [Hopkins p.52]

Let us ponder this statement carefully. The phrase 'what is going to occur .. can be not going to occur' shows us that Anselm must be talking about events, of which it is possible that they would not occur, even though they actually do occur. This in turn shows us that the kind of possibility, respectively necessity, in question must be antecedent possibility. For the occurrence of an event entails its subsequent necessity, and hence, in that sense it cannot be going not to occur. We repeat the formula used earlier on to capture the kind of possibility at stake:

(2) $F(1)p \wedge \sim N_pF(1)p$

Now let us apply these observations to the statement 'what is past can never become not-past'. Given that we are talking about antecedent necessity, it must be interpreted in the following way:

(D1') $P(x)A \supset N_pP(x)A$

Since Anselm rejected the general version of (D1), he must have presupposed some constraints on the type of propositions

which can be accepted as *A* in (D1'). In order to explain which constraint is natural from the Anselmian point of view one should note that Anselm did not say that any past tense *proposition* is necessary, but rather he made his assertion about past events. We do not think that he would accept God's foreknowledge in the past as a past event, for according to him divine knowledge is different from human knowledge. In *De Concordia* he says:

> We should also understand that like foreknowledge, predestination is not properly attributed to God. For there is no before or after in God, but all things are present to Him at once. [Hopkins p.68]

So according to Anselm the fact that God knew something in the past cannot be properly characterised as a past event. Following Anselm God's knowledge should be understood as timeless knowledge, but it is also true that he assumed that this divine knowledge can be transformed into the temporal dimension. This seems to be how prophecy works.

THOMAS AQUINAS

The idea of viewing God's knowledge as timeless was suggested by Boethius (480 - 524), and since then it has been discussed many times (see e.g. [Lucas 1989, p. 209 ff.]). During the Middle ages it became common to appeal to this idea in attempts at solving the problem of the logical tension between the doctrines of human freedom and divine foreknowledge. The medieval philosopher who contributed the most to the elaboration of this solution was Thomas Aquinas (1225-1274). In Aquinas' opinion, God's eternity is timelessly simultaneous with all parts of time. He compared this view with the relation between the center and the circumference of a circle. The relation between the center and the circumference is the same

all the way round; in a similar manner, God relates in the same way to all times. In his own words:

> Furthermore, since the being of what is eternal does not pass away, eternity is present in its presentiality to any time or instant of time. We may see an example of sorts in the case of a circle. Although it is indivisible, it does not co-exist simultaneously with any other point as to position, since it is the order of position that produces the continuity of the circumference. On the other hand, the center of the circle, which is no part of the circumference, is directly opposed to any given determinate point on the circumference. Hence, whatever is found in any part of time coexists with what is eternal as being present to it, although with respect to some other time it be past or future. [*Summa contra gentiles* I, c. 66]

As Marilyn McCord Adams [1987 II, p.1121] has pointed out, Aquinas apparently assumed not only that God and His knowledge are timeless, but also that time should be regarded as a system in which the basic relations of succession and simultaneity are given in a timeless way - owing to the fact that time is given to God in a timeless way. But Aquinas also maintained that the divine knowledge can be transformed into the temporal dimension by means of prophecies. In [*Summa contra gentiles* I, c. 67] he emphasised this possibility quoting the biblical statement "I foretold thee of old, before they came to pass I told thee" [Isaias 48:5]. So the conceptual difference between past, present, and future is relevant only when humans are involved, either as the subjects of cognition or as participants in communication. Furthermore, Aquinas clearly stated that a temporal being cannot have any certain knowledge of future contingents at all. Thus Aquinas was suggesting a distinction between time as it is for temporal beings such as humans, and time as it is for God, who is eternal.

WILLIAM OF OCKHAM

Ockham discussed the problem of divine foreknowledge and human freedom in his work *Tractatus de praedestinatione et de futuris contingentibus* [Ockham 1969]. He asserted that God knows all future contingents, but he also maintained that human beings can choose between alternative possibilities. In his Tractatus he argued that the doctrines of divine foreknowledge and human freedom are compatible.

Ockham was aware that the concept of communication was essential to this discussion - especially, of course, the communication coming from God to human beings. He claimed that God can communicate the truth about the future to us. Nevertheless, according to Ockham divine knowledge regarding future contingents does not imply that they are necessary. As an example Ockham considered the prophecy of Jonah: "Yet forty days, and Nineveh shall be overthrown" (Jonah ch. 3 v. 4).

This prophecy is a communication from God about the future. Therefore, it might seem to follow that when this prophecy has been proclaimed, then the future destruction of Nineveh is necessary. But Ockham did not accept that. Instead, he made room for human freedom in the face of true prophecies by assuming that "all prophecies about future contingents were conditionals" [Ockham 1969, p.44]. So according to Ockham we must understand the prophecy of Jonah as presupposing the condition 'unless the citizens of Nineveh repent'. Obviously, this is in fact exactly how the citizens of Nineveh understood the statement of Jonah!

Ockham realised that the revelation of the future by means of an unconditional statement, communicated from God to the prophet, is incompatible with the contingency of the prophecy. If God reveals the future by means of unconditional statements, then the future is inevitable, since the divine revelation must be true. The concept of divine communication (revelation) must be taken into consideration, if the belief in divine foreknowledge is to be compatible with the belief in the freedom of human actions. So Ockham understood that the compatibility can only

be established within a framework which duly considers what we in the introduction called sociotemporal notions.

Ockham attempted to clarify the issue as much as possible. About the divine foreknowledge, he stated:

> ... the divine essence is an intuitive cognition that is so perfect, so clear, that it is an evident cognition of all things past and future, so that it knows which part of a contradiction [involving such things] is true and which part is false. [Ockham, 1969, p.50]

However, he had to admit that this is not very clear. In fact, he maintained that it is impossible to express clearly the way in which God knows future contingents. He also had to conclude that in general the divine knowledge about the contingent future is inaccessible. God is able to communicate the truth about the future to us, but if God reveals the truth about the future by means of unconditional statements, the future statements cannot be contingent anymore. Hence, God's unconditional foreknowledge regarding future contingents is in principle not revealed, whereas conditionals can be communicated to the prophets. Even so, that part of divine foreknowledge about future contingents which is not revealed must also be considered as true according to Ockham.

Richard of Lavenham made a remarkable effort to capture and in a clear way to present the logical features of Ockham's system as opposed to Aristotle's solution. Lavenham described some examples. In his view the propositions

> 'Antichrist will be',
> 'The Day of Judgement will be', and
> 'The resurrection will be'

are all about future contingent facts. Then, Lavenham maintained, they are neither determinately true nor determinately false on Aristotle's account. To substantiate that interpretation

Lavenham referred to the following consequentia, which he held to be the crucial claim of Aristotle's theory:

> 'If a proposition is about a future contingent fact, then the proposition is not determinately true.'

The Christian faith, however, goes against the acceptance of the consequentia, since a Christian person must believe that God foreknows all future contingent facts. If Antichrist will indeed be, then God knows that Antichrist will be, and for this reason it is determinately true that Antichrist will be.

It will be recalled that Richard of Lavenham enumerated four possible approaches to solving the apparent conflict between God's foreknowledge and human freedom. He rejected the three classical opinions corresponding to the first, second, and third solutions, and then formulated his own answer to the problem - which was also the opinion of many of his contemporaries. Lavenham held that the doctrines of divine foreknowledge and human freedom are compatible. He considered two versions of the inference from God's prescience to the necessity of the future, and he explained why they should be rejected. Let us with Lavenham consider the first version. The starting point is this example:

> q: 'The Day of Judgement will be.'

The proposition q is regarded as being about a future contingent fact. The following consequentia can now be formed:

> (C) 'God knew from eternity that q; therefore q'.

This consequentia is obviously valid. The argument now proceeds by utilising the principle:

> (P) 'A true proposition about the past, the truth of which does not depend on the future, is necessary'.

The principle is formulated such as to deal only with sentences, which are genuinely about the past. (P) seems to be equivalent to the idea that what has already been (or is now) the case cannot be undone. It might appear to follow from (P) that the antecedent of (C) is necessary, that is, that we should have

(A) 'Necessarily, God knew from eternity that *q*'.

(As you shall see, this is the step which Lavenham rejected, but which was crucial to the argument.) We can now apply a well known principle of modal logic (medieval as well as modern):

(M) 'If '*q* follows from *p*' is a valid consequentia, and *p* is necessary, then *q* will also be necessary'.

Hence, the consequent of (C) will also be necessary. In short, the argument goes as follows:

(1) 'The Day of Judgment will be'.
(2) 'God knew that the Day of Judgment will be'.
(3) 'It is necessary that God knew that the Day of Judgment will be'.
(4) 'It is necessary that the Day of Judgement will be'.

But Lavenham rejected the inference from (2) to (3). He claimed that (P) cannot be used in order to justify this inference, precisely because the truth of (2) depends on the future. For if the Day of Judgement will not be, then (2) must also be false! Lavenham's answer to this argument obviously depends on Ockham's view in *De praedestinatione et de praescientia Dei*.

In considering the other version of the argument, Lavenham used the examples:

p: 'Antichrist will be'.
g: 'God wills that Antichrist will be'.

Within the framework of the dogmas of the Christian faith, the consequentia from *g* to *p* is clearly valid So if *g* can be proved to be necessary, then *p* will also be necessary in virtue of (M). To see how one might go on to establish the necessity of *g*, Lavenham investigated the following syllogism:

> *Major premise: g* is unchangeably known to God.
> *Minor premise:* What is unchangeably known to God is necessarily known to God.
> *Conclusion: g* is necessarily known to God.

In this syllogism the minor premise is valid on grounds of the principle that whatever is unchangeable is also necessary. The major premise is shown by a proof ad absurdum: if *g* were known to God, but not unchangeably known to him, then *g* would be changeably known to God. But this is absurd. Thus it is proved that *g* is necessarily known to God. But more than that is needed, since it should be demonstrated that *g* by itself is necessary. It seems that Lavenham forgot to mention the following premise:

> 'What is necessarily known to God is necessary'.

However, there is no doubt that this premise is presupposed in his reconstruction of the argument. By means of this extra premise it is easily shown that *g* is itself necessary, and consequently *p* is also necessary. Thus goes the second version of the argument as rendered by Lavenham. But Lavenham himself of course rejected the argument. He pointed out that the minor premise of the syllogism above is not valid. For we might just as well assume that what is unchangeably known to God could after all have been different, and therefore it does not have to be necessary! It can therefore be said that in appealing to this minor premise, the argument was in a sense presupposing that which it was going to demonstrate, a fact which Lavenham apparently realised.

As we have seen Lavenham identified four possible approaches to solving the tension between our two apparently

conflicting dogmas. However, these approaches were not the only ones to be considered by medieval philosophers. At least one important position seems to have been left out. That is the opinion of St. Thomas Aquinas and others who claimed that the knowledge of God abstracts from the difference between past, present and future. According to this view it might be said that all events are 'always' present to God - in an atemporal sense of 'always'!

It was mentioned earlier that Leibniz worked out a metaphysics of time, which from a systematical point of view is very similar to the thoughts of Anselm and Ockham. We shall now for a passage leave the Middle Ages in order to examine his system.

LEIBNIZ

Leibniz accepted the doctrine of divine foreknowledge as well as that of human freedom. He of course knew the standard arguments that can be constructed in order to prove the incompatibility of the two doctrines, but he claimed that those arguments were invalid:

> Nor does the foreknowledge or preordination of God impose necessity even though it is also infallible. For God has seen things in an ideal series of possibles, such as they were to be, and among them man freely sinning. By seeing the existence of this series He did not change the nature of things, nor did he make what is contingent necessary. [Rescher 1967 p.39]

Leibniz's central idea was that God had chosen the best of all possible worlds and made it actual. But in actualising the creatures of that world He did not change their free natures. So it is not necessary for a man to do that which he will in fact be doing according to the foreknowledge of God. It would have been

possible for him to make different decisions leading to different acts. But if this is so, how can the foreknowledge of God be infallible? Leibniz' solution to this problem is very close to Anselm's solution. Like Anselm, Leibniz introduced a distinction between two kinds of necessity:

> For we must distinguish between an absolute and a hypothetical necessity. [Alexander p.56]

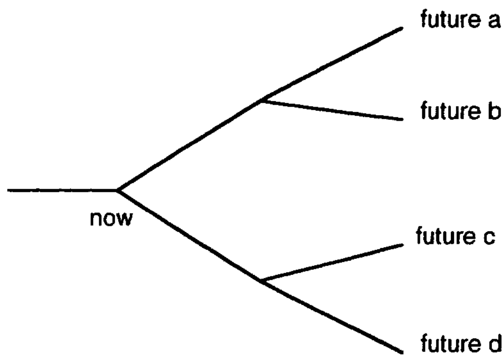
These two concepts of necessity correspond exactly to Anselm's antecedent and subsequent necessity, respectively. With respect to the argument from God's foreknowledge to necessity of the future, Leibniz would have no objection so long as the necessity in question is the hypothetical necessity - just like Anselm accepted the argument when interpreted as referring to succedent necessity. Leibniz observed that

> Hypothetical necessity is that, which the supposition or hypothesis of God's foresight and pre-ordination lays upon future contingents. [Alexander p.56]

This statement is equivalent to Anselm's 'What will be, necessarily will be', i.e.

$$F(x)p \supset N_s F(x)p$$

- where N_s may be interpreted as subsequent as well as hypothetical necessity. It is important to realise that this statement does not provide any information at all about the number of future possibilities. Perhaps this is more easily seen, if we once again allow ourselves to illustrate the thoughts involved by means of the modern notion of 'branching time'. For ease of reference, we here repeat the illustration which we used in the previous discussion of the Master Argument:



Suppose that, say, future *b* is the course of events that is actually going to be. Then to assert hypothetical necessity of some future event *E* can be interpreted as saying simply that the event *E* 'is on the *b*-branch'. But that is exactly the same as simply saying that E is going to occur, which makes the formula above a simple tautology. And in fact it is very likely that Leibniz considered this type of statement as outright tautological, that is, as stating what we would now express with the formula

$$N(F(x)p \supset F(x)p)$$

It should be obvious, then, that this kind of statement does not convey any information as to the number of possible futures. On the other hand, Leibniz would certainly reject the validity of the formula

$$F(x)p \supset NF(x)p,$$

where *N* represents necessity in general, that is, it also includes absolute necessity. In terms of the branching time model above, absolute necessity in effect quantifies over all branches expanding from the given 'now'.

That rejection is a consequence of his refusal to accept 'past-necessitation'- (D1) - for arbitrary statements. Although Leibniz rejected the generalised version of (D1), he was willing to accept a limited version of that principle. In his *Theodicy* [II §

170] he explained that there is a difference in modality between the past and the future. For while it is not possible to cause a past event, it is now possible to cause some of the future events. Therefore, if p is a statement variable such that $P(x)p$ is genuinely about the past, it follows that $P(x)p$ is also necessary in the general sense, i.e.

$$P(x)p \supset NP(x)p$$

Regarding the contingent future there are statement variables, e.g. the variable q, such that it is possible to make $F(x)q$ false, in spite of the fact that it will be true - that is

$$F(x)q \wedge M{\sim}F(x)q$$

This means that while there is no alternative to the actual past, there are alternatives to the future. These alternative futures correspond to the Leibnizian concept of possible worlds.

The connection which Leibniz established between modality and the multitude of possible futures is the one which is also commonly used within present-day modal logic and possible world semantics: what is necessary is that which holds in all possible futures, and what is possible is that which holds in at least one possible future. The concept of modality involved here is clearly of a temporal nature. This means that a proposition which describes some event is necessary (in the absolute sense) if and only if the proposition follows from a proposition about the past or the present. It seems that the implication in question is a kind of causal implication. Thus, an alternative formulation would be to say that a future event is necessary if and only it is unchangeably caused by present or past events. It follows that in terms of our branching time model a necessary event must be true in all future branches.

The possible worlds of Leibniz represent the ways in which the entire history might have been different from what it is. Therefore it seems to be reasonable to identify a possible world with a possible history.

Leibniz claimed that among all the possible worlds God has chosen the best one:

> It follows from the supreme perfection of God that he has chosen the best possible plan in producing the universe, a plan which combines the greatest variety together with the greatest order ... For as all possible things have a claim to existence in God's understanding in proportion to their perfections, the result of all these claims must be the most perfect actual world which is possible. Without this it would be impossible to give a reason why things have gone as they have rather than otherwise. [Leibniz 1969, p.639]

Leibniz' idea of possible worlds can in the context of temporal logic be viewed as a number of sequences of events. In each of these chronicles the future events follow logically from the present. In this connection it should be noted that all relevant information about the present also includes information about all past events. Leibniz formulated his position as follows:

> For everything has been regulated in things, once and for all, with as much order and agreement as possible; the supreme wisdom and goodness cannot act except with perfect harmony. The present is great with the future; the future could be read in the past; the distant is expressed in the near. One could learn the beauty of the universe in each soul if one could unravel all that is rolled up in it but that develops perceptibly only with time. [Leibniz 1969, p.640]

It may seem that Leibniz in this way left no room for the idea of free choice. That would, however, be an erroneous conclusion. In dealing with the question of human freedom he stated:

> Since the individual concept of every person includes once and for all everything which can ever happen to him, one sees in it a priori proofs or reasons for the truths of each event and why one has happened rather than another, but

> these truths, however certain, are nevertheless contingent, being based on the free will of God and of creatures. It is true that their choice always has its reasons, but these incline without necessitating. [p.310]

Leibniz took the person of Julius Caesar as an example. The concept of this person involves already at his birth (and in fact, ever before) a future crossing of the Rubicon, a future dictatorship etc. Nevertheless, the assumption that the person who was crossing the Rubicon on a certain day in 52 B.C., and who has also done exactly everything Caesar did before the crossing, will choose not to be a dictator, does not imply a contradiction. Therefore Caesar's becoming a dictator is not necessary, but merely certain as foreseen by God.

THE ANSELM-OCKHAM-LEIBNIZ SOLUTION

It is evident that the solutions presented by Anselm, Ockham, and Leibniz, have very much in common. In spite of minor differences it is meaningful to speak about 'the Anselm-Ockham-Leibniz solution'.

The analysis of the relation between the dogmas of human freedom and God's omniscience, which led to the 'the Anselm-Ockham-Leibniz solution', has proved to be very important also for the development of modern tense-logic. Within the modern discipline the problems concerning determinism and the status of the contingent future are normally not thought of in theological terms, but rather it is discussed at a purely tempo-modal level: What does it mean for an event E to take place? How shall we solve the problems of determinism versus indeterminism? What is the relation between time and modality in general?

In part 2 and 3 we intend to examine some different theories for the future operator in an indeterministic tense-logic, theories which form modern counterparts of the various medieval approaches we have been discussing. In this modern context, we shall argue that 'the Anselm-Ockham-Leibniz

solution' is consistent as well as plausible. We shall also qualify our own heading of 'the Anselm-Ockham-Leibniz solution' by showing that from a strictly formal point of view, the solution can be differentiated into two slightly different systems. This is so because Leibniz' ideas can give rise to a model of time which differs slightly from what could be called an Ockhamistic model of time.

1.10. THE DOWNFALL OF MEDIEVAL TENSE-LOGIC

In a short but thought-provoking sketch of the history of logic with a special view to tense-logic, A. N. Prior has argued that the central tenets of Medieval logic with respect to time and tense can be summarised in the following way:

> (i) tense distinctions are a proper subject of logical reflection, and
> (ii) what is true at one time is in many cases false at another time, and vice versa. [1957a, p.104]

Prior admitted that he had not actually documented these claims in his sketch. However, as we have seen in the preceding chapters there are many concrete examples in support of his claims. Prior's statement can be made more precise, though, by mentioning its two main points in the reverse order, since (ii) can be seen as a natural presupposition for (i). One can hardly imagine a logical system based on the first claim which rejects the second. That is to say: if, in accordance with a rejection of (ii), logic is to treat timeless truths only, then it seems rather futile to establish theories for tensed propositions. On the other hand there is no inconsistency in recognising that the truth value of propositions can in principle vary with time, but finding work on this subject uninteresting for logic. And in fact the waning of tense logic began with a gradual loss of interest in temporal structures, that is, it was (i) which was first abandoned by the different schools of logic, and (ii) came to be rejected only afterwards. We shall now sketch a few major points of this gradual transformation of logic as a discipline.

The downfall of Scholasticism was a process unfolding with the rise of the Renaissance Humanism. One of the losses was the logical studies practised within the Scholastic discipline of dialectics. The Scholastic disputation, which can be seen as a method of unravelling logical intricacies, came to be particularly despised. It was perceived as expressive of an abstract philosophy, which could not lead to anything constructive, and which did not have any worthwhile qualities in its own right.

The struggle against the Scholastic tradition initiated by the dawning Renaissance lasted for at least a century, although E. J. Ashworth [1982] is probably correct in suggesting that the most significant phase of the battle took place in the years around 1530. In the early period some of the major critics of Scholasticism were Laurentius Valla (1407-1457) and Rudolf Agricola (1443-1485). In the footsteps of Cicero and Quintilian, Valla and Agricola wanted to study and define logic within the discipline of rhetoric. For instance Agricola defined logic as "the art of expressing yourself convincingly of anything" [Dumitriu p.232]. This considerable change in the conception of logic was to a large extent a reaction against the perceived maltreatment of the Latin language by Scholastic logicians.

In the decisive phase of the strife the most important humanists were Juan Luis Vives (1493-1540) and Peter Ramus (1515-1572), who also introduced the new 'humanist logic', based on the same ideas as those of Valla and Agricola. Vives went to the university of Paris in 1509 to study within the Scholastic tradition, but when he left it again in 1512, he was totally convinced that Scholastic logic had very little going for it, if indeed anything at all. He especially reacted against the sophisticated, almost artificial, language of the Scholastic logicians. That language was actually semi-artificial in much the same manner as the verbiage of present-day philosophical logic, as seen also in the preceding pages - think of phrases such as 'it will always have been the case that...', etc. However, modern logicians do not have to rely on this kind of language, because we have actual symbolic logic at our disposal. But the Scholastics had no other means than this quasi-formal language in order to make their ideas precise, and for that reason it became an important and pervasive part of their logical tradition. Against that tradition Vives maintained, like Valla and Agricola before him, that contemporary logicians ought to stick to ordinary language. In this connection they fielded the extra argument that it had been possible for Aristotle and Cicero to describe their logical rules in everyday Greek and Latin. In passing it is worth noting that there are striking similarities between the Humanist criticism of Scholastic logic,

and the modern 'pragmatical' opposition against the project of formal semantics for natural language. In both cases there is a reaction against the (real or imagined) regimentation of natural language, and a turn to pragmatical phenomena instead. Especially the downfall of Logical Atomism in the face of Ordinary Language Philosophy in the Fourties, which is recorded in [Urmson 1967], makes out a striking parallel.

The Renaissance perception of the Scholastic tradition is reflected in strong terms in Vives' *Adversus pseudodialecticos* of 1520, in which passages like this one can be found:

> One is nowadays ashamed of speaking of 'incipit' and 'desinit'. Who, by chance, passed on this subtle rigourism, these futile examples, these inane [examples]. [Vives p.59]

In this arrogant way Vives dismissed the attempts of the previous centuries to build a conceptual apparatus, which amongst other things should provide an account of temporal continuity and limits. He also ridiculed Scholastic distinctions between propositions such as:

> (I) 'Antichristus qui fuit erit' (Antichrist who was, will be) and
> (II) 'Antichristus erit, qui fuit' (Antichrist will be the one, who was).

According to the Scholastic analysis the first proposition is false, because it implies that Antichrist has already lived, whereas the second one is considered to be true, since Antichrist - according to the Bible - will come, whereafter he will be the one who was! For a modern tense-logician this is familiar as the distinction between $(Fq \wedge Pq)$ and FPq, respectively. Vives regarded this discussion primarily as an example of bad Latin, and did not realise that there was indeed a significant logical difference underlying the discussion.

Vives was greatly applauded for his endeavours. Erasmus of Rotterdam, for example, wrote that Vives was more suited than

any other person for the task of refuting Scholastic logic, due to his previous service for several years within that tradition.

It is possible, however, that the Scholastic tradition was also to some extent dissolving from within. Ashworth [1982] has discussed whether there was a general recognition of the inadequacy of their logic among the better known logicians at the University of Paris. That question must still be said to be open, although it is obvious that in their work these logicians were getting close to the limit of what linguistic formulations could bear in order to gain more insight by logical analysis. In an ironical manner, this is the very same problem which also gave rise to Vives' type of accusation, namely that of unnecessary sophistry and maltreatment of Latin.

It is hardly possible to find any real progress, or any real novelties, in the modified logic of the Renaissance, as Robert Adamson [1911] has remarked. The same is true of the only logician of 'the Crunch period', Peter Ramus, whose works were very popular in the 16th and 17th centuries. He became the main proponent of the so-called humanist logic. The result of the leading Renaissance logician's work was not a recreation of logic, but an amputation. The emphasis on rhetoric and the significance attributed to Roman logic, which mainly accentuated elegance and simplicity, turned logic into a science of the art of argumentation, or an 'ars docendi' as seen by Melanchthon (1497-1560).

As a consequence of Humanistic logic the temporal dimensions of logic became progressively more neglected. During the 16th century interest in temporal constructions such as those discussed in the previous sections nearly disappeared, although a few logicians continued to work along the lines of the Scholastic tradition (see [Trentman 1982]). 'Ampliatio' was among the temporal constructions which attracted the attention of logicians for the longest period of time (see [Ashworth 1982]).

By the 17th century, the interest in such temporal constructions had nearly disappeared among logicians. Nevertheless there were a number of logicians who felt that the truth value of propositions must in principle be looked upon as

varying with time, as Nuchelman [1980 p.131 ff] has shown in his thorough analysis. But in the famous and representative work *La Logique ou l'art de penser*, which was first published in 1662, Antoine Arnauld (1612-1694) and Pierre Nicole (1625-1695) presented a persuasive and coherent logical theory, in which little room was left for the medieval approach to the logic of tensed propositions. The idea of temporally varying truth was not categorically rejected in the text, but on the other hand it just did not play any rôle. In the following chapter we shall briefly consider the new understanding of logic developed by Leibniz and others.

1.11. Logic as a Timeless Science

In his thorough history of logic Anton Dumitriu [1977 p.11 ff] puts much emphasis on the significance of Francis Bacon (1561-1626). In Bacon's attempt to establish experimental science and define its methods, he presented logic as a tool to be applied within the respective scientific disciplines, as well as a more general tool for analysing the conditions of each discipline. Thus Bacon emphasised the rôle of logic as methodology. This emphasis would eventually lead to the dissociation of logic from language, that very connection which in the Scholastic times had inter alia legitimised the study of propositions with time reference. Dumitriu attributes almost the same importance to the rôle of René Descartes (1596-1650) within post-Scholastic logic. In Descartes' methodology, mathematics becomes a model for all of science. Since mathematical truths are in general considered to be independent of and without reference to time, Descartes' point of view also seemed to motivate that time be neglected in logic. One of the great Cartesians, Malebranche (1638-1715) wrote, in his *Recherche de la vérité*:

> La vérité est incréée, immuable, éternelle, au-dessus de toutes choses. [Risse 1970, p.110]

Let us now consider Gottfried Wilhelm Leibniz (1646-1716), who was of paramount importance in the history of logic. Leibniz must be considered to be the founder of symbolic logic. He is also one of the logicians responsible for the definitive abandonment of tense-logic. In 1679 he presented a subject-predicate logic, in which the study of the copula (English 'be', or Latin 'esse') was not significant (see [Leibniz 1969 p.235]). In so doing he effectively distanced himself from a considerable number of the subjects with which the Scholastic logicians had been concerned.

Leibniz was clearly influenced by Peter Ramus and Melanchthon (see [Leibniz 1969 p.464 & p.471]), and followed them in finding Scholastic logic inadequate. He also mentioned the logicians Jacobus Zabrella (1533-1599) and Joachim

Jungius (1587-1657) as persons who revealed the inadequacy of Scholastic logic. Nevertheless, Leibniz' approach to logic was very different from the methods known within 'humanist logic'. He wanted to mathematicize logic, and to construct a calculus which could be used as a mathematical description of logical structures and inference. In this endeavour, he left out the copula and its conjugation, as well as other auxiliaries, for instance modal ones. In Leibniz' symbolic logic it is implicitly understood that a copula has to appear in the present tense (and an omnitemporal sense).

An important person in the development was Gabriel Wagner, who in 1696 settled in Hamburg,. Here he began to publish the journal 'Vernunftübungen', in which he led a bitter fight against contemporary Scholasticism, roundly attacking logic. This prompted Leibniz, in a letter to Wagner in the same year, to defend the discipline as extremely valuable (see [Leibniz 1969 p.462 ff]). In his letter Leibniz tried to determine what exactly logic was. He established that logic ought to be looked upon as an art, which can make the knowledgeable more secure. This happens not only by evaluating the truth values of given propositions, but also by logical investigations and methods leading to new and hitherto hidden truths. Leibniz thus regarded logic as a science of thought and method.

In his attempt to determine what logic is, Leibniz pointed to it as a science important for all kinds of intellectual work. In his opinion, it had to be considered the key to all intellectual evaluations, and hence to all of science. Since Wagner was unwilling to draw this conclusion he must either have disagreed with Leibniz' definition of logic, or else he must have considered the state of logic as well as its results so far to be pretty poor. Some of Wagner's remarks indicate that he did indeed hold the latter, and actually, so did Leibniz himself, at least to some extent. He recognised that logic at present was 'but a shadow' of what he wanted it to be. But even though he thus partly agreed with Wagner, he thought it was wrong to reject the entire logical tradition out of hand, since he considered much traditional logic as both thought-provoking and useful.

The logic which Leibniz himself wanted to promote was a timeless logic. But that is not to say that Leibniz did not take an interest in questions involving time, and in fact, questions which we would call tense logical. Indeed, we have argued that Leibniz' philosophy regarding the relation between God's foreknowledge and man's free will is in tune with Medieval as well as modern tense-logic.

Nevertheless, in his actual logical works Leibniz formulated his logic in such a way that it did not take time into account. We shall here suggest what we believe to be the main reasons for Leibniz' deliberate neglect of time within his logic.

Firstly, it was Leibniz' ambition to bring mathematics and logic close together. Leibniz admitted that mathematics is identical with logic, but he maintained that it is 'one of the eldest sons' of logic. In particular, he emphasised what he considered to be an important discovery, namely the insight that many of the advantageous features of algebra can be traced back to the august science of logic! (See [Leibniz 1969 p.469 ff]) Given Leibniz' clear interest in the relation between mathematics and logic, and especially in the use of logic within mathematics, it is easy to understand that he would favour the timeless variety of truth.

A second and more philosophical reason for Leibniz' preference for a tenseless logic can be found in his concept of the individual substance. The complete or perfect notion of an individual substance on his view includes everything which can be said of the substance with respect to past, present, and future (see [Leibniz 1969 p.268 ff]). We have already discussed the example of the concept of Julius Caesar. Leibniz also mentioned the Apostles Peter and Judas as examples: it is inherent in the complete notion or concept of Peter that he was going to deny Jesus, and likewise is inherent in the complete notion of Judas that he was lost. Therefore, according to Leibniz any argumentation or investigation concerning complete notions does not need to make any reference to time. Predicates belong, or do not belong, to the complete notion irrespective of temporal relations. It should however be noted that the temporal aspects

are clearly incorporated into the very formulation of the concepts!

Leibniz' conception of logic was continued by people like Wolff and the Dane Jens Kraft, who in their books on logic referred to mathematical examples, although they did not use actual symbolic logic. Examples of actual symbolic logic were rare in the 18th century and the first part of 19th century. Important exceptions were Johan Heinrich Lambert, Gottfried Ploucquet, Leonard Euler, and J. D. Gergonne [Kneale 1962 p.348 ff.]. But the typical attitude of the time was that there was no need for a further development of logic. Kant, notably, had in the preface to *Critique of Pure Reason* stated that logic had been unable to make any substantial advances since Aristotle, and logic appeared to him as "allem Ansehen nach geschlossen und vollendet zu sein" [2. ed. p.VIII]. Kant defined logic as the discipline concerned with the formal rules for any kind of thinking ("die formalen Regeln alles Denken"). In his opinion, the study of these rules, i.e. logic as such, had already been completed by Aristotle. This of course did not exclude that there could still be work to be done on the foundation of logic, in order to be able to formulate the conditions for 'pure reason' - this was exactly what Kant himself was doing. But the set of formal rules of thought, that is, logic itself, was considered to have been already exhaustively determined. Therefore attention was turned away from a supposedly futile study of actual logic, and instead directed towards a discussion of the application of logic, primarily within reasoning and scientific methodology. Thus logical studies were concerned with general truths, and logic became timeless.

2.1. THE 19TH CENTURY AND BOOLEAN LOGIC

In the 19th century temporal distinctions were usually considered to be irrelevant to logic. The timeless character of logic was often argued for by reference to philosophy of science: the primary interests of science should be timeless (or omni-temporal); and since logic was thought to be a tool for sciences, it too had to follow suit. Thus Alexander Pfänder, who worked within the phenomenological tradition, asserted that the non-historical sciences had exclusively to aim at true propositions described with "eine durch alle Zeiten hindurchgehende Gegenwart" [1921, p.269] (app. "a present stretching through all times"). Pfänder was by no means the first person to state such views. On the contrary, they must be said to have been prevalent during the previous two centuries. The perception of logic as timeless was still a common-place around the turn of the century. In the following we shall as one example show how this conception was expressed by one Danish logician of that period.

K. Kroman was one of the most prominent logicians of his day in Denmark. His work on logic entitled *Tænke og Sjælelære* (app. "Textbook on Thought and the Soul") [1899] was concerned with the nature of scientific statements:

> [To say] that N. N. has a horse, which had a fall yesterday, is not [to make] a scientific statement. But the observation that the horse is a solid ungulate hoofed mammal, which is normally used as a domestic animal, is on the other hand a scientific statement. [1899, p.5] (our translation)

Kroman's understanding of the rôle of logic as a tool for the other sciences is apparent in the following quotes:

> ...it is with the aid of logic, that we build any other science... Logic is ... the science of correct thinking. [1899, p.5] (our translation)

Thus logic is, according to Kroman as well as most of his contemporaries, a tool to be used in connection with the formulation of any science. The underlying assumption is that scientific theories are essentially systems of propositions. Logic itself consists of three parts since it studies concepts, propositions, and inferences:

> The basic items of logic are... concepts, and with concepts the first part of logic is thus concerned [p.15];
> Words refined into concepts are not sufficient as a basis for logic, but logic must, whilst following everyday language, also seek a refined expression for the life and movement of conceptual content, corresponding to the [everyday language] sentence; this expression it finds in the proposition. [p.22];
> ...it (is) the task of logic to teach us how we form inferences, or how we should correctly derive new propositions from given ones. [p.23] (our translation)

This trichotomy is historical and obvious in Scholastic logic, too. There, however, temporal considerations were interwoven with all three elements. But Kroman only discussed the rôle of time explicitly in connection with the formulation of propositions:

> Science is furthermore concerned essentially with conditions and activities of a general character, conditions and activities which are not depending on any specific present moment of time, but which are of a lasting validity; hence the different 'tenses' of the verb would as a rule also be superfluous, and we could in general abide by the present tense only. [p.31-32] (our translation)

There is thus no room for considerations concerning tense inflected propositions in Kroman's logic. The reason for this is clearly that logic is to serve science as Kroman and his contemporaries saw it. Here time comes into the picture, partly in connection with the basis for prediction, and partly in connection with his ideas on the development of knowledge.

Kroman's understanding of logic as a timeless discipline was continued in Høffding's logic, in which the copula of propositions is seen to be always in the present tense: 'is'. Høffding did allow that time relations can be considered, but only when the expression indicating time occurs either as the logical subject or as a predicate. Not until Jørgen Jørgensen, who emphasised Boole's inclusion of time and Łukasiewicz's analysis of propositions relating to the future, did time-logic find a modest place within Danish logic.

Kroman's view on logic was certainly typical of his period, not only in Denmark but in Western logic as a whole. The most important logician of the nineteenth century was Gottlob Frege (1848-1925). For Frege, truth in logic was completely timeless: the time at which an utterance is made is considered as part of the thought which is being expressed [Klemke, p.361]. If somebody wants to say the same today, as he said yesterday then he must substitute the expression 'today' with the expression 'yesterday' [Prior 1957]. In Frege's logical system there is no room for the conception of a proposition as a function with time as a variable. Likewise, the study of tensed propositions is not considered to be interesting. While this without doubt holds for his symbolic logic and its concomitant conception of what logic is about, it is paradoxical that Frege at the same time was keenly aware of intensionality in language. In his famous article "Über Sinn und Bedeutung" [Frege 1969] (On Sense and Reference), he actually did invite a conception of propositions as functions. His observations in that paper played a crucial rôle for the later development of intensional logic (and possible world semantics), where propositions are ordinarily construed as functions with time as one of their crucial parameters [Montague 1976a].

In Prior's overview of the history of logic it is described how many 19th century logicians - for instance R. Whately, H.L. Mansel, Francis Bowen, and Thomas Fowler - denied that tensed propositions were important or at all relevant in logical analysis. There are some notable exceptions, though; these include J. S. Mill (1806-73), George Boole (1815-64) and C. S. Peirce (1839-1914). In the next section we shall examine

Peirce's rôle, but for now we shall concentrate on George Boole, who constructed a logical formalism modelled on algebra. This formalism proved vital for the development of modern symbolic logic. But Boole is also perhaps the first 19th century logician to have included the concept of time explicitly in his theories (although only in a few passages). Naturally, we shall here concentrate on this element of Boole's logic and leave aside a more exhaustive exposition.

Some interesting considerations regarding the relation between time and logic can be found in the manuscript entitled *Sketch of a Theory and Method of Probabilities Founded upon the Calculus of Logic*, which Boole must have written between 1848 and 1854. Boole here discussed elementary propositions such as "The Thermometer falls" and "It will rain". Boole made this observation:

> Accordingly I have ... interpreted the symbols x, y, z, as expressing the cases in which those elementary propositions are true [x corresponding to 'it rains' and y corresponding to 'it hails']. This is in agreement with the ordinary doctrine of the 'Reduction of Hypotheticals'. But more exact analysis has led me to another conclusion. And without stopping here to assign the reason upon which that interpretation is founded, I shall simply state that it consists in regarding the symbols as representing the times in which the elementary propositions to which they refer are true. [LP, p.146]

From this passage it is not clear what convinced Boole that a temporal approach is preferable to the approach based on an analysis of the given situation. But an appendix to the manuscript shows that he recognised the importance of ordinary language as a guide for - or perhaps a test-bed of - logical considerations (especially with respect to the study of universals and particulars):

> The language of common discourse, which in many respects outstrips the limits within which the logicians

> would fain have restricted it, recognizes, however, the particular as well as the universal in hypothetical judgements, - and it distinguishes them by the particle 'sometimes'. The system which I have endeavoured to establish introduces the same element, *Time*, and in the same manner. This I was not aware of when I was led to form that system, and I accordingly esteem it an interesting verification. [LP, p.162-63]

Boole obviously held that a proposition refers to one or more durations. If two propositions refer to the duration x and the duration y, respectively, then the conjunction between two propositions equals xy (i.e. the intersection between the two durations). It thus becomes especially interesting to examine the numerical constants *0* and *1*:

> .. the numerical values 0 and 1 will be equally admissible with this system of interpretation, the former as the representative of the nothing of time or never: the latter as the Universal of time, which when unlimited is *Eternity*, when limited the duration to which our discourse refers. [LP, p.146]

For example, Boole represented propositions such as:

'If it rains, it hails'

by an equation of the type:

$$x = v\,y$$

Let us render the ideas involved in this example in modern terms: y is to be understood as the function defined on the set of times, yielding the value *1* when 'it hails' is true, and *0* otherwise; x is the analogous function corresponding to 'it rains'. The third function of the equation v, is according to Boole "the representative of time partially indefinite and is a symbol of the same kind as x and y". The set theoretical rôle, which v plays in

the equation is to ensure that the set of truths for x is a subset of the set of truths for y. Similarly, one could say that 'it hails' and 'it rains' can respectively be represented by the equations $y=1$ and $x=1$. Here the implication 'if it rains, it hails', has been captured by an equation. Alternatively, one could represent it by the function $(1 - x + vy)$.

In his major work *An Investigation of the Laws of Thought, on which are founded the Mathematical Theories of Logic and Probability* [1854], Boole presented some of the same thoughts which appear in the earlier manuscript. He considered the so-called secondary propositions [p.159], verdict functions which relate to other verdicts. It is characteristic of secondary verdicts that they involve a time relation. This means that logic must be about relations between 'valid times'. Jørgen Jørgensen has pertinently characterised Boole's theory for secondary verdicts as a "time interval calculus" [1937, p.48]. This kind of theory as was unique in nineteenth century logic.

It must be admitted that the introduction of time into a logic for secondary verdicts does not seem to have been one of Boole's chief concerns (judged on the basis of the number of pages in his works devoted to the treatment of this question). But his remarks on the matter are on the other hand clear enough. In any case they caused John Venn [1894, p.451-52] to realise as a consequence of Boole's theory that tense inflected propositions must be considered in logic, even though Venn himself did not like the idea much. Half a century later, Boole's inclusion of time in logic became one of the inspiring factors for the founder of modern tense-logic, A.N. Prior[1957], who strongly emphasised Boole's suggestions.

2.2. C.S. PEIRCE ON TIME AND MODALITY

Time has usually been considered by logicians to be what is called 'extra-logical' matter. I have never shared this opinion. But I have thought that logic had not yet reached the state of development at which the introduction of temporal modifications of its forms would not result in great confusion; and I am much of that way of thinking yet. C.S. Peirce [CP 4.523]

To Charles Sanders Peirce (1839-1914) semiotics (or to use his own expression: 'semeiotic') gradually became identical with logic in a broad sense. In defining this relation between semiotics and logic, Peirce was no doubt highly influenced by the way the scholastic realists understood science, as for instance demonstrated by Emily Michael [1977]. He got his inspiration first and foremost from the medieval juxtapositon of three of the seven free arts into the so-called trivium. The trivium consisted of the disciplines Grammar, Dialectics (or: Logic), and Rhetoric. As demonstrated by Max H. Fisch [1978], Peirce's work from 1865 to 1903 shows a constant development of reflections on the content and application of this tripartition. In the Spring of 1865 he subdivided the general science of representations into 'General Grammar', 'Logic' and 'Universal Rhetorics'. In May the same year he called this division 'General Grammar', 'General Logic', and 'General Rhetorics', and in 1867 it was presented as 'Formal Grammar', 'Logic' and 'Formal Rhetorics'. Twenty years later, in 1897, it had become 'Pure Grammar', 'Logic Proper' and 'Pure Rhetorics'. In 1903 Peirce - within his own now more matured framework - determined the tripartition as 'Speculative Grammar', 'Critic', and 'Methodeutic'. By then it was also clear to him that semiotics - subdivided in that way - can in fact be understood as logic in the broad sense. Altogether Peirce's semiotics can be looked upon as a modernisation of the understanding of logic from the late Middle Ages.

Not only the tripartion, but also several other elements of medieval logic had an impact on Peirce's analyses and his development of semiotics. One example is the tripartition of the subjects of logic into terms, propositions and arguments - a division, which can be found in almost every mediaeval introduction to logic. It was clear to Peirce that this classification was relevant not only within logic (in the narrow sense), but also within both grammar and rhetoric, a fact which had also been recognised by the ancients and the medievals. It should be mentioned, however, that Peirce rejected the idea of completely non-assertoric terms. In his opinion even terms are in general assertoric [CP 2.341].

One of the very obvious differences between mediaeval logic and the logic of later centuries is the rôle of time in logic. In mediaeval logic time was taken very seriously. Words and terms with a temporal content such as 'begin', 'end', 'while' were analysed, and the tenses of the verbs were made the object of endless logical/semantical analyses. Peirce was certainly aware of this, and there are many indications that he realised as one of the earliest modern philosophers and logicians that time could and even should be gradually included in logic.

As the introductory quote above makes evident, Peirce made himself a spokesman for an open and undogmatic understanding of logic. This openness, which was obviously due to his extensive knowledge of classic and scholastic logic, also meant that he would not accept logic as an untemporal science. He could well imagine a new development of a logic, which would take time seriously. Peirce, however, held that logicians around the turn of the century were not ready to (re)introduce time into logic without creating great confusion; not until later would it be possible to introduce the logic of time.

Peirce's prophetical vision of a temporal logic proved to be correct. In the 1950's and 60's A. N. Prior succeeded in re-establishing the logic of time as a proper part of logic. It is obvious that the study of Peirce's philosophy meant a great deal to Prior. In Prior's first great time logical work *Time and Modality* [1957], he gave a brief presentation of the history of the modern logic of time in an appendix; about one fourth of this exposition is

devoted to the importance of Peirce with respect to the development of the new logic of time.

Peirce's philosophy contains features, which could well be interpreted as an emergent logic for events. For example, he defined the notion of a 'Token' as applying to "A Single event which happens once and whose identity is limited to that one happening or a Single object or thing which is in one single place at any one instant of time" [CP 4.537]. As we have already seen Peirce was hesitant about advancing a formal logic of time himself, but nevertheless it is relatively easy in his authorship to find clear ideas which can be used in a presentation of a formal time logic.

In the following we shall first discuss Peirce's conception of time. Then we shall examine those rudimentary elements of a time logic, which can be found in Peirce's work after all. This examination will be followed by a preliminary discussion of Prior's formalisation of those elements. In a later chapter we shall compare these Peircean answers regarding future contingents with a formal version of the Ockham answer discussed in Part One.

PEIRCE'S UNDERSTANDING OF TIME

It is reasonable first to discuss Peirce's understanding of time within mathematics. Peirce was fully aware of the fact that one of his greatest sources of inspiration, the philosophy of Immanuel Kant, had in Anglo-Saxon thinking given rise to an extraordinary linkage of the concept of time with mathematics:

> However, Sir William Rowan Hamilton and De Morgan influenced (the latter only indirectly) by Kant defined mathematics as the science of time and space. This definition never had very wide vogue. It is one of the very worst any science ever received. [NEM, p.594].

Inspired by Kantian thinking, Hamilton had felt that if geometry could be understood as a pure mathematical study of space, then a similar pure mathematical study of time ought to exist. The research program emerging from this conviction can be described as an attempt to establish algebra as the 'science of pure time'. Hamilton encountered many difficulties in that endeavour. In fact, there are several indications that he actually gave up the fundamental idea himself [Øhrstrøm 1985a]. Peirce was very categoric in his rejection of Hamilton's program. He even stated that "it must be an unclear head that cannot see that number and counting have nothing in particular to do with time." [NEM, p.594] However, the validity of that rejection is doubtful. Aristotle had already determined time as "the number of motion with respect to earlier and later" [Physica IV, 220b]. In fact, there seems to be an etymological connection between the Greek words for rhythm and number, respectively. This connection apparently strengthened the belief within Greek philosophy that these two concepts are interdependent, or at least semantically related. At any rate, there is ample historical proof that time and numbers are closely interwoven. Immediate examples are the calendar and the clock. As a matter of fact, in Peirce's own work there are enough examples to support the viewpoint that time is relevant to mathematics. Peirce's first rejection of Hamilton's program was rather injudicious, but as we shall see below the following observation is more convincing:

> Hamilton called algebra the science of Time. But the most remarkable characteristic of time, namely, that the passage from the past to the future is qualitatively different from the passage from the future to the past is not represented in algebra. [NEM, p.9]

According to Peirce, Hamilton's program failed because it did not in its algebra incorporate the temporal asymmetry between the past and the future.

In his comprehensive work *New Elements of Mathematics* [NEM] (which was a rewriting. or perhaps a paraphrase, of his father Benjamin's manuscript), Peirce included a brief chapter

concerned with time. In a letter to William E. Story (dated the 22nd of March 1896), Peirce's stated his motivation for including this chapter as follows: "The science of Time receives a brief chapter, chiefly because it affords an opportunity for studying true continuity" [NEM, p.vi]. The mathematical character of time is defined in this chapter in the following way:

> Time is that by the variations of which individual things have inconsistent characters. Thus, to be alive and to be dead are inconsistent states; but at different times the same body may be alive and dead. [NEM, p.248]

Obviously, this definition is not merely mathematical, but also substantially logical; this can be seen from the way it uses the notion of (in)consistency, and also from the implicit reference to assertions. Moreover, the kind of logic implicit in the definition is a time logic, since it involves assertions which are true at some times, but false at other times. In general, it is clear that to Peirce time was to be understood in relation to events, and it is unlikely that his framework should leave any room for representations of an 'empty time', wherein no change at all would take place. These observations are supported by the following quotation from 1892:

> Time, as the universal form of change, cannot exist unless there is something to undergo change and to undergo change continuous in time there must be a continuity of changeable qualities. [CP 6.132]

Further Peirce defines the past, the present and the future in the following way:

> The present is the existing state of things .... The past is that part of time with which the memory is concerned ... The future is that part of time with which the will is concerned. [NEM, p.248-49]

Obviously, these definitions are not themselves exactly what we would call pure mathematics. Nonetheless, they invite a mathematical discussion of the concept of time, as witnessed by the appeal to the notion of continuity; evidently, such a discussion should on the other hand be influenced by cognition and psychology. The chapter on time is mainly concerned with considerations about temporal continuity. The crucial point is here the gradual change in the course of time. In that connection it becomes very important to distinguish between 'instant' corresponding to the mathematical time and 'moment' which is an infinitesimal duration and which can be used in a mathematical description of the gradual change.

In Peirce's philosophy, experience was a crucial notion, and in that connection he naturally had to discuss time. Any realisation process, as for instance the change from doubt to belief, must involve something temporal, he stressed [CP 7.346]. As Sandra B. Rosenthal [1987] noted, Peirce was aware of the fact that no experience is so limited as not to contain a flow of continuity. Peirce put it like this:

> There is no span of present time so short as not to contain ... something for the confirmation of which we are waiting. [CP 7.675]

In 'The Law of Mind' (1892) he tried to determine the salient features of how we as human beings understand time:

> One of the most marked features about the law of mind is that it makes time to have a definite direction of flow from past to future. The relation of past to future is different from the relation of future to past. [CP 6.127]

This temporal asymmetry is clearly in opposition to the laws of mechanics, which are fully symmetrical with respect to the time co-ordinate - the two temporal directions being no more different in relation to mechanics than two spatial directions. Nevertheless, Peirce maintained that our experience of time is

asymmetrical. Our cognitive relation to the past is certainly different from our relation to the future.

The fact that "future conduct is the only conduct that is subject to self-control" [CP 5.427] was very important for him with respect to a theory of meaning:

> The rational meaning of every proposition lies in the future. How so? The meaning of a proposition is itself a proposition... it must be simply the general description of all the experimental phenomena which the assertion of the proposition virtually predicts. [CP 5.427]

Peirce believed that the power or principle shaping the history of nature is neither coincidence nor necessity, but rather it is love, *agape*: the divine love which the Creator expresses towards creation in the course of time. In this way nature can be viewed as a continuous flow. But it should also be clear that according to Peirce man is not only living in this progressive time. Human time is also tense-oriented, that is, the concepts of past, present and future are essential to the human mind. The past can be characterised as 'facts'. According to Peirce a fact should be understood as a "fait accompli; its esse is in praeterito" [CP 2.84]. Such facts should be viewed as 'now-unpreventable'. But with the future it is a different matter:

> Being in futuro appears in mental forms, intentions and expectations. Memory supplies us a knowledge of the past by a sort of brute force, a quite binary action without reasoning. But all our knowledge of the future is obtained through the medium of something else. [CP 2.86]

The medium mentioned here could for instance be the laws of physics, or nature in general. That is, in some cases the future can be present in its causes, and in these cases we can have knowledge of the future. But in other cases we must confine ourselves to other kinds of law-like statements. It should, however, be mentioned that Peirce did not consider natural laws to be quite as compelling as logical laws. Natural laws he saw as

'habits of nature', and he even accepted the possibility of a "sudden stoppage of everything" [CP 4.547]. He did not consider the possibility of a law of nature against such a sudden stoppage as an argument which should be taken very seriously!

To which extent would Peirce allow that a scientific law can be a medium through which aspects of the future can be known? This is, as far as we can see, an open question. Peirce's views on the relation between time and cognition as well as his idea of time in general are very complex, and we must admit that his statements do not unambiguously point in any single direction. A few quotations should illustrate that:

> For time is itself an organized something, having its law or regularity; so that time itself is a part of that universe whose origin is to be considered. We have therefore to suppose a state of things before time was organized. Accordingly, when we speak of the universe as "arising", we do not mean that literally. [CP 6.214]
>
> The idea of time must be employed in arriving at the conception of logical consecution; but the idea once obtained, the time element may be omitted, this leaving the logical sequence free from time. That done, time appears as an existential analogue of the logical flow. [CP 1.491]

Statements like these are typical for Peirce's philosophy. They form a good inspiration for further speculation regarding the concept of time. It must be admitted, however, that his ideas of time become very complicated when it is added that Peirce apparently believed in what Milic Capek [1991, p.265] has termed a 'self-returning nature of time'. Peirce stated:

> The other question is whether time is infinite in duration or not. If it has no flaw in its continuity, it must, as we shall see in Chapter 4 return to itself. This may happen after a finite time, as Pythagoras is said to have supposed, or in infinite time, which would be a doctrine of consistent pessimism. [CP 1.498]

Peirce formulated similar views in [CP 1.498] and in [CP 6.210]. It is hard to see how the idea of eternal recurrence can fit with the rest of the Peircean thinking. Peirce may in fact have had problems with this question himself, since the 'Chapter 4' to which he refers in the above quotation was apparently never written. We shall leave this issue here.

TIME AND MODALITY

The concept of possibility has always played a great rôle in philosophy, and Peirce is no exception to this rule - on the contrary, it was one of his essential goals to find a suitable definition of 'possibility'. Early in his authorship his attempts at a definition were characterised by semantically negative expressions, but later he emphasised the positive character of the notion. On the 18th of March 1897 Peirce wrote:

> ... my old definition of the possible as that which we do not know not to be true (in some state of information real or feigned) is an anacoluthon. The possible is a positive universe, and the two negations happen to fit in, but that is all ... I found myself arrested until I could form a whole logic of possibility, - a very difficult and laborious task. [CP 8.308]

Later he formulated the positive character of possibility in still stronger terms:

> *Potentiality* is the absence of Determination (in the usual broad sense) not of a mere negative kind, but a positive capacity to be Yea and to be Nay; not ignorance but a state of being ... *Actuality* is the Act which determines the merely possible ... *Necessitation* is the support of Actuality by reason ... [Ms 277, 1908; quoted from Fisch and Turquette 1966:78]

It is a natural consequence of Peircean thought that he in some contexts related modality, including the definition of the

possible, to time or temporality. In order to examine this relation closer we must first scrutinise Peirce's view on the relation between time and reality, which means that we have to start with the ontological foundation for his ideas about the logic of time. Peirce distinguished between three modes of being, which can be understood from the following quotations:

> My view is that there are three modes of being. I hold that we can directly observe them in elements of whatever is at any time before the mind in any way. They are the being of positive qualitative possibility, the being of actual fact, and the being of law that will govern facts in the future. [CP 1.21-1.23]

Thus the three modes of being in Peirce's philosophy are: actuality, possibility and necessity. In 'temporal terms', 'actuality' (understood as the 'given') will cover both the past and the present. The future is thought of as a possibility sphere with certain predetermined incidents (logically necessary or determined by natural law). In this way possibility as well as necessity are both related to the future; and conversely, future events are subdivided into either necessary ones or merely possible ones. In accordance with this binary subdivision, Peirce rejected the idea that the truth about the contingent (and undecided) future could be known beforehand - or indeed, that assertions about the contingent future could at all be meaningfully regarded as having a truth-value. The following statement sums up essential features of Peirce's views:

> That time is a particular variety of objective Modality is too obvious for argumentation. The Past consists of the sum of *faits accomplis*, and this Accomplishment is the Existential Mode of Time. ...the Mode of the Past is that of Actuality. Nothing of the sort is true of the Future.... (The future) is not Actual, since it does not act except through the idea of it, that is as a law acts; but is either Necessary or Possible... [CP 5.459]

Peirce did not define the past as 'necessary', but reserved this definition for 'the preordained'. However, he maintained that at a cognitive level the relation of the present to the past is decisively different from the relation of the present to the future. Whilst in principle any past event belongs to the domain of the memory, we have no possibilities of obtaining a similar insight into future events. On the contrary, the future lies open before us, thus enabling us to influence the forming of the future within certain limits. A similar possibility of influencing the past does not exist:

> I remember the past, but I have absolutely no slightest approach to such knowledge of the future. On the other hand I have considerable power over the future, but nobody except the Parisian mob imagines that he can change the past by much or by little. [CP 6.70]

Peirce furthermore wrote:

> A certain event either will happen or will not. There is nothing now in existence to constitute the truth of its being about to happen, or of its being not about to happen, unless it be certain circumstances to which only a law or uniformity can lend efficacy. But that law or uniformity, the nominalists say, has no real being, it is only a mental representation. If so, neither the being about to happen nor the being about not to happen has any reality at present ... If, however, we admit that the law has a real being, and of the mode of being an individual, but even more real, then the future necessary consequent of a present state of things is as real and true as the present state of things itself. [CP 6.368]

Peirce saw himself as a realist. Truth and reality were for him objective, albeit in a sense differing from classical 'naïve realism'. As pointed out by for instance Harry R. Klocker [1968, p.80 ff], truth is according to Peirce that viewpoint upon which everybody examining the state of things eventually agree, and reality is the object represented by this viewpoint. (In fact, this

should probably be modified even further, into approximately 'that viewpoint upon which everybody examining the state of things *by scientific method* eventually agree'.)

THE DETERMINISM PROBLEM

The determinism problem was one of the problems of primary importance to Prior, and in this problem is included the question of free choice or free will. Strictly speaking Peirce did not leave much latitude for the will itself. On the 18th of March 1897 he stressed in a letter to William James that the will as such is not free to any important extent. The freedom rather antecedes the will and is being established in a state of unstable equilibrium:

> The freedom lies in the choice which long antecedes the will. There a state of nearly unstable equilibrium is found. [CP 8.311]

In the history of logic the problems concerning the freedom of man have often been discussed in theological terms as a tension between man's putative freedom of choice in the face of divine foreknowledge. Now and then Peirce discussed these questions in that context, too. In 1893 he wrote:

> That is to say, they suppose that a man is perfectly free to do or not to do a given act; and yet that God already knows whether he will or will not do it. This seems to most persons flatly self-contradictorary; and so it is, if we conceive God's knowledge to be among the things which exist at the present time. But it is a degraded conception to conceive God as subject to Time, which is rather one of his creatures. [CP 4.68]

In this way Peirce accepted the view of the church fathers that the world is not created in time, but that God in the beginning created the world as well as time. Peirce continued:

> Literal fore-knowledge is certainly contradictory to literal freedom. But if we say that though God knows (using the word knows in a trans-temporal sense), he never did know, does not know, and never will know, then his knowledge in no wise interferes with freedom. [CP 4.68]

The year before Peirce had in the Monist published a survey of what he called 'The Doctrine of Necessity', which he described as "the common belief that every single fact in the universe is precisely determined by law" [CP 6.36], or alternatively:

> The proposition in question is that the state of things existing at any time, together with certain immutable laws, completely determine the state of things at every other time [CP 6.37]

This doctrine he traced back to the Stoics, who according to Peirce linked the doctrine with materialism. He pointed out that the later advances in mechanical physics gave an impetus to the doctrine. On the other hand, it was not generally accepted, exactly because it appeared irreconcilable with the belief in the freedom of the will and the possibility of miracles. Peirce himself argued against mechanical determinism and insisted that in the description of courses of events there would have to be a decisive element of probability, spontaneousness and real possibility. As indicated by John E. Smith [1987] this position fits very well with the viewpoints published by William James some years earlier.

Peirce rejected that conception of science upon which the doctrine of necessity rests by stressing the observation that conclusions drawn from science are never more than probable. He argued that the doctrine presupposed the idea that physical quantities do in fact have mathematical values, an idea which just like the doctrine of necessity itself could not be established by

means of observation. Peirce could not come to terms with a universal mechanical determinism. He felt that especially the origin of life in its infinite complex forms fitted very badly with the doctrine of necessity. Moreover, reflections on consciousness also undermined the doctrine. If the doctrine is accepted without modification, then one must reject the idea of consciousness as a source or ground of real choices and decisions. For all functions of the mind must then be understood as parts of the physical universe, and thus the perception that we are free to do a given act will be reduced to an illusion. In rejecting the doctrine Peirce believed to have made room for consciousness, or in his own words: "room to insert mind into our scheme, and put it into the place where it is needed, into the position, which as the sole self-intelligible thing, it is entitled to occupy, that of the fountain of existence ..." [CP 6.61]

It is worth mentioning that Peirce's refutation of the doctrine of necessity and his accentuation of indefiniteness, coincidence and spontaneousness can be seen as a forerunner of the philosophy prevalent today in the interpretation of quantum physics, as pointed out by Peder Voetmann Christiansen [1988 p.38 ff].

Peirce rejected the notion that indefiniteness should be seen as a degeneration from definiteness. To him, indefiniteness was of primary importance:

> Get rid, thoughtful Reader, of the Ockhamistic prejudice of political partizenship that in thought, in being, and in development the indefinite is due to a degeneration from a primal state of perfect definiteness. The truth is rather on the side of the scholastic realists that the unsettled is the primal state, and that definiteness and determinateness, the two poles of settledness, are, in the large, approximations, developmentally, epistemologically, and metaphysically [CP 6.348]

These observations are, in fact, an ontological counterpart of the position that statements regarding the contingent future cannot be true now (perhaps an additional reason why Ockham is mentioned explicitly in the quote). In a Peircean system, truth

cannot be the only basic concept - if indeed, it can be basic at all. Some kind of theoretical vagueness also has to be involved. Peirce gave this example:

> Again, statisticians can tell us pretty accurately how many people in the city of New York will commit suicide in the year after the next. None of these persons have at present any idea of doing such a thing, and it is very doubtful whether it can properly be said to be determinate now who they will be, although their number is approximately fixed. [CP 4.172]

Even though statisticans can predict the *number* of suicides in New York pretty accurately, they cannot tell *which persons* will commit suicide in the year after the next. It is sufficiently evident that in Peirce's opinion, a proposition like 'Mr. Smith is going to commit suicide in the year after the next' cannot be true now, since Mr. Smith has not yet made up his mind - or, if he had, he might change it. (The only possible exception to this rule would be for us to establish some kind of natural law, which would in the fixed amount of time lead inevitably to Mr. Smith's suicide. But apart from the extreme unlikelihood of finding such a law, if we found it we would not be dealing with the contingent future any more, but with the necessary future.)

THE FORMALISATION OF THE PEIRCEAN IDEAS

Peirce did not make any attempt to formalise his ideas on temporal and modal logic in terms of an operator calculus in the modern sense. However, he made some interesting attempts of relevance in this field, using his so-called existential graphs. This however was not recognised, or at any rate did not had any rôle to play in the development of tempo-modal logic during the 1950s and 1960s. But later it has become widely recognised that Peirce in fact established a general calculating

technique, which comprehends what we today call predicate logic. In part 3 we shall outline the ideas involved, and also discuss how they can be utilised in a computational context. In the following we shall however concentrate on those Peircean ideas, which came to play a rôle for A. N. Prior in his development of tempo-modal logic.

Peirce maintained that logical assertions become true by representing facts. If 'a fact' is understood as 'a portion of reality' small enough to be represented in one single (atomic) assertion, then the assertions of logic will stand as figures representing features of reality.

But how does this conception of logic stand in relation to a changing world? If assertions represent features of reality, then the perception of the single fact must be linked with time. Peirce's insistence on a correspondence between logic and reality presupposes a conception of logic which takes time seriously. In his introduction to the logic of time A. N. Prior analysed Peirce's position, especially with respect to future statements. The analysis showed that in Peirce's framework, a proposition like (i) 'Tomorrow Jane chooses to go to Copenhagen' is equivalent with either (ii) 'Tomorrow Jane necessarily chooses to go to Copenhagen', or (iii) 'Tomorrow Jane possibly chooses to go to Copenhagen'. The system thus makes no room for any concept of plain truth 'in between' 'necessarily' and 'possibly'.

On the basis of his studies of Peirce's philosophy Prior put forward a tense logical system, with which he, by the way, declared himself to be very satisfied. In his own words:

> .. C.S. Peirce's description of the past (with, of course the present) as the region of the 'actual', the area of 'brute fact', and the future as the region of the necessary and the possible. That is why I call this system 'Peircean'. [Prior 1967, p.132]

There is hardly any doubt that Prior's rendition of Peirce's ambitions as regards the logic of time and modality is correct.

We are even able to support this with Peirce quotations, which Prior, in all likelihood, did not have at his disposal:

> A simply assertory Proposition differs just half as much as the assertion of a Possibility, or that of a Necessity, as these two differ from each other. For as we have seen above, that which characterises and defines an assertion of Possibility is its emancipation from the Principle of Contradiction, while it remains subject to the Principle of Excluded Third; while that which characterizes and defines an assertion of Necessity is that it remains subject to the Principle of Contradiction, but throws of the yoke of the Principle of Excluded Third; and what characterizes and defines an assertion of Actuality or simple existence, is that it acknowledges allegiance to both formulae, and is just midway between the two rational 'Modals' as the modified forms are called by all the old logicians. [The Art of Reasoning elucidated, 1910; quoted from Fisch and Turquette 1966, p. 78]

Obviously, this statement makes a distinction between three types of assertions. For ordinary assertions both 'the Principle of Excluded Third':

$$p \vee {\sim}p$$

and 'the Principle of Contradiction'

$${\sim}(p \wedge {\sim}p)$$

are valid. That is also rather to be expected, for by the ordinary laws of (bivalent) logic the two principles are equivalent. But in modal contexts matters are not that simple. For instance, as a rule

$${\sim}(Np \wedge N{\sim}p)$$

applies for assertions containing the necessity-operator N. However, the following is not valid:

$$Np \vee N{\sim}p$$

As for the possibility-operator M it is the other way round:

$$Mp \vee M{\sim}p$$

is valid, but

$${\sim}(Mp \wedge M{\sim}p)$$

in not valid in general. - By analysing Peirce's way of thinking and transferring this into the modern logic of time, we arrive at the conclusion that the following formula must hold for any proposition p:

$${\sim}(F(x)p \wedge F(x){\sim}p),$$

whereas its 'excluded middle' analogue

$$F(x)p \vee F(x){\sim}p$$

does not hold in general. - This is due to the fact that both assertions, $F(x)p$ and $F(x){\sim}p$, can be false, if they represent a pair of statements about the contingent future. On the other hand, if they are taken to represent statements about the necessary future, involved precisely one of them is true - that is, the law of excluded middle holds in that case.

It seems, however, that this theory gives offence to the intuition on which everyday language is based. We normally accept a concept of future which is logically between possible future and necessary future, and which certainly makes no distinction between $F(x){\sim}p$ and ${\sim}F(x)p$. For instance, we may well wish to assert that 'Tomorrow Jane chooses to go to Copenhagen' without saying that this choice is necessary. And if Jane the next day does choose to go to Copenhagen, we shall feel justified in having made the assertion yesterday - that is, we would be inclined to consider it as having been true, when we made it.

In any case it is clear that Peirce did not depart from the classical logic, but rather added a great deal to it. This attitude is shown clearly in the following remarks:

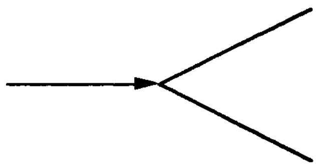
> I have long felt that it is a serious defect in existing logic that it takes no heed of the limit between two realms. I do not say that the Principle of Excluded Middle is downright false; but I do say that in every field of thought whatsoever there is an intermediate ground between positive assertion and positive negation which is just as Real as they. Mathematicians always recognize this, and seek for the limit as the presumable lair of powerful concepts, while metaphysicians and old fashioned logicians, the sheep and goat separators - never recognize this. The recognition does not involve any denial of existing logic, but it involves a great deal of addition to it. [Letter to William James, dated Feb. 26, 1906]

It would have been interesting to learn more about what kind of 'additions' to 'the existing logic' Peirce had in mind. From what he said about the principle of excluded middle and the law of contradiction it seems very likely that he had in mind some kind of operator logic, as the one we have presented here. One might even with some justification conjecture that Peirce realised how a distinction between $\sim F(x)p$ and $F(x)\sim p$ would become necessary, when it comes to formulating a logic of time. Perhaps he was thinking of the need for this kind of distinction, when he stated that the introduction of temporal modifications of the forms of logic would result in great confusion, and that logic had to be developed further before it could be done [CP 4.523].

SOME FORMALITIES OF THE PEIRCEAN SOLUTION

We shall now finally sketch Prior's 'Peircean system' (it will also be described in chapter 2.5 and subsequent chapters in increasing detail). The essential feature of this system can be

explained in terms of the old Aristotelian example about the possible sea-fight tomorrow. According to Peirce's ideas we can model the future in the following way:



In the Peirce-model it makes no sense to speak about 'the true future' as one of the possible futures. There is no future yet. Let s stand for 'there is a sea-battle going on', and let us make the reasonable assumption that 'tomorrow it will be the case that s' is contingent. Then in this model $F(1)s$ as well as $F(1){\sim}s$ are false, whereas ${\sim}F(1)s$ and ${\sim}F(1){\sim}s$ are both true. There is certainly a tension between this hall-mark of the system and the intuition normally involved in everyday reasoning.

Prior realised that according to the Peirce-solution we cannot infer that *there was going to be a sea-battle* from the fact that *there is a sea-battle going on*, although it certainly does follow that *there will have been one*. [Prior 1957a, p.95]

That is, in the Peirce-model one must accept that even if s, 'there is a sea-battle going on', is given (true), we cannot infer $P(1)F(1)s$. Therefore, $q \supset P(x)F(x)q$, is not a thesis in the Peirce system. On the other hand, it should be obvious that in this system, the proposition schemas

$$F(x)P(x)q \supset q$$
$$P(x)F(x)q \supset q$$

are generally valid.

We may sum up these features by noting with Prior that in the Peirce model $F(x)q$ is understood in the strong way, i.e. as "it is bound to be the case after x time units that q" [Prior 1969, p.329]. Moreover, future contingents cannot be known 'now', and

hence there cannot be any true statements about the contingent future. We have seen that in this system the statement 'there will be a sea-battle tomorrow' cannot be true today, for there is no unique future, but rather a number of alternative, possible futures.

The basic question concerns the interpretation of expressions regarding the future: can it be maintained with conceptual and logical consistency that '(some event) *E* will happen', whilst distinguishing this statement from '*E* could happen', and '*E* will necessarily happen'? We have shown how in the Peircean System the plain 'tomorrow' becomes equivalent to 'necessarily tomorrow', or 'possibly tomorrow'. But we have also previously seen how the Ockham view on statements permits a differentiation between actual, possible and necessary future, and hence a differentiation between 'tomorrow', 'possibly tomorrow' and 'necessarily tomorrow'. This fundamental question of the status of the contingent future has certainly not been definitively settled (and perhaps can never be). The debate between Ockhamists and Peirceans goes on. Many things indicate that the discussion about which position to prefer is actually a question about the very understanding of logic. Later we shall also compare Peircean answers regarding future contingents with the Ockhamistic answers.

2.3. ŁUKASIEWICZ'S CONTRIBUTION TO TEMPORAL LOGIC

In a series of articles during the 1920's and 30's the famous Polish logician Jan Łukasiewicz advocated a particular interpretation of Aristotle's discussion of the status of sentences about the contingent future, as developed in his 'sea-fight' example (from *De Interpretatione* chapter IX). Łukasiewicz' interpretation crucially rests on a rejection of the principle of bivalence. In fact, this kind of interpretation was not new, but had been formulated as early as by the Epicureans. However, Łukasiewicz presented this position more clearly than had ever been done before, and developed it with the aid of modern symbolic logic.

Now it is clear that philosophical determinism goes nicely with some tempo-modal logical systems, and conversely; but on the other hand, a tempo-modal system can be constructed so as to allow for indeterminism. Łukasiewicz used his interpretation of Aristotle and the status of sentences about the contingent future as an argument against logical determinism and in favour of logical indeterminism, for which he declared his wholehearted support. He defined (logical) determinism as the assumption that

> If A is B at time t; then it is true at any time before t, that A is B at t. [McCall 1967, p. 22]

Generally speaking, determinism thus becomes equivalent to a thesis of omnitemporal truth, since 'A is B at time t' is identified with 'it is true for any time t_1, that A is B at time t'; the restriction in the above quote that t_1 be earlier than t disappears in a fuller development, as will be shown below.

Let p stand for the statement 'A is B', the expression $T(t,p)$ for 'p is true at time t', and $(t_1 < t)$ for 't is earlier than (before) t'; then Łukasiewicz's rendition of determinism can be symbolised as

(D) $(T(t,p) \wedge (t_1 < t)) \supset T(t_1,T(t,p))$

On Łukasiewicz' interpretation, Aristotle's considerations in the sea-fight example were intended to show that (D) follows from two common (logical) presuppositions. Firstly, the principle of bivalence which can be expressed in the following way:

> (B) For any time t and any proposition p: either $T(t,p)$ or $T(t,\sim p)$, but not both.

If (B) holds p and $\sim p$ cannot both be true at the same time, but one of them has to be true and the other false. This means that the following two formulae

(B1) $\sim (T(t,p) \wedge T(t,\sim p))$
(B2) $T(t,p) \vee T(t,\sim p)$

hold for any p and any t. Consequently

(C1) $\sim T(t,p) \equiv T(t,\sim p)$
(C2) $\sim T(t,T(t_1,p)) \equiv T(t,T(t_1,\sim p))$

Łukasiewicz furthermore considered the principle expressed in the following Aristotelian statement:

> ... if [a certain thing] was white or was not white, then it is true to confirm or deny it. [18a39].

The Aristotelian assumption that if it was true that *X* is *Y* then it is true that is true that *X* was *Y*, can according to Łukasiewicz be translated as the following principle:

(P) $(T(t,p) \wedge t< t_1) \supset T(t_1,T(t,p))$

The difference between (P) and (D) is thus merely rooted in the before/after relation between t and t_1. The proof that (B) and (P)

implies (D) can be done indirectly. Assume that (D) is invalid i.e. that

$$T(t,p) \wedge t_1 < t \wedge \sim T(t_1, T(t,p))$$

holds for a proposition p and for two times t and t_1. By (C2) this is equivalent to

$$T(t,p) \wedge t_1 < t \wedge T(t_1, T(t,\sim p)).$$

By applying (P) we get:

$$T(t,p) \wedge t_1 < t \wedge T(t, T(t,\sim p))$$

which assuming $T(t,T(t,p)) \equiv T(t,p)$ must be equivalent to

$$T(t,p) \wedge t_1 < t \wedge T(t,\sim p).$$

This clearly contradicts (B1), so we conclude that it is possible to infer (D) from (P) and (B1-2)!

Łukasiewicz suggested that the principles embodied by the above theorems (D, B, and P) were the underlying tenets, respectively the implications of the Aristotelian text. This means that Aristotle in order to avoid determinism had to restrict the validity of bivalence. Nevertheless, Łukasiewicz had to admit that the putative limitations of this sacred principle are by no means self-evident in Aristotle's discussion; the very need for interpretation and 'reconstruction' bears witness to this observation! Indeed, Aristotle does not seem to have been definitive in his attitude towards the principle of bivalence.

Whichever way Aristotle himself is to be understood, Łukasiewicz's solution to the problem of sentences about the contingent future and the associated problems with determinism was very much inspired by the Aristotelian analysis. His solution, then, was to consistently reject the principle of bivalence by introducing a third truth value. This

truth value, 'undetermined', is applied to contingent propositions regarding the future [McCall 1967, p. 64]. For instance, a proposition stating that there will be a sea-fight tomorrow can be assigned the truth value 'undetermined' today. This is because today it is not given or definitely determined whether the sea-fight is actually going to take place tomorrow or not.

Łukasiewicz' interpretation was disputed by Prior [1962, p. 240 ff.], who pointed out a significant difference between Łukasiewicz's trivalent logic and Aristotle's text: according to Aristotle it is true already today, that there *either* there will *or* there will not be a sea-fight tomorrow, even though the truth value of the two constituents of the disjunction are separately unknown, or possibly 'undetermined'. This contradicts one important and presumably inevitable property of Łukasiewicz' three-valued logic: namely the fact that the truth value of the disjunction of two (separately undetermined) propositions is according to Łukasiewicz 'undetermined', i.e. $(p \vee q)$ is undetermined for p undetermined and q undetermined. For this reason it must be concluded that Łukasiewicz' trivalent logic does not provide a convincing basis for the interpretation of Aristotle's sea-fight example and the associated logical and philosophical problems. As we have seen in part I, Nicholas Rescher's [1968] so-called 'medieval interpretation' seems far more promising, although it has to be admitted that a distinction between the principles of bivalence and 'tertium non datur' can be read into the Aristotelian text (see e.g. [Andersen & Faye 1980]).

We have to follow Prior in his refutation of Łukasiewicz' interpretation, and we may add that on quite different grounds we ourselves - like many others - are uncomfortable with the idea of a trivalent logic. But we could not possibly mention the name and part of the work of Jan Łukasiewicz without saying a word in praise of this great logician - and wise philosopher, too.

First of all, Łukasiewicz made a number of specific contributions to the development of formal and mathematical logic - the most well-known one being his 'Polish Notation', which is superior to standard logical (infix) notation by being syntactically unambiguous without the aid of parentheses. Even

though this notation (which Prior also used) has failed to establish itself as a standard, it has proved its practical worth at least within computer science.

Secondly, although Łukasiewicz can not be said to have developed a genuine temporal logic, he was the first logician to actually work out a symbolic calculus sensitive to some of the logical and philosophical problems associated with time and tense; to that extent he anticipated Prior's work by 30-40 years.

Thirdly, and in our opinion most importantly of all, Łukasiewicz was one of the first significant logicians to relate modern symbolic logic to Classical and Medieval discussions of logic. Together with a number of other prominent logicians, including A. Tarski, he managed to establish a fruitful Polish environment which took a particular interest in the history of logic, especially Ancient and Scholastic. It is worth noting the manner wherein modern symbolic logic was related to the historical sources: Łukasiewicz and his associates did not merely apply the former *to* the latter, they also took direct inspiration *from* the latter for the development of the former. The way in which Łukasiewicz related Aritotle's text in *De Interpretatione*, Chapter 9, to a trivalent logic is a first class example of this. Thus these Polish logicians established the very paradigm, within which Prior himself obviously worked - and on which, we may add, this book is also based.

Perhaps Łukasiewicz's most remarkable achievement within this paradigm was his investigation of the history of the propositional logic [1935]. One of Łukasiewicz's students during the 1930s was J. Salamucha, who in 1935 published a book on Ockham's propositional logic. We may conclude this section by quoting a passage from the German translation of that book, which neatly typifies the way in which this group of Polish logicians thought and worked:

> Wir haben bei Ockham, wie bei fast allen mittelalterlichen Autoren, neben der katagorischen Syllogistik, in der Aussagen und Aussagenfunktionen von Typos: A ist B, auftreten, noch andere Syllogistiken; vor allem entwicklet sich nach Aristoteles die Syllogistik der modalisierten

> Aussagen, ferner die Syllogistik der Aussagen, in denen das Zeitwort die Form der Zukunft oder der Vergangenheit hat.... [Salamucha 1950]

The quote is typical for this group of pre-war Polish logicians. There was an awareness of the fact that logicians in the Ancient times and in the Middle Ages had analysed tensed propositions and arguments, and that such analyses were still relevant; at the time, however, these were given less priority than other kinds of logical analysis.

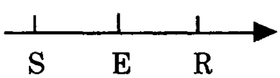
2.4. A THREE-POINT STRUCTURE OF TENSES

In his *Elements of Symbolic Logic* [1947], Hans Reichenbach put forth a description of tenses which was to have a significant impact upon the linguistic community. Reichenbach suggested that in order to understand how tenses work we must consider not only the time of utterance, and the time of the event in question, but also a 'point of reference'.

To understand the idea of this three-fold distinction, it is probably best first to consider the future perfect, as in 'I shall have seen John'. This sentence clearly speaks of a certain event, namely 'my seeing John'; but it is also clear that it directs us to a future time *different* from the time of the (expected) event - namely a time prior to which the event has already occurred. Thus, we must distinguish between the time of the event and the time to which the sentence refers. Reichenbach called the former 'point of the event' and the latter 'point of reference', symbolised by E and R, respectively. Furthermore, both must of course be determined with respect to the time of utterance, the 'point of speech' S.

Armed with these distinctions Reichenbach could give the following diagram for the future perfect:

Future Perfect
I shall have seen John

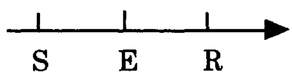


A quite similar analysis can be given for the past perfect 'I had seen John'. These two tenses, then - the past perfect and the future perfect - establish the prima facie case for distinguishing between E, S, and R in the description of tenses. However, if the difference between E and R is crucial in explaining the past perfect and the future perfect, it is precisely the *coincidence* between one or more of E, R, and S, which is crucial in explaining some of the other tenses. Indeed, what particularly impressed linguists was the elegant and concise account of the

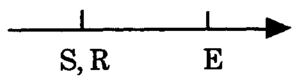
difference between the simple past and the present perfect which Reichenbach could give on the basis of the three-fold distinction.

In grammars of English, six tenses are standardly recognised; the diagram for each of these can be seen in this figure (cf. [Reichenbach 1947, p. 290]):

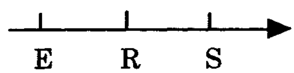
Future Perfect
I shall have seen John



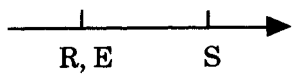
Simple Future
I shall see John



Past Perfect
I had seen John



Simple Past
I saw John



Present Perfect
I have seen John

Present
I see John

On this account, the crucial difference between the simple past and the present perfect is determined by the relative 'position' of the reference point. In the case of the simple past, the diagram clearly suggests that the point of reference coincides with the point of the event. Thus the sentence 'I saw John' clearly refers to the past, but it makes no discernible distinction between the time of the event - E - and the time from which this event is seen, i.e. the reference time R. In the case of the present perfect, the event is also situated in the past, but here, the point of reference coincides with the point of speech.

Reichenbach's system makes a rather strong prediction about the notion of tenses, logically as well as grammatically. If tenses are in general to be construed as a three-point structure, the

possible arrangements of this kind of structure must exhaust the set of possible tenses. In principle, Reichenbach's systematisation allows for 13 different tenses; he only regarded nine of these as significantly different, though:

> If we wish to systematize the possible tenses we can proceed as follows. We choose the point of speech as the starting point; relative to it the point of reference can be in the past, at the same time, or in the future. This furnishes three possibilities. Next we consider the point of the event; it can be before, simultaneous with, or after the reference point. We thus arrive at 3 • 3 = 9 possible forms, which we call *fundamental forms*. Further differences of form result only when the position of the event relative to the point of speech is considered; *this position, however, is usually irrelevant* [our italics] [p. 296]

The fact that Reichenbach considered the relative positions of E and S as basically irrelevant explains a slight oddity about his diagram for the future perfect. The sentence 'I shall have seen John' would also seem to be true even if the speaker has in mind an event which has already occurred - that is, the structure would be E----S----R (this is perhaps a less natural reading, but quite possible). However, the above quotation makes it clear that according to Reichenbach, there is no important difference between E----S----R and S----E----R. Indeed, in summing up the possible tenses he explicitly aligns

S----E----R
S, E----R
E----S----R

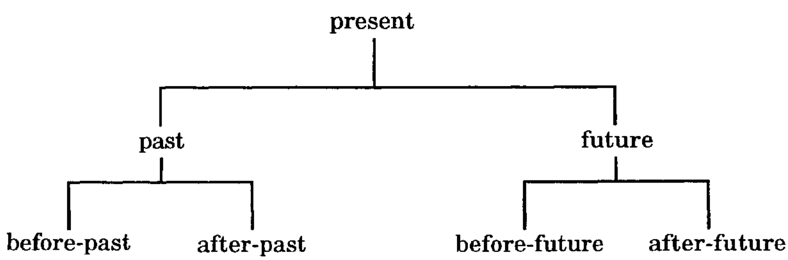
under the common heading of 'future perfect'. A similar account is given for R----E----S, R----S----E, and R----S,E, which he collects under the heading 'posterior past'. None of the six traditional tenses corresponds to posterior past, but it can be stated by some transcription, as in 'I was to see John once more' or 'the letter was to cause her great anxiety'.

OTTO JESPERSEN ON TIME AND TENSE

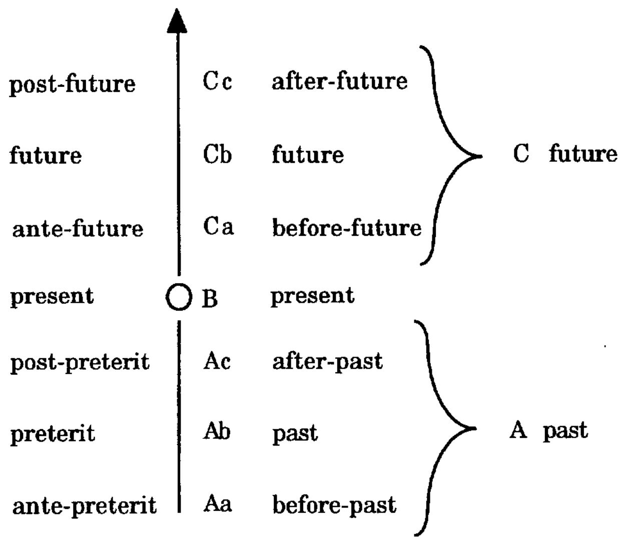
According to Reichenbach, the idea of a three-point structure for tenses had already been suggested by the great Danish linguist Otto Jespersen (1860-1943):

> In J. O. H. Jespersen's excellent analysis of Grammar (*The Philosophy of Grammar*, H. Holt, New York, 1924) I find the three-point structure indicated for such tenses as the past perfect and the future perfect (p. 256), but not applied to the interpretation of the other tenses. This explains the difficulties which even Jespersen has in distinguishing the present perfect from the simple past (p. 269). He sees correctly the close connection between the present tense and the present perfect, recognizable in such sentences as 'now I have eaten enough'. But he gives a rather vague definition of the present perfect and calls it 'a retrospective variety of the present'. [Reichenbach 1947, p. 290]

In fact, when looking into Jespersen's text it takes some consideration to see how the three-point structure can be said to be suggested here. Jespersen's book deals with time and tense in two chapters. In these chapters there are no explicit statements that such a three-fold distinction has been made, nor do they make any clear qualitative - let alone terminological - distinction between speech time and reference time. However, Jespersen first suggests that we should basically consider seven different possible tenses. For these he introduces this diagram, which must be what Reichenbach sees as the first suggestion of a three-point structure:



However, the diagram is immediately reworked into the one below, since "it is much better to arrange the seven "times" in one straight line... For there can be no doubt that we are obliged (by the essence of time itself, or at any rate by a necessity of our thinking) to figure to ourselves time as something having one dimension only, thus capable of being represented by one straight line" [Jespersen, p. 256]. Such a representation - in which an indication of a three-point structure is easier to see - is given on p. 257:



Jespersen here uses the terms before-past, past, etc., in an ontological sense, i.e. concerning the 'essence of time', whereas the terms ante-preterit, preterit, etc., are the corresponding grammatical terms. The four 'subordinate times' can be briefly described as follows:

a) before-past (ante-preterit): corresponds to the past perfect;
b) after-past: is described by periphrastic forms such as 'The letter was to cause anxiety';
c) before-future: corresponds to the future perfect;

d) after-future: is described by periphrastic forms such as 'I shall be going to see John'.

The closest Jespersen gets anywhere in his text to describing a Reichenbach-like three-point structure is in his explanation of the figure above:

> This figure, and the letters indicating the various divisions, show the relative value of the seven points, the subordinate "times" being orientated with regard to some point in the past (Ab) and in the future (Cb) exactly as the main times (A and C) are orientated with regard to the present moment (B). [p. 257]

Clearly, in addition to the present (the time of utterance), two more 'points of orientation' are brought into the picture. However, Reichenbach's implementation of this idea differs from Jespersen in three important respects:

1) Obviously, Jespersen considers three points - rather than just two - to be relevant only for the subordinate tenses. This is crucial, of course, for "the difficulties which even Jespersen has" in explaining the difference between the simple past and the future perfect - in the manner suggested by Reichenbach.

2) Jespersen makes no qualitative or terminological distinction between the two points besides the present.

3) An exhaustive system of tenses, respectively, times, cannot be constructed on the basis of the above distinctions. Jespersen says: "The system thus attained seems to be logically impregnable, but, as we shall see, it does not claim to comprise all possible time-categories nor all those tenses that are actually found in languages" [p. 257]. This is obviously in contrast to Reichenbach, who, as we have already seen, proposes his three-point system as a comprehensive account of all possible tenses.

It is a very interesting fact that one tense which Jespersen considers to be 'beyond' his seven-tense system is exactly the present perfect:

> The system of tenses given above will probably have to meet the objection that it assigns no place to the perfect, *have written*... This, however, is really no defect of the system, for the perfect cannot be fitted into the simple series, because besides the purely temporal element it contains the element of result... it represents the present state as the outcome of past events, and may therefore be called a retrospective variety of the present. [p. 269]

Furthermore, Jespersen points out another significant difference between the simple past and the present perfect, namely that the former is about some definite point in the past, as opposed to the latter. Indeed, in English this difference is taken so strictly that it "does not allow the use of the perfect if a definite point in the past is meant, whether this be expressly mentioned or not" [p. 270]. This is in contrast to some other languages, e.g. German and Danish, the latter of which tolerates combinations like "jeg har set ham igår (I have seen him yesterday)" [p. 271].

It appears, then, that Jespersen considered the present perfect to be a tense which could not be fitted into his general 'structure of time' with corresponding tenses (the diagram on p. 257). Or perhaps we should rather say that in this structure it would not be *wrong* to place the present perfect under *Ab*, together with the simple past (preterit) - but this would not be *sufficient* to describe it. The reason for this is that the present perfect bring in an element which is not strictly temporal (the element of result). Now Reichenbach, on the other hand, in his generalised framework did manage to give a clear formal distinction between the simple past and the present perfect. Therefore, his system seems to be an improvement of Jespersen's ideas. But of course, this only holds provided that his generalisation is also otherwise logically and linguistically tenable. We shall now try to assess these questions.

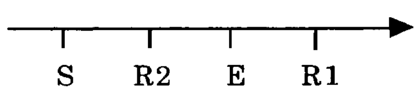
PRIOR, JESPERSEN AND REICHENBACH

For all its intuitive elegance, it is clear that Reichenbach's formalism is very limited. It is certainly not a complete calculus, but at best it could be seen as a suggestion of some guidelines along which such a system could be constructed. However, even when measured on its own terms the system harbours severe difficulties.

As we have seen there is a difference between Jespersen and Reichenbach in that the latter makes a sharp distinction between 'point of reference' and 'point of event'. This is the very move on which the general viability of Reichenbach's systematisation rests - as well as its accounts of the individual tenses. One who clearly saw this was Prior, who in [1967] discussed the precursors of tense logic. Herein he gave Reichenbach some credit for his observations, but then went on to state that "Reichenbach's scheme, however, will not do as it stands; it is at once too simple and too complicated" [1967, p. 13]. The main target of Prior's attack was exactly the sharp distinction between 'point of reference' and 'point of event'. Consider a complicated future tense like this one:

'I shall have been going to see John'.

This sentence is perhaps not very natural, but it is grammatically correct, and it does express a tense-relation for which we must be able to account. It is not too hard to see that to describe this tense, we in fact need two points of reference. Prior's 'Reichenbachian' diagram for this case looks like this:



So, for such a tense the Reichenbachian framework would have to be extended to allow for two points of reference; and in

general, an arbitrary number of 'reference points' might be needed. Prior could therefore observe that

> ... once this possibility is seen, it becomes unnecessary and misleading to make such a sharp distinction between the point or points of reference and the point of speech; the point of speech is just the *first* point of reference. (This, no doubt, destroys Reichenbach's way of distinguishing the simple past and the present perfect; but that distinction needs more subtle machinery in any case.) [1967, p. 13]

It is crucial for Reichenbach's system that three points of time should always be taken into consideration. But we have just seen that this may sometimes be too little; and, as the quotation also suggests, it is sometimes too much. For in the account of, say, the simple past - in terms of an R,E----S diagram, where R = E - why should we accept that there is really more than two temporal indicators involved? And even more so, why should we accept such a thing for the present S,R,E (where S=R=E)? Only cogent logico-linguistic reasons should make one accept that there are three temporal indicators at play in these cases. But referring to the fact that Reichenbach's account apparently explains the difference between the simple past and the present perfect is at best circumstantial evidence; for it *explains* this difference only if the distinctions are valid beforehand.

Incidentally, these observations also show that the Reichenbach framework really ought to distinguish between on one hand the *temporal indicators* - or *concepts* - of 'event', 'reference' and 'speech', and on the other hand the *points of time* which they 'indicate'. Thus for instance, if the event E occurs at t, we might say that $\tau(E)=t$. Only thus can a diagram like

$$\tau(R), \tau(E)\text{----}\tau(S)$$

make a meaningful distinction between more than two indicators. Here, R and E are co-extensive with respect to their time-parameter, but they must be assumed to be intensionally different (i.e. $\tau(R)=\tau(E)$, but $E \neq R$).

As we have already seen Jespersen did not assume that three points of time were relevant for the tenses in general; rather, in his system such a structure is applicable only to the subordinate times. For the past and for the future, only one point of 'orientation' besides the present is taken into consideration - and for the present, only the present is relevant. Moreover, even when a three-point structure becomes relevant, there is no suggestion that the two points besides the present qualitatively differ from each other. In all these respects, it is Prior - rather than Reichenbach - who is in agreement with Jespersen.

To be true, Jespersen's system does not foresee a multiple-point structure as Prior does; but then again, Jespersen explicitly stated that his arrangement of the seven-tense system was not exhaustive. The divisions which he does make are, however, more naturally expressed in terms of tense logic than in terms of the three-point structure: for instance, we have the following correspondences (assuming linear discrete time):

Aa: before-past (past perfect)	PPq
Ab: past (simple past)	Pq
Ac: after-past	PFq

These tense-logical forms are really closer to Jespersen's system than the three-point structures. In each case, the number of tense-operators clearly agrees with the number of 'points of orientation' considered relevant by Jespersen. Forms where still more 'points of orientation' are needed, as in 'I shall have been going to see John', can be represented by tense-logical formulae with a corresponding number of operators, e.g. $FPFq$. Obviously, these tense-logical forms also agree with Jespersen in making no qualitative differences between the corresponding 'points of orientation'.

One minor discrepancy should be mentioned: the tense-logical form PFq differs slightly from the category *Ac*, which in Jespersen's diagram seems unambiguously situated in the past. PFq, on the other hand, may also be true if q takes place at the present moment or even at a future moment. However, this does not *contradict* *Ac*, but is simply more general. As far as we can

see, *PFq* actually describes tense-constructions of the form 'it was to be that q' better than *Ac*. For it is clear that such linguistic forms *can* also refer to an event q which is in the 'absolute future' (even though this may be rare in actual language use). But if somebody insists that this category *Ac* must refer strictly to the past, we could obtain this by a metrical tense-logical form, as in $P(n)F(m)q$, where $n > m$.

We mentioned earlier three differences between Jespersen and Reichenbach - having to do with (a) whether three-point structures should always be used, (b) whether there are any qualitative differences between the different points involved, and (c) whether the respective systems are exhaustive or not. The previous paragraphs should have made it clear that on the first two points Prior is obviously much closer to Jespersen - they both answer in the negative - than is Reichenbach, who confirms both of these points. As for (c), it is at least clear that Jespersen did not consider his seven tense system to be linguistically complete, but there are some indications that he considered it to be logically, or conceptually, exhaustive. Nevertheless, he clearly did not think that all linguistically realised tenses could be uniquely captured by his system, as opposed to Reichenbach's belief in his 9 tenses. But neither Jespersen nor Reichenbach had available formal tense logic. In this discipline, it has been made clear that the number of tenses depends on several assumptions about the structure of time (one result by Prior and Hamblin, yielding 30 different tenses on certain given assumptions, is mentioned in the next chapter). Jespersen's openness in this respect, however, goes better with Prior's findings than Reichenbach's strong prediction of just 9 (or 13) possible tenses.

Reichenbach was a brilliant mind, and many of his results - also on the philosophy of time - have had lasting value. Fairness demands that this be acknowledged, and in the case of his 'three-point structure' it must at least be admitted that for its day it was an elegant and advanced proposal. But its real deficiencies together with its very success made it counter-productive - Prior considered Reichenbach's work in this respect as an impediment rather than a help in the development of tense logic.

Apparently, Prior did not question that Reichenbach's ideas had essentially originated in Jespersen's work. In fact, he even accepted that "Jespersen only used this 'three-point structure' to explain these two tenses [future perfect and past perfect]..." [1967, p. 12]. However, the discussion of Jespersen should have made it clear that this is oversimplifying matters somewhat, since all four "subordinate times" depend on some kind of three-point structure. It is true, however, that only two of these times correspond directly to traditionally recognised tenses. Nevertheless, the fact of the matter is that Jespersen's ideas in many ways seem more compatible with tense logic than with Reichenbach's system, a fact which Prior could well have put to good use.

2.5. A.N. PRIOR'S TENSE-LOGIC

The history of tensed logic proper began with Prior's insight: Tensed propositions are propositional functions, with times as arguments. [Bas C. van Frassen 1980]

A. N. Prior must be said to have laid the foundation for modern tense-logic. He revived the medieval attempt at formulating a temporal logic for natural language. Therefore his work also established a paradigm applicable to the exact study of the logic of natural language. Prior held that logic should be related as closely as possible to intuitions embodied in everyday discourse, and his tense logic can indeed account for a large number of linguistic inferences. In the 1950's and 1960's he laid out the foundation of tense-logic and showed that this important discipline was intimately connected with modal logic. Prior also argued that temporal logic is fundamental for understanding and describing the world in which we live. He regarded tense and modal logic as particularly relevant to a number of important theological problems. Using his temporal logic Prior analysed the fundamental question of determinism versus freedom of choice.

Arthur Norman Prior was born in Masterton, New Zealand, on December 4th., 1914. His mother died a fortnight after his birth. His father was a doctor and a medical officer during the First World War, and Prior was brought up by his aunts and grandparents. Both of his grandfathers were Methodist ministers.

Prior went to Otago University at Dunedin in 1932. He set out to study medicine, but after a short time he instead went into philosophy and psychology. In 1934 he attended Findlay's courses on ethics and logic. Through Findlay Prior became interested in the history of logic and was introduced to Prantl's textbooks. His M.A. thesis was devoted to this subject. In 1949 Prior wrote about Findlay: "I owe to his teaching, directly or indirectly, all that I know of either Logic or Ethics" [Kenny p. 323].

Prior was brought up as a Methodist, but while he was a student he came to consider Methodistic theology too unsystematic,

and he became a Presbyterian. He also became a very active member of the Student Christian Movement (SCM). In the years about 1940 he found himself in a crisis of belief. During these years he wrote the article 'Can religion be discussed?'(1942), in which he advocated an almost atheistic position. This view, however, does not seem to have lasted very long. He continued to treasure his theological library and to join the work of the SCM [Kenny p. 326]. Later in his life, however, he became an agnostic.

It is very likely that Prior's abandoning of Christianity and his becoming an agnostic was related to the problems concerning freedom and time. He was acutely aware of the fact that a number of significant Christian thinkers in the course of history had attacked or criticised the idea of free will. In a paper entitled 'Determinism in Philosophy and Theology' [DPT] (probably written in his Calvinist period), he formulated this in the following way:

> It is extremely rare for philosophers to pay any great attention to the fact that a whole line of Christian thinkers, running from Augustine (to trace it no further back) through Luther and Calvin and Pascal to Barth and Brunner in our own day, have attacked free will in the name of religion. [DPT, p. 1]

Prior added that for instance Jonathan Edwards, who produced a novel defence of Calvinism in 18th-century New England, did it by "demonstrating the absurdity of free will itself" [DPT p. 1]. However, even if we accept that the idea of free will is illusory (and at the time of writing DPT, Prior seems to have accepted this, in contrast to his later convictions), the ordinary perception of freedom and of guilt has to be explained:

> Even those of us who accept a straightforward determinism have to give some account of men's feeling of freedom, and their feeling of guilt; ... [DPT, p. 2-3]

> This state of inner conflict between two parts of the self, in which we feel both responsible and enslaved, is also one to which no one can be a stranger ... [DPT, p. 4]

Even so, Prior felt that a Christian had to be a determinist, and that the believer must accept that we are guilty of that which we are totally helpless to alter [DPT, p. 2]. It appears that Prior accepted Edward's and others' argument that the doctrine of God's infallible and complete foreknowledge is incompatible with the contingency of future events. "I must confess I can't see that foreknowledge is compatible with preventability", he said [IWB, p. 12]. Prior clearly understood that foreknowledge should not itself be seen as the cause of that which is foreknown, but rather as an effect. But what has got so far as to have effects is surely "beyond stopping", he pointed out [IWB, p. 12]. The only way out of this for anyone who wants to accept the doctrine of divine foreknowledge appeared to be Thomas Aquinas' idea of atemporal knowledge. Thomas "taught that God doesn't experience time as passing, but has it present all at once. In other words, God sees time as tapestry" [SFTT, p. 2]. This solution was not at all attractive to Prior, since it seems to be in conflict with the reality of tenses. Moreover, atemporal knowledge cannot be foreknowledge in the strict sense. Prior's own view was that God "cannot know the answer to the question "How will that person choose?" because there isn't any answer to it until he has chosen" [SFTT, p. 3]. This position of course suggests that Prior at some stage had adopted an indeterministic position. That in turn would mean that a full Christian faith could no longer be held, provided that Christianity implies a full forekonowledge by God, and that such foreknowledge is incompatible with the notion of free will.

In 1943 he married Mary. From 1946 to 1958 he taught philosophy at Canterbury University College in New Zealand. In 1953 he became a professor of philosophy. In 1949 his book *Logic and the Basis of Ethics* had been published. After that time he became even more interested in logical problems. During 1950 and 1951 he wrote a manuscript for a book with the working title *The Craft of Logic*. This book was, however, never

published as a whole, but in 1976 P. T. Geach and A. J. P. Kenny edited parts of it. In the first chapter of the book, *Propositions and Sentences*, the author among other things analysed Aristotle's view on some of the problems concerning time and tense. Prior found that according to the ancient as well as the medieval view a proposition may be true at one time and false at another. He described this view in the following way:

> ... the statement or opinion that someone is sitting will be true so long as the person in question is in fact seated, and will become false - if it is persisted in - as soon as he rises. [Prior 1976b, p. 38]

In the following years Prior worked mainly on questions in the history of logic. From 1952 to 1955 he had seven articles on the history of logic published. Four of these were concerned with Medieval logic and one with Diodorean logic. His interest in the history of logic is also evident in his *Formal Logic*, published in 1955. According to Mary Prior his resurging interest in the history of logic was very much due to the fact that the university library bought Bochenski's *Précis de Logique Mathématique* (1948).

It seems that a short article by Benson Mates [1949] made Prior even more aware of the interesting relation between time and logic. The paper was concerned with Diodorean logic, primarily Diodorus' definition of implication. Prior seemed to realise that it might be possible to relate Diodorus' ideas to contemporary works on modality by developing a calculus which included temporal operators analogous to the operators of modal logic. Mary Prior has described the first occurrence of this idea: "I remember his waking me one night, coming and sitting on my bed, and reading a footnote from John Findlay's article on Time, and saying he thought one could make a formalised tense logic." This must have been some time in 1953 [Kenny p. 336]. The footnote which Prior studied that night was the following:

> And our conventions with regard to tenses are so well worked out that we have practically the materials in them

> for a formal calculus... The calculus of tenses should have been included in the modern development of modal logics. It includes such obvious propositions as that
>
> x present = (x present) present ;
>
> x future = (x future) present = (x present) future;
>
> also such comparatively recondite propositions as that
>
> (x).(x past) future; i.e. all events, past and future will be past. [Gale p. 159-60]

To be sure, Findlay's considerations on the relation between time and logic in this footnote were not very elaborated, but it gave Prior the idea of developing a formal calculus which would capture this relation in detail. For this reason Prior called Findlay "the founding father of modern tense logic" [Prior 1967, p. 1]. But there are, in our opinion, certainly not sufficient reasons for viewing Findlay as the founder of tense logic. The honour of being the founder must without doubt be attributed to Prior himself. With his many articles and books on questions in tense logic he presented a very extensive and thorough corpus, which still forms the basis of tense logic as a discipline. Findlay's major merit in tense logic is, as Jean-Louis Gardies [1975, p. 40] has remarked, to have had the luck of inspiring Prior to initiate the development of formal tense logic.

In fact, Findlay's footnote was certainly not the only source of inspiration for Prior's incipient formal study of the logic of time. Prior highly valued various parts of Polish logic like Łukasiewicz's three-valued logic. And of course, from the previous stages of his career he was well acquainted with a huge historical material on questions related to temporal logic. A persistent feature throughout Prior's works is a clear interest in the history of logic. Indeed, Prior took an interest in the history of logic not only as a subject in its own right, but he also saw the works of ancient and medieval logicians as a significant contribution to the contemporary development of logic. He was particularly interested in Aristotle, Diodorus, and the Scholastics, but his interest also extended to more recent logicians such as Boole and Peirce, whom he called "the greatest of all symbolic logicians" [1957c].

It is likely that Prior was already in the early fifties acquainted with McTaggart's considerations on time [1908], and Reichenbach's examinations of the tenses of verbs [1947]. However, he made no reference to those ideas in his introductions to tense logic during the 1950's. The reason may be that he thought of these philosophers as adversaries. At least, he himself declared that at first he considered McTaggart an 'enemy', until Peter Geach made him aware of the importance and relevance of McTaggart's distinction between the so-called A- and B-series conceptions of time [1967, p. vi]. The A-series conception is based on the notions of past, present, and future, as opposed to a 'tapestry' view on time, as embodied by the B-series conception of time. Prior later formally elaborated McTaggart's distinction, and showed that we can discuss time using either a tense logic, corresponding to the A-series conception, or using an earlier-later calculus, corresponding to the B-series conception; we shall show in detail how he related the two to one another in chapter 2.8. Prior's interest in McTaggarts observations was first aroused when he realised that McTaggart had offered an argument to the effect that the B-series presupposes the A-series rather than vice versa [1967, p.2]. Prior was particularly concerned with McTaggart's argument against the reality of tenses. He pointed out that the argument is in fact based on one crucial assumption, namely that tenses should be explicated in terms of a non-temporal 'is', attaching either an event or a 'moment' to a 'moment'. That assumption is certainly very controversial. Nevertheless, since Prior's studies brought renewed fame to McTaggart's argument, this so-called McTaggart's paradox has been very important in the debate about various kinds of temporal logic and their mutual relations. In the next part of this book, we shall discuss the kind of reasoning involved in McTaggart's paradox.

With regard to Reichenbach's ideas, however, he did not change his mind. As we have seen in chapter 2.4, Reichenbach made some elegant observations, but the formalism he constructed was very limited. Indeed, its crucial idea of a three-point structure directly resists some required logical generalisations. It is also dubitable whether its capacity for linguistic

generalisation really goes very far - in spite of considerable initial success in the linguistic community. In consequence, Prior regarded Reichenbach's analysis as in some ways "a hindrance rather than a help to the construction of a logic of tenses" [1967, p. 13].

Prior shared the medieval view on statements. He presented this view in *Past, Present and Future*, quoting Peter Geach, who had formulated it as early as 1949:

> Such expressions as 'at time t' are out of place in expounding scholastic views of time and motion. For a scholastic, 'Socrates is sitting' is a complete proposition, *enuntiabile*, which is sometimes true, sometimes false; not an incomplete expression requiring a further phrase like 'at time t' to make it into an assertion. [Prior 1967, p. 15]

Prior continued to examine the Scholastic sources himself, and in his writings he clearly demonstrated the validity of Geach's formulation of the Scholastics' view on propositions.

Prior was invited to Oxford as 'John Locke Lecturer' in Philosophy in 1955-56. This led on to the Prior family moving in 1959 to Manchester and a few years later to Oxford, where Prior worked at Balliol College.

The John Locke lectures gave Prior an excellent opportunity to present his new findings regarding time and modality. The lectures were held on Mondays. Among the participants were John Lemmon, Ivo Thomas, and Peter Geach [Kenny p. 337]. The lectures were later published as the book *Time and Modality* (1957). It was this work which made Prior internationally known. After the publication of *Time and Modality* he received a number of important and interesting letters from various logicians. One of the logicians who wrote to Prior was Saul Kripke. In two letters to Prior in September and October 1958 Kripke put forth some very stimulating ideas regarding temporal logic. In the next section we shall examine Kripke's ideas and their impact on Prior's work.

According to Peter Geach, Prior regarded his own research into the logic of ordinary language constructions as a continua-

tion of the medieval tradition [Geach p. 188]. His attitude was congenial to that of the young Russell in *Principles of Mathematics:* ordinary language is not a logician's master, but it must be his guide [Geach p. 187]. After all logic in Prior's opinion "is not primarily about language, but about the real world" [TR, p. 1]. For this reason he strongly opposed the formalistic view on logic:

> Formalism, i.e. the theory that logic is just about symbols and not about things, is false. [TR p. 1]
> I cannot see how any statement whatever can be made true simply by using language in a particular way... [WL, p. 2]

Prior's own answer to the question about the nature of logic ran as follows:

> Logic deals, at bottom, with statements - it enquires into what statements follow from what - but logicians aren't entirely agreed as to what a statement is. Ancient and medieval logicians thought of a statement as something that can be true at one time and false at another. [SFTT, p. 1]

It is an obvious consequence of the ancient and medieval view that time should not be ignored in logic. Following this view Prior stressed that "the tense of a statement must be taken seriously" [SFTT, p. 2]. To Prior, all logic was in a sense tense logic: "... tenseless statements of modern logic are just a special case of statements in the old sense ..." [SFTT, p. 2]. He argued that tense logic is based on two fundamental assumptions [Prior 1957a, p. 104]:

> 1) tense-distinctions are a proper subject of logical reflection,
> 2) what is true at one time is in many cases false at another time, and vice versa.

Prior observed that ancient and medieval logicians took these assumptions for granted, but that they were eventually denied

(or simply ignored) after the Renaissance. Prior himself can be said to have realised the possibility of formulating a logic based on these old assumptions. In fact, he took the assumptions even further and also claimed the reality of tenses:

> So far, then, as I have anything that you could call a philosophical creed, its first article is this: I believe in the reality of the distinction between past, present, and future. I believe that what we see as a progress of events is a progress of events, a coming to pass of one thing after another, and not just a timeless tapestry with everything stuck there for good and all. [SFTT, p. 1]

It was Prior's conviction that tense logic was not merely a formal language together with rules for purely syntactic manipulations. It also embodied a crucial ontological and epistemological point of view according to which "the tenses (it will be, it was the case) are primitive; only present objects exist." [Prior & Fine, 1977, p. 116] To Prior, the present and the real were one and the same concept. Shortly before he died he formulated his view in the following way:

> ...the present simply is the real considered in relation to two particular species of unreality, namely past and future. [Prior 1972]

It is obvious that Prior was strongly attracted by questions concerning the relation between time and existence. In *Time and Modality* he proposed a system called 'Q' which was specifically meant to be a 'logic for contingent beings' [1957a, 41 ff.]. System Q can deal with a certain kind of propositions, which are not always 'statable'. Such propositions are particularly interesting from a tense logical point of view.

Consider the proposition r: 'x exists'. If the object x is a contingent being, then we may assume that it has come into existence at some past time (or perhaps it is coming into existence right now). Before its coming into existence the proposition r was not statable. One consequence is that in a tense

logic which is sensitive to this problem, such as Q, we cannot in general assume that Pr and $\sim H\sim r$ are equivalent. This makes things very complicated, so it is understandable that Prior subsequently chose to leave aside that problem in his first pioneering development of symbolic tense logic. However, late in his work he again took up this challenge and formulated a tense-logic for non-permanent existents [1968, p. 145 ff.].

In 1958 Prior entered into a very interesting correspondence with Charles Hamblin of The New South Wales University of Technology in Australia. Their correspondence led to important results, especially on implication relations among tensed propositions. Prior and Hamblin discussed two central issues in tense logic: the number of non-equivalent tenses, and the implicative structure of the (non-metric) tense operators. In a letter to Prior dated 18th April 1958 Hamblin suggested a set of axioms with P and F as monadic operators, corresponding to "a simple interpretation in terms of a two-way infinite continuous time-scale". Hamblin's axioms are:

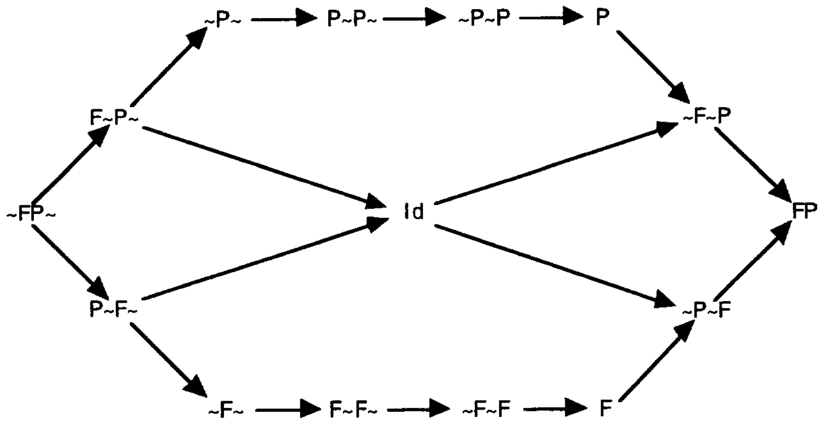
Ax1: $F(p \vee q) \equiv (Fp \vee Fq)$
Ax2: $\sim F\sim p \supset Fp$
Ax3: $FFp \equiv Fp$
Ax4: $FPp \equiv (p \vee Fp \vee Pp)$
Ax5: $\sim F\sim Pq \equiv (q \vee Pq)$

Hamblin also assumed 3 rules of inference:

R1: If A is a thesis, then $\sim F\sim A$ is also a thesis.
R2: If $A \equiv B$ is a thesis, then $FA \equiv FB$ is also a thesis.
R3: If A is a thesis, and A' is the result of simultaneously replacing each occurrence of F in A by P and each occurrence of P in A by F, then A' is also a thesis. (A' is the socalled mirror-image of A.)

When these axioms and rules are added to the usual propositional calculus a number of interesting theorems can be proved. In fact, Hamblin could prove that "there are just 30 distinct tenses", which can be formed using only P, F and negation.

Hamblin also suggested a certain implicative structure for the tenses. His suggestion can be illustrated like this:

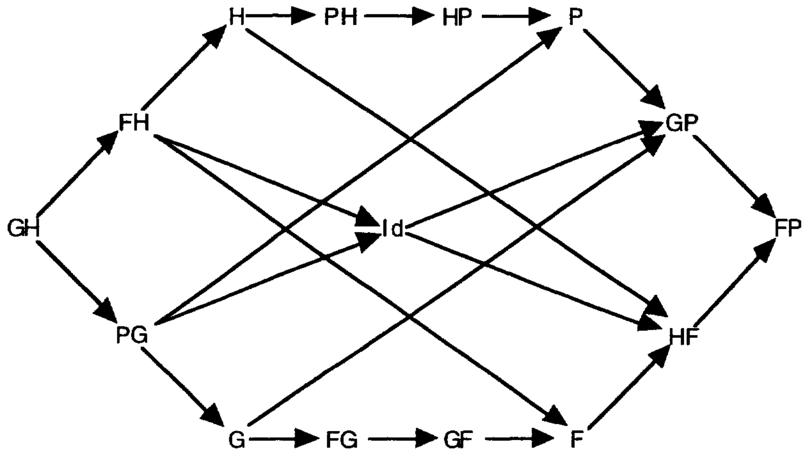


These results became even more appealing when Prior started to use the operators *G (= ~F~)* and *H (= ~P~)*. (We have not been able to find any explicit explanation as to why Prior chose exactly those two letters. However, M. J. Cresswell has by personal communication suggested to us that G was inspired by the phrase 'is always going to be', and H by the phrase 'has always been'.)

Using *G* and *H*, Hamblin could summarise the reduction of tenses with more than two adjacent tense-operators into the following diagram:

	GH	FH	PH	HP	GP	FP	HF	GF	FG	PG
P	GH	PH	PH	HP	P	FP	FP	GF	FG	PG
H	GH	H	PH	HP	HP	FP	HF	GF	FG	HG
F	GH	FH	PH	HP	FP	FP	F	GF	FG	FG
G	GH	GH	PH	HP	GP	FP	GF	GF	FG	G

Finally, in 1965 Hamblin and Prior ended up with the following nice implicative structure for the non-metrical tense-operators, which according to Hamblin is "a bit like a bird's nest" [Hamblin, letter of 6th July 1965]:



In 1967 Prior published his major work, *Past, Present and Future*, in which his approach to tense logic had reached a very convincing form. The decade of intense work in the field since the John Locke lectures had brought him a lot further. Also he had been able to benefit greatly from the correspondence with logicians like Kripke and Hamblin.

As a teacher Prior was very inspiring. He was always able to find nice and understandable illustrations of the logical systems he wanted to introduce. For instance, he would illustrate the fact that *LMp* cannot be deduced from *Mp* in the following way (where Mp in this context means '*p* either is or will be true', and *Lp* stands for 'p is and always will be true'):

> ...it is or will be that Uncle Joe's car is running, but it will not always be true that this is or will be true; so in this sense Mp does not imply *LMp* [1957c]

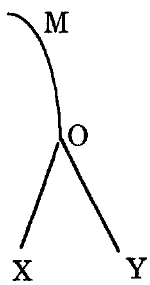
It seems clear that he very much liked teaching and lecturing. He was not 'the Oxford type', but it appears that he almost immediately build up a reputation as one of the best lecturers in Oxford.

Prior died on October 6th., 1969, whilst on a lecture tour in Scandinavia. On the day of his death he was visiting Trondheim in Norway. Prior had by then accomplished an impressive production. The bibliographical overview of Prior's philosophical works comprises more than 150 titles [Flo 1970]. In this overview one can follow how Prior's interests developed in the course of his work. Summarising the main trends it can be said that his work until the middle of the 1950's was characterised by a preoccupation with ethics and the history of logic. From the mid-fifties and onwards he devoted himself mainly to the study of the relation between time, modality, and logic. That should be seen as a natural consequence of his endeavour to develop the formal calculus of tense logic, a task which he took up around 1953 (at the time of being inspired by Findlay's footnote). Nevertheless, we hope to have also made clear that there is no sharp distinction between Prior's philosophical and historical concerns on one hand and his work as a formal logician on the other.

2.6. THE IDEA OF BRANCHING TIME

If the determinist sees Time as a line, the indeterminist sees it as a system of forking paths...
John P. Burgess [1978, p.157]

The straight line and the circle, respectively, are the traditional geometrical representations of time. According to the linear conception time is progressive. Strictly speaking, nothing will stay as it was, everything will change. Even if a phenomenon appears to be stable, say, the whiteness of an object, it is still not seen to be identical with 'same' phenomenon one moment ago - since that phenomenon does not really exist as opposed to the phenomenon we are contemplating 'now', and which does exist. According to the circular conception of time nothing is really new. Any event is a repetition of previous events, and will be repeated indefinitely in the future. These two geometrical images of time have been dominant within the philosophy of nature and other strands of systematic thinking from the antiquity and up to this century. However, during the last decades a number of intellectuals have suggested a new kind of time models. According to these models time is viewed as a branching system - a tree-structure. Since branching time models are very important in the modern analysis of temporality, it is worth trying to understand this new image of time in relation to the history of ideas. Consider this figure:



One of the first philosophers of time to formulate the idea of branching time in a precise manner was Henri Bergson (1859-

1941) in his book from 1889 *Essai sur les données immédiates de la conscience*. In this book Bergson considered the problems regarding time and free will. As a possible illustration of the process of deliberation he discussed the above figure [Bergson 1950, p. 176]. Bergson considered a interpretation of this illustration like the following: The person in question has traversed a series, MO, of conscious states. At the state O he finds the two directions, OX and OY, equally open for him. - However, Bergson argued that this geometrical representation of the process of coming to a decision is deceptive:

> This figure does not show me the deed in the doing but the deed already done. Do not ask me then whether the self, having traversed the path MO and decided in favour X, could or could not choose Y: I should answer that the question is meaningless, because there is no line MO, no point O, no path OX, no direction OY. To ask such a question is to admit the possibility of adequately representing time by space and a succession by a simultaneity. [Bergson 1950, p.180]

In our century the idea of branching time has become more acceptable than it was in the 19th century. In this connection the authorship of Borges stands out prominently. Apparently, Borges was the first intellectual to give a detailed description of the new model of time, namely in his short story from 1941 *The Garden of Forking Paths* [in Borges 1962] (which in some of its elements appears almost a thriller). In the following we shall account for the new understanding of time anticipated and compellingly unfolded by this story.

Borges' story is set during World War I. The Chinese Yu Tsun is a spy for the Germans in England. But a certain English counter-intelligence officer, captain Richard Madden, has just managed to quash the spying network to which Yu Tsun belongs. Yu Tsun himself has not yet been taken, but captain Madden is right on his heels. The Chinese spy, however, still has one important task to accomplish for his German superiors in Berlin. He must point out to them the town of Albert, where the

English are building a new strategically important artillery park. In the circumstances he sees no other way of achieving this goal than to kill some person with the name of 'Albert'; this incident, then, should reach the headlines of the English newspapers, where the Berlin office regularly looks for clues from its spies. Yu Tsun searches for possible victims in a telephone directory, and the only possible victim turns out to be one Stephen Albert, who lives about half an hour's travel by train from Yu Tsun's home. Yu Tsun plans the murder and leaves his place; he starts carrying out his plan, whilst trying to observe the following maxim:

> Whosoever would undertake some atrocious enterprise should act as if it were already accomplished, should impose upon himself a future as irrevocable as the past. [p. 92]

Yu Tsun now sees the murder of Albert as something inevitable. Albert is already dead in this planned future, which has been given the glow of necessity by Yu Tsun. And yet the murder is no necessity. This is strongly suggested by the fact that captain Richard Madden was right on the trail of Yu Tsun and went after him on his way to the train, but "by an accident of fate" he does not reach him [Borges, p. 92]. Madden is only a few minutes late for the train, but this "small victory" [Borges, p. 92] is the difference between life and death for Stephen Albert - and for the fate of Yu Tsun himself. Already in this introductory sequence the branching between different future courses of events is evident to the attentive reader. The past, on the other hand, is irrevocable, necessary and unchangeable. Planning is an attempt to assign to the future the same characteristics as those of the past, even though on grounds of principle this may only succeed to a certain extent. And Yu Tsun's maxim, it may be added, is a recipe for soothing one's conscience by projecting the properties of the past onto the future.

References to bifurcations in time pervade the story. When Yu Tsun leaves the train at Ashgrove and has to walk the last stretch to Stephen Albert's house, some local children give him the following piece of guidance:

> The house is a good distance away, but you won't get lost if you take the road to the left and bear to the left at every crossroad. [p. 93]

The children's' instruction about turning always to the left reminds Yu Tsun that "such was the common formula for finding the central courtyard of certain labyrinths" [p. 93]. His thoughts are thus led on to his great-grandfather Ts'ui Pên, who for thirteen years worked on the construction of a maze, "in which all men would lose themselves". [p. 93]. The analogy between a labyrinth and time now becomes explicit to Yu Tsun's mind:

> ... I thought of a maze of mazes, of a sinuous, ever growing maze which would take in both past and future and would somehow involve the stars. [p. 94]

These thoughts are being mirrored by nature itself: "... overhead the branches of trees intermingled..." [p. 93]. For a moment Yu Tsun feels as if he is allied with eternity - as a spectator to the totality of temporal courses of events:

> For an undetermined period of time I felt myself cut off from the world, an abstract spectator. [p. 94]

When approaching Albert's home, Yu Tsun to his surprise hears Chinese music coming from the garden. Stephen Albert at first mistakes Yu Tsun for a Chinese consul, who was apparently expected to come round some time to see Albert's garden. Thus a conversation is started, and Yu Tsun eventually learns that his victim-to-be is a sinologist, who holds a profound knowledge about his forefather's Ts'ui Pên's universe of ideas. For this reason Yu Tsun decides to postpone the execution of his otherwise irrevocable decision to kill Albert for about an hour. He lets Albert know that he is a descendant of Ts'ui Pên. The two together take a stroll through the garden. Also here the suggestions of the concept of branching time are clear:

> The damp path zigzagged like those of my childhood. [p. 95]

A philosophical - and at the same time highly existential - conversation then takes place in Albert's library. Stephen Albert is sitting with his back to a large circular clock. Yu Tsun is seated facing the clock. The conversation is about Ts'ui Pên, who for thirteen years lived remote from the world in order to write a book and construct a labyrinth. After the death of Ts'ui Pên his heirs found only a mess of chaotic manuscripts, which were published only because the executor of his will insisted. Yu Tsun himself never understood the book. That is evident from his remarks about it:

> Such a publication was madness. The book is a shapeless mass of contradictory rough drafts. I examined it once upon a time: the hero dies in the third chapter, while in the fourth he is alive. [p. 96]

Yu Tsun holds that the book is by no means characterised by the logical rules which in his view every author should obey. Thus for instance he thinks that the third chapter of the book should respect chapter two as a phase which has been concluded. It just has not occurred to him that the logic of the book could be quite new. Due to his close studies Stephen Albert, however, has been able to see through the mystery. A fragment of a letter from Ts'ui Pên has proved to hold the decisive key to the right understanding of the book:

> I leave to various future times, but not to all, my garden of forking paths. [p. 97]

The book and the labyrinth were not to be considered as two separate pieces of work to be carried out by Ts'ui Pên. They were in essence the same thing. The book was to be constructed as a labyrinth of time. The apparent conflict between the different parts of the book is simply due to the fact that Ts'ui Pên wanted to describe all the possible futures concurrently. The

book therefore does not respect the usual logic but defines its own logic - a new kind of temporal logic:

> This is the cause of the contradictions in the novel. Fang, let us say, has a secret. A stranger knocks at his door. Fang makes up his mind to kill him. Naturally there are various possible outcomes. Fang can kill the intruder, the intruder can kill Fang, both can be saved, both can die and so on and so on. In Ts'ui Pên's work, all the possible solutions occur, each one being the point of departure for other bifurcations. Sometimes the pathways of this labyrinth converge..... [p. 98]

In other words the novel depicts time as an infinite branching system. Thus Ts'ui Pên has handed over his proposal for a solution of the enigma of time to posteriority. That is to say, he has handed it over to the different futures after his death - even though it will in fact not be received in some of them. For instance, the solution is not received in the case of those possible futures, in which the executor of the will has the manuscripts burnt in order to prevent their publication.

Throughout Borges' short story the description of time as a gigantic branching system gets still more precise. Towards the end of the short story he lets Stephen Albert say:

> The explanation is obvious. The Garden of Forking Paths is a picture, incomplete yet not false, of the universe such as Ts'ui Pên conceived it to be. Differing from Newton and Schopenhauer, your ancestor did not think of time as absolute and uniform. He believed in an infinite series of times, in a dizzily growing, ever spreading network of diverging, converging and parallel times. This web of time - the strands of which approach one another, bifurcate, intersect or ignore each other through the centuries - embraces every possibility. [p. 100]

Borges' conception of time bears many similarities to Leibniz' idea of possible worlds. The different futures represent different

possibilities, and this aspect assumes a particular importance with respect to the existence of persons. Even though a person exists in one series of time, it cannot at all be taken for granted that he or she exists in another series of time. Borges lets Stephen Albert emphasise the fact that in most times - we might say, courses of events - neither Yu Tsun or he himself (Albert) exist. Moreover, the question about the existence of persons in the different series of time gives rise to some considerations on the extremely difficult philosophical problems concerning temporal and counterfactual identity:

> Once again I sensed the population of which I have already spoken. It seemed to me that the dew-damp garden surrounding the house was infinitely saturated with invisible people. All were Albert and myself, secretive, busy and multiform in other dimensions of time. [p. 100]

Yu Tsun experiences the counterfactual, yet in a sense real and simultaneous existence of other 'editions' of himself and Albert - varied infinitely as in a nightmare. And it is understandable that this should appear like a nightmare, for if the number of Yu Tsuns is infinite, who then is the real Yu Tsun? A complementary question to this philosophical and existential problem is this one: what exactly does it mean that a number of different possible persons are in some sense all Yu Tsun? Perhaps in his short story Borges presupposed the answer later to become prevalent in philosophy, namely that two possible persons are identical in a 'simultaneous' sense if they have a common history (indistinguishable histories) up to a certain point of time. Borges increases the dramatic effect of the idea of simultaneous identity by letting Albert say:

> Time is forever dividing itself toward innumerable futures, and in one of them I am your enemy. [p. 100]

This utterance answers a warped remark by Yu Tsun, who claims that in all possible times he would appreciate and be grateful to Albert for his reconstruction of the garden of forking

paths. Albert's answer stands out in dramatic contrast to Yu Tsun's proclaimed gratefulness, since Yu Tsun shortly after fires the killing shot at Albert in accordance with the 'irrevocable decision' made before he even met Albert.

Borges new idea about time is represented in a literary figure, and therefore it is no wonder that a number of philosophical and logical problems remain unanswered. In particular, the question about the branching towards the past is conspicuous. How can Borges accept an idea about a branching past? What does it mean when "the web of time - the strands of which approach one another ..., intersect" [p. 100], and "Sometimes the pathways converge" [p. 98]? Does Borges actually mean that it makes sense to talk about alternative possibilities of the past in the same way as one may operate with alternative possibilities of the future? Clearly, it is meaningful to talk about an alternative past in an epistemological sense, since we do not have a full or definite knowledge about the vast majority of questions about the past. This epistemological limitation is different from an ontological assumption that there be several different courses of events in the past, which are equally real. However, there is hardly any evidence of such a distinction being made in Borges' story. On the other hand it is difficult to believe that Borges would really make room for a liberty of choice regarding the past. The story repeatedly stresses the observation that the past is irrevocable. The ethical tension arises exactly out of the wilful and forced projection of this property of the past onto the future. Neither Yu Tsun nor anybody else can repeat or alter the past, and this fact in the end influences Yu Tsun's attitude towards the murder which he has committed. Immediately after the deed, Yu Tsun is apprehended by Richard Madden, who has somehow managed to trace him to Albert's home. Afterwards, writing in his cell, where he is awaiting execution, Yu Tsun expresses his anguish at his deed:

> What remains is unreal and unimportant. Madden broke in and arrested me. I have been condemned to hang. Abominably, I have yet triumphed! The secret name of the city to be attacked got through to Berlin. Yesterday it was

> bombed...He [the German superior in the Berlin headquarters] does not know, for no one can, of my infinite penitence and sickness of the heart. [p. 101]

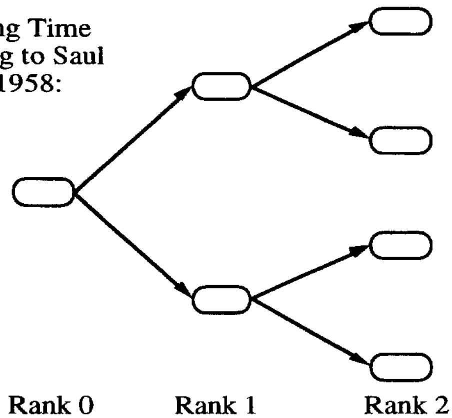
Those last words conclude the story. The feeling of repentance and fatigue expressed in them seems to be the nearest a human being can come in an attempt to change the past. - A solution to the question about alternative pasts in Borges short story can be based on the observation that the text seems to contain two concepts of eventuality (possible courses of events). One is connected with the situation of the human being. Our alternatives (of choice) with respect to eventuality regards the future only, since the past has already been settled. The other concept of eventuality is related to the conceivable or the consistent. It is a very comprehensive concept, since everything that does not directly involve a logical self-contradiction is regarded as possible. (Specific causal restrictions might be superimposed on this notion.) Apparently, Borges is relying on the latter concept of eventuality in his depiction of a temporal branching system. From the viewpoint of the present state of things it is possible to imagine different past courses of events, which have in various ways led to the present situation. These different pasts would be possible in so far as they among themselves make no (recognisable) contrast to the present state of affairs, for if they did we could rule some of them out.

It is a quite striking fact that Borges wrote his short story already in 1941, the very same year when J. Findlay's article *Time: A Treatment of Some Puzzles* was published in the Australasian Journal of Psychology and Philosophy. This article is normally considered to be the starting point for the modern logic of time (although Łukasiewicz trivalent logic might be seen as an even earlier forerunner of temporal logic, as we have shown in a chapter 2.3). There can be no doubt that *The Garden of Forking Paths* is a compelling picture of the very same basic intuitions which also underlie the later formal development of branching time. Nevertheless, it is hard to establish any direct impact of Borges' ideas in the development of the formal logic of time - in spite of the fact that many leading logicians and

philosophers within the study of time have evidently been aware of Borges' short story. In order to understand the more or less simultaneous appearance of Borges' short stories about time and the incipient study of temporal logic, we should perhaps rather focus on a general desire of understanding the nature of time in a more satisfactory way than the classical models could provide. There appears to have been a widespread concern with fundamental questions about time among intellectuals in the 1940's and the 1950's. Both the early logic of time and Borges' literary description of time can be said to have had the purpose of stressing the reality of time. Time is seen as an aspect of the real world and not an illusion. But what does this mean and how do we work out these ideas in detail? The idea of branching time is a framework within which we can begin to answer some of those questions. At least as an experiment, we can with Borges take on the rôle of an 'abstract spectator' of the world and try to understand the infinite temporal branching structure of possible events.

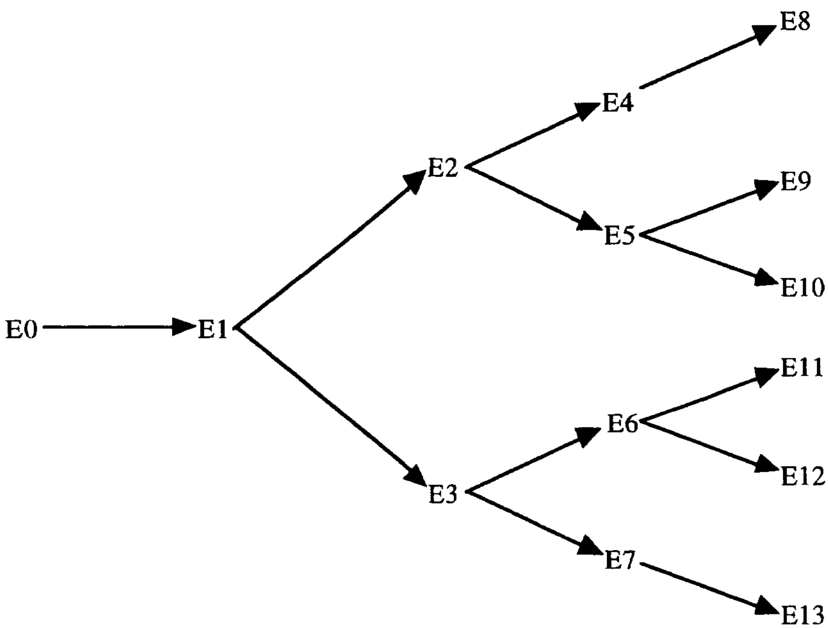
The idea of branching time was not realised in early work on temporal logic. Indeed it had not yet been formulated in Prior's *Time and Modality* (1957), which otherwise marked the major breakthrough of the new logic of time. As an explicit (or formalised) idea, branching time was first suggested to Prior in a letter from Saul Kripke in September 1958. This letter contains an initial version of the idea and a system of branching time, although it was of course not worked out in details. Kripke suggested that we may consider the present as a point of 'rank 0', and possible 'events' or 'states' at the next moment as points of 'rank 1'; for every such possible state in turn, there would be various possible future states at the next moment from 'rank1', the set of which could be labelled 'rank2', and so forth. In this way it is possible to form a tree structure representing the entire set of possible futures expanding from the present (rankO) - indeed a set of possible futures can be said to be identified for any state, or node in the tree. In this structure every point determines a subtree consisting of its own present and future.

Branching Time according to Saul Kripke, 1958:



Prior clearly found this view of time highly interesting, and in the following years he substantially developed it. He worked out the formal details of several different systems, which constitute different and even competing interpretations of this idea, as you shall see below. Eventually, he incorporated the idea of branching into the concept of time itself.

We may refine the intuitive picture of branching time by the figure below. In this picture, it makes sense to say that for every event there is one unambiguous past. For instance, in relation to the event E_5, the past contains the linear arrangement of events represented by E_0, E_1, and E_2. In relation to E_5 considered as the present time the events E_9 and E_{10} are alternative future possibilities. Relative to E_5, the events E_4, E_6 and E_7 will be counterfactual; that is, if E_5 is ever 'realised', E_4, E_6 and E_7 are indeed 'by now' (E_5) beyond possible realisation. Each E-node really represents a set of events and facts; if two facts both 'belong to' one and the same node, say E_5, they are of course genuinely simultaneous at E_5. E_4, E_6 and E_7, on the other hand, represent a pseudo-simultaneity with E_5 for what would have been real under different and counterfactual conditions.



It is, however, still possible to interpret this general idea in various ways. Prior himself worked out two different interpretations, inspired respectively by Ockham and Peirce [Prior 1968, p.122 ff.]. This fundamental work has led to a large number of articles in various journals. A significant number of these papers are concerned with the problem of determinism versus indeterminism, and we shall in part 3 examine in detail how indeterministic tense logics based on the idea of branching time can be worked out.

In order to shed light on the concept of time, Prior's procedure basically was to work out different temporal systems and then to examine their logical consequences. Other researchers have taken a more 'ontological approach', focusing on the concept of time itself; from an analysis of that concept, one can then construct the corresponding logic. (Needless to say, the two procedures cannot be kept strictly apart, but they do differ somewhat in their methodological consequences.) Nicholas Rescher [1968], for one, has reacted against Prior's rendition of branching time,

arguing that time itself is not really branching, in spite of the fact that a wealth of possibilities for the future course of events can be found (as seen from the present). To Rescher, we have a "branching in time", but not "branching of time" [1971, p.173]. Storrs McCall [1976], on the other hand, has argued that the passing of time is genuinely related to the understanding of time as a branching system: the passing of time is equivalent to a loss of possibilities! This observation emphasises how the branching of time is directed towards the future only, that is, for any point in the system there exists only one possible past. Of course, the problem of the ontological status of the possible futures is a very difficult one. Should we consent to what Borges lets Yu Tsun say immediately before he kills Stephen Albert: "The future exists now -" [p. 101]? Prior would certainly disagree; he repeatedly stated the conviction that only the present exists. The tension between these two creeds is in fact also manifest in *The Garden of Forking Paths*. Before Yu Tsun plans the murder and embarks on his chosen mission, he ponders his probable fate in the near future, namely the ordinary punishment meted out to spies: execution. But reflecting on the importance of the present as constituting reality, he finds some solace:

> Then I reflected that all things happen, happen to one, precisely *now*. Century follows century, and things happen only in the present. [p. 90]

Thus Yu Tsun comforts himself with an observation exactly opposed to the maxim with which he later tries to justify his deed. The above words bear a striking resemblance to some of Prior's remarks on the present as reality. In tense logic, the picture of branching time unfolded in the story is actually compatible with the identification of the present with the real. Nevertheless, while Borges' story certainly depicts and apparently advocates the branching view of time, it is not quite so clear whether it also agrees with the notion that 'only present objects exist' [Prior & Fine 1977, p. 116]. Even so, the fact that the story also pays attention to the special rôle of the present bears yet more witness to its profundity.

Finally, it may be worth considering the fact that the whole course of the story is itself what we would ordinarily consider as extremely unlikely. It is quite a bit of a coincidence that Yu Tsun's only possible victim should turn out to be a sinologist, indeed a sinologist who happens to have studied intensely the work of Yu Tsun's great-grandfather. In this circumstance one might seek evidence to the effect that after all, courses of events are seen as governed by Fate, or Providence. But on the other hand, it might also be seen as a suggestion that no future possibility should be ruled out or considered 'too unlikely' (excepting those which would violate the laws of logic or physics). The latter interpretation would be in good accordance with general features in Borges' work, to the best of our knowledge.

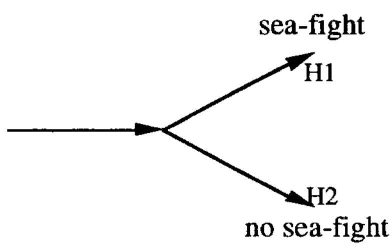
Several models of branching time have been proposed. The main difference between these models has to do with the status of the future. The models fall into a small number of groups, where the basic ideas can be shown in a very intuitive way: consider once again the old Aristotelian example about the possible sea-fight tomorrow. How should we define truth for statements like $F(1)p$?

One particular line of answer to this question can be based on a simple but radical assumption, namely the rejection of the principle of bivalence. As we have seen Jan Łukasiewicz maintained that we should view the logic of time as three-valued, attaching a third truth-value: 'indeterminate' to statements about the contingent future. A comparable line has been taken by Richmond H. Thomason [1970], according to which the truth-value of statements about the contingent future are in general undefined. Thomason's theory is certainly consistent, and it is also interesting that he has been able to use it in the context of deontic logic (i.e. the logic of moral obligation) [Thomason 1981, pp. 165 ff.]. The crucial problem with this approach as well as that of Łukasiewicz is the circumstance that the usual truth-functional technique breaks down for these theories. This condition is a source of serious formal problems as well as highly counter-intuitive features. For instance, if $F(1)p$ and $\sim F(1)p$ are both 'indeterminate' (or 'undefined'), it is very hard to explain how statements like the conjunction $F(1)p \wedge \sim F(1)p$ and the dis-

junction $F(1)p \vee \sim F(1)p$ can be anything else than 'indeterminate' (or 'undefined') [Prior 1967, p. 135]. We think that the introduction of 'indeterminate' or 'undefined' statements is an unnecessary complication. For this reason we shall leave aside further discussion of solutions based on the rejection of bivalence.

Let us consider the four bivalent answers which have been given in the literature. For the sake of simplicity, we shall use metrical time in our examples; but the results can be generalised into non-metrical time, if each branch defines an equivalence class of futures.

1) The first answer is that the two possibilities, sea-fight and no sea-fight, are both future, and that none of them has any superior status relative to the other. This answer can be represented graphically in the following way:



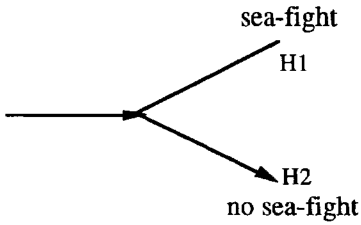
The arrows on end of the two future branches indicate that the statements 'there is going to be a sea-battle (tomorrow)' and 'there is not going to be a sea-battle (tomorrow)' are both true in this picture of branching time. That is, if we let p stand for 'there is a sea-battle going on', and $F(1)p$ stand for 'there is going to be a sea-battle tomorrow', then

$$F(1)p \wedge F(1)\sim p$$

is true. The corresponding tense-logical system is called K_b after Saul Kripke.

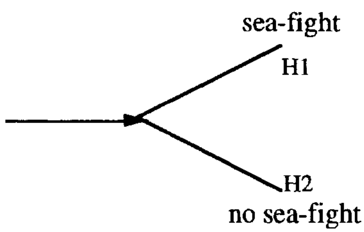
2) According to the Ockham-model only one possible future is the true one, although we as human beings do not know which of them it is. Let us assume that there is in fact going to be no

sea-fight tomorrow. In this case the future should be represented graphically in the following way, where a line not ending in an arrow indicates that it will be false to assert that the corresponding state-of-affairs will be the case tomorrow:



So, *~F(1)p* ∧ *F(1)~p* is the true description of this situation, even though we may be unable to know this at the present moment (p etc. being defined as above).

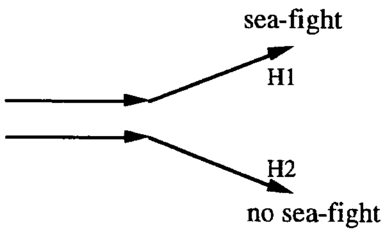
3) According to the Peirce-model - which Prior himself adopted as covering his own view - it makes no sense to speak about the true future as one of the possible futures. There is no future yet, just a number of possibilities. Hence, the future - or perhaps rather, the 'hypothetical future' - should be represented graphically in this way:



Neither *F(1)p* nor *F(1)~p* are true on this picture. However, if some proposition q holds tomorrow in all possible futures - that is, if the truth of q tomorrow is regarded as necessary - then *F(1)q* is true.

4) The possibility of the first three answers mentioned above were realised by A. N. Prior. However, later Hirokazu Nishimura [1979] formulated a new temporal model which turned out to

be slightly different from the Ockham-model which Prior had considered. Nishimura's model involved not only times, but also histories defined as linear subsets of the set of times. In fact, it is natural to view Nishimura's model of time as a union of disjoint histories. According to the model the tenses (past, present, future) are always relative to a history. Relative to one possible history there is going to be a sea-fight tomorrow, and relative to another history there is not going to be a sea-fight tomorrow. Graphically, this model can be presented in the following way:



Here, $F(1)p$ is true with respect to H1, whilst $F(1){\sim}p$ is true with respect to H2.

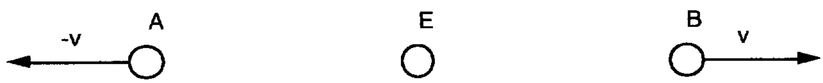
Clearly Nishimura's model has to involve some relation of identity of histories before certain events. H1 and H2 may be identical in all past times except for the fact that $F(x)p$ is true at all such times in H1 (for some x), while it is false in H2. Therefore, in order to achieve such an identity relation future statements must be disregarded. In dealing with the model, it is natural to consider the full set of histories as pre-defined. As we have seen, this view is similar to Leibniz' concept of creation of a temporal world. In general, it is interesting that the constructions in Nishimura's model come very close to ideas that can be found in Leibniz' *philosophy*, in spite of the fact that Leibniz himself ruled out time from his endeavour to establish a *symbolic logic*. Nishimura's ideas can be incorporated into a formal branching time model, which we shall call the *Leibniz System*, to be presented in due course. This system seems to be very close to the Ockhamist one, but it turns out that there are certain statements which are true from an Ockhamistic point of view, but false within the Leibniz System - as we shall see later.

2.7. TENSE LOGIC AND SPECIAL RELATIVITY

According to Prior many philosophers and scientists who accept the tapestry view of time have claimed that "they have on their side a very august scientific theory, the theory of relativity, and of course it wouldn't do for mere philosophers to question august scientific theories" [SFTT, p. 3]. Prior early became aware of the conflict between tense logic and special relativity. It was mentioned by Saul Kripke in a letter to Prior as early as 1958. Prior described the conflict in a very clear way:

> The trouble arises when we come to compare another's experiences, when, for example, I want to know whether I saw a certain flash of light before you did, or you saw it before I did. ... It could happen that if I assumed myself to be stationary and you moving, I'd get one result - say that I saw the flash first - and if you assumed that you were stationary and I moving, you'd get a different result ... And the conclusion drawn in the theory of relativity is that this question - the question as to which of us is right, which of us really saw it first - is a meaningless question ... Now I don't want to be disrespectful to people whose researches lie in other fields than my own, but I feel compelled to say that this just won't do. [SFTT, p. 3-4]

It is easy to understand what Prior means. Suppose that two observers, A and B, are moving with velocities v and $-v$, from an emitter E, both leaving E when the E-clock reads $t=0$.



According to special relativity the following transformations for the time co-ordinates hold:

$$t_A = L\,(t_E + vx_E)$$
$$t_B = L(t_E - vx_E)$$

where $L= (1 - v^2)^{-1/2}$ and the speed of light is taken as unity ($c = 1$).

A flash is emitted from E and received simultaneously by A and B, yielding same readings, t_E, on the E-clocks. The time coordinates for seeing the flash on A ($x_E = -vt_E$) and B ($x_E= vt_E$) can be calculated in A's system in the following way:

$$t_{A,A} = L(t_E + vx_E) = L(1-v^2)t_E$$
$$t_{A,B} = L(t_E - vx_E) = L(1+v^2)t_E$$

Clearly according to this A is the first to see the flash. The arrivals of the light signals can also be calculated in the B-system:

$$t_{B,A} = L(t_E - vx_E) = L(1+v^2)t_E$$
$$t_{B,B} = L(t_E - vx_E) = L(1-v^2)t_E$$

According to this calculation B sees the flash before A. For this reason some physicists would say that the question as to which of the two observers really saw a certain flash first can only make sense if an inertial frame is specified relative to which the calculation should be carried out.

However, Prior thought that the question as to which of the two observers really saw a certain flash first is indeed a meaningful one. He stated that what it means is simply this: "When I was seeing the flash, had you already seen it, or had you not?" [SFTT, p. 5] Of course, it might be doubted that a physicist committed to the ordinary interpretation of special relativity would be convinced by that definition. He would probably say that this is begging the question. As a precondition for accepting the question as a meaningful one he would instead demand some experimental procedure, by means of which the question can be settled.

Prior admitted that we cannot in all cases know whether a given event is present or not, i.e. whether it is really taking place 'now' or not, but he maintained that this epistemological question is very different from the corresponding ontological question. He wanted to make it clear that all what physics could show would be that "in some cases we can never know, we can

never *physically find out* [our italics], whether something is actually happening or merely has happened or will happen" [Prior 1972, p. 323]. Nevertheless, many modern physicists want to go even further, and claim with Albert Einstein:

> There is no irreversibility in the basic laws of physics. You have to accept the idea that subjective time with its emphasis on the now has no objective meaning. [Letter to Michele Besso; quoted from Prigogine 1980, p. 203]

On the other hand, Prior could also note - without doubt with some pleasure - that not even Einstein was quite content with this view. Einstein once said to Carnap that the problem of the Now worried him seriously, explaining that "the experience of the Now means something special for men, something different from the past and the future, but that this important difference does not and cannot occur within physics" [Prior 1968, pp. 133-134]. Following this kind of reasoning, Prior maintained that questions concerning the human Now make sense, even though we cannot be sure that such questions can ever be decided by physical means. On logical and philosophical grounds Prior maintained that when an event X is happening, another event Y either has happened or has not happened. He strongly rejected the idea of treating 'having happened' as a property that can attach to an event from one point of view whilst not from some other point of view:

> So it seems to me that there's a strong case for just digging our heels in here and saying that, relativity or no relativity, if I say I saw a certain flash before you, and you say you saw it first, one of us is just wrong - is misled it may be, by the effect of speed on his instruments - even if there is just no physical means whatever of deciding which of us it is. [SFTT, p. 5]

There seems to be two different ways of solving the conflict between tense logic and special relativity. We can either reject (or adjust) the fundamental beliefs underlying tense logic, or we

can reject (or adjust) the basic assumptions of special relativity. In the following we shall prefer to adjust the philosophical assumptions of special relativity in such a way that no empirical (or measurable) consequence of the theory is denied.

The paper [Øhrstrøm 1990] analyses a number of conceptual possibilities for upholding at the same time the assumptions of the Special Theory of Relativity and Prior's equating reality with the present. The analysis shows that this can be done in various ways. One of the most obvious ways presupposes the selection of a privileged inertial system, to whose time-coordinates special meanings are attributed. If such a selection is not to be made ad hoc, then it must be possible to list the reasons (preferably cosmological ones) for it. It should be pointed out that the principle of relativity does not exclude a cosmological time (that is, a 'natural' inertial system, which distinguishes itself through the distribution and movement of matter in the universe). However, even on the assumption of a homogeneous universe it can be doubted that cosmic time can actually be viewed as an ontological feature of the universe; Whitrow, sharing the assumption of a homogeneous universe, stated:

> It is doubtful whether there exists a precise definition which has so great merits that there would be sufficient reason to consider the time thus obtained as the true one. [Whitrow 1980, p. 304]

This point of view is not shared by all researchers. As Mogens Wegener has pointed out in his *Simultaneity and Weak Relativity* [1992, pp. 10 ff.] some scientists think that the cosmological evidence supports the existence of a universal substratum relative to which a cosmic and absolute simultaneity can be introduced. At least, it is clear that it is possible to hold Prior's very strong tense-logical position without violating any of the empirical consequences of special relativity, as long as we conceive the tenses as relative to one privileged observer. Arguing from a theological point of view, J. R. Lucas [1989, p. 220] has come to the same conclusion. Lucas points out that "the canon of simultaneity implicit in the instantaneous acquisition

of knowledge by an omniscient being" is not incompatible with the special theory of relativity, since there may be "a divinely preferred frame of reference".

If there is some privileged frame of reference, then the temporal co-ordinates in other systems do not strictly speaking represent proper time. For this reason Prior claimed:

> we may say that the theory of relativity isn't about real space and time... the time which enters into the so-called space-time of relativity theory ... is just part of an artificial framework which the scientists have constructed to link together observed facts in the simplest way possible... [SFTT, p. 5]

Prior did not mind playing that parlour game, too. He realised that the non-linear structure of space-time points, ordered with absolute before-after relations, possibly of a causal nature, constitutes an interesting object of study for the tense logician. The structure branches both forwards and backwards, so it is not immediately clear how the corresponding tense logic is to be axiomatised. He argued [Prior 1967, p. 203ff.] that the characteristic axioms for relativistic space-time are:

$FGq \supset GFq$
$PHq \supset HPq$

His argumentation was thorough and detailed, although a more systematic investigation of the relation between special relativity and tense logic was not carried out until 1980 (see [Goldblatt 1980]). A decade earlier on, Professor Gerald Masseyfrom Michigan State University had directed a frontal attack on tense logic as a new discipline. He had specifically referred to results from the Special Theory of Relativity, accusing Prior of promoting "bad physics and indefensible metaphysics" [Massey 1969]. However, in the light of amongst other things Goldblatt's results, Massey's attack was somewhat unreasonable.

Regarding a tense logical approach to relativity, Prior also pointed out that there is a logic of such functors as 'It appears from a certain point of view that -'. Hence, it is possible to make good sense out of talk about an infinity of different 'apparent' time-series. Prior suspected that the infinity of 'local proper times', which figure in relativistic physics, amounts simply to what appears from various points of view, or what appears to be the course of events in various 'frames of reference'. If the physicist wants to obtain a more general picture, he can "indicate what features of the course of events (what temporal orderings of those events) will be common to all points of view, and one can work out a tense logic for that too" [Prior 1968 p. 133]. Prior himself made some important contributions to the development of such a relativistic tense logic [Prior 1967, p. 203 ff.] even - though he felt that the project of a relativistic tense logic was on the whole a bit strange.

Although some results regarding relativistic tense logic have been obtained by Prior and his followers, J. P. Burgess [1984] in his overview of tense logic had to observe that a tense logic for special relativity had not yet been worked out fully - indeed that the results which had been produced so far had been sparse. In our opinion this is still the case.

2.8. Some Basic Systems of Temporal Logic

Temporal reasoning is captured in one manner by tense logic, and in another manner by the logic of instants; the tension between the two approaches was reflected in relation to the traditional interpretation of the special theory of relativity, which was analysed in the previous chapter. In terms of McTaggart's time-series we can say that tense logic is A-logical, whereas the logic of instants (or dates) is B-logical. Thus, we can speak about two kinds of temporal logic (A and B). In this chapter we shall study the relation between these kinds of temporal logic from a formal point of view.

Unlike most other disciplines in modern logic, temporal logic and its symbolic calculi were first developed entirely outside of the field of mathematics. This stands out in contrast to the comparable discipline of, say, modal logic, which also has clear philosophical motivations and implications, but in whose development regular mathematicians played an important rôle from the beginning. A. N. Prior, however, who was himself a philosopher by training, established temporal logic as a part of philosophical logic. In consequence, the emphasis was put on conceptual investigations rather than studies of purely mathematical aspects of temporal logic. This should not be misconstrued as a failure in mathematical competence, for Prior was clearly aware of the importance of meta-mathematical questions concerning general properties and mutual relations of logical systems, and contributed to these issues in developing tense logical systems. However, in constructing the systems conceptual considerations would take priority over mathematical neatness. The strength of philosophical logic lies in its self-imposed obligation to take the logical intuitions embodied in everyday language into serious consideration. On the other hand, it is also clear that until recently, temporal logic has lacked the kind of mathematical glamour exhibited in many other fields of symbolic logic.

In most presentations of temporal logic there is a very clear distinction between axiomatics and proof theory on one hand

and semantics and model-theory on the other. A-logic is viewed as axiomatics, and B-theory mainly as a kind of semantics, dealing with truth-conditions and temporal models. Prior's approach to temporal logic was different. Elaborating some observations by McTaggart, he maintained that A-logic is basic and that B-logic can be derived from it. In this chapter and the next one we shall expound some of the basic systems of temporal logics (i.e. A- and B-logics), largely following Prior's ideas. We shall present some of his most important results regarding temporal logic.

Any A-logic, i.e. tense logic, is based on the primitive tense-operators P and F; its axiomatisation is often formulated in terms of the derived operators H and G (as we have pointed out earlier, H and G are inter-definable with P and F, respectively, so either pair of operators can in fact be chosen as primitives). A very fundamental system has been named K_t (where the 'K' is probably in honour of Saul Kripke). This tense logic can be presented as an axiomatic system with the following axiom schemes [Prior 1967 p. 176; McArthur 1976, p. 17 ff.]:

(A1) p, where p is a tautology of the propositional calculus
(A2) $G(p \supset q) \supset (Gp \supset Gq)$
(A3) $H(p \supset q) \supset (Hp \supset Hq)$
(A4) $p \supset HFp$
(A5) $p \supset GPp$

In (A2) - (A5), p and q are arbitrary, well-formed formulas. All axioms are said to be immediately provable, while other theses can be proved by inference. In K_t, Modus Ponens is the basic rule of inference:

(RMP) If $\vdash p$ and $\vdash p \supset q$, then $\vdash q$.

In addition we have two rules, which introduce tense-operators:

(RG) If $\vdash p$, then $\vdash Gp$.
(RH) If $\vdash p$, then $\vdash Hp$.

From K_t, other tense logical systems can be defined by adding more axioms to the above list, (A1) - (A5), as we shall see in the following.

In order to introduce a logic of instants or dates, i.e. a B-logic, we need a set *TIME* of instants (or dates) with a relation, < , which attributes to *TIME* some structure. The relation '<' is called the before-after-relation. For any temporal instant t and any statement p, $T(t,p)$ is a new statement, which can be read 'p is true at t'. In most B-logics it is assumed that

(T1) $T(t,p \wedge q) \equiv (T(t,p) \wedge T(t,q))$
(T2) $T(t,\sim p) \equiv \sim T(t,p)$

Note that in principle we should make a distinction between two kinds of conjunction (and also between two kinds of negation) in (T1)-(T2). The reason is that in most B-logics, p and q are treated as propositional functions rather than full-fledged propositions such as $T(t,p)$. This means that the two kinds of expressions would be of different types. On the other hand, it is also possible in a B-logic to put both types of expressions syntactically on a par, as you shall see in the next chapter. So we shall neglect this complication, since it is after all rather clear how the conjunctions, negations etc. should be read in each case.

Now, the definitions

(DF) $T(t,Fp) \equiv_{def} \exists t_1\colon (t<t_1 \wedge T(t_1,p))$
(DP) $T(t,Pp) \equiv_{def} \exists t_1\colon (t_1<t \wedge T(t_1,p))$

would allow us to evaluate any tense logical formula p, in terms of $T(t,p)$. From the definitions $Hp \equiv_{def} \sim P\sim p$ and $Gp \equiv_{def} \sim F\sim p$ it immediately follows

(DG) $T(t,Gp) \equiv \forall t_1\colon (t<t_1 \supset T(t_1,p))$
(DH) $T(t,Hp) \equiv \forall t_1\colon (t_1<t \supset T(t_1,p))$

We shall say that a structure $(TIME,<,T)$ is a *B-logical structure* , if T satisfies (T1-2) and the definitions (DF), (DP), (DG), and (DH). T is called the *T-operator* (or the valuation operator) of the structure.

It is easy to see that the axioms (A1) - (A5) are all true at any t in any B-logical structure. Let us consider the case of (A4). In this case the following proof can be given:

(1)	$(t<t_1 \wedge T(t_1,p)) \supset (t<t_1 \wedge T(t_1,p))$
(2)	$(t<t_1 \wedge T(t_1,p)) \supset \exists t_2:(t<t_2 \wedge T(t_2,p))$
(3)	$(t<t_1 \wedge T(t_1,p)) \supset T(t,Fp)$
(4)	$(T(t_1,p) \wedge t<t_1) \supset T(t,Fp)$
(5)	$T(t_1,p) \supset (t<t_1 \supset T(t,Fp))$
(6)	$T(t_1,p) \supset \forall t: (t<t_1 \supset T(t,Fp))$
(7)	$T(t_1,p) \supset T(t_1,HFp)$
(8)	$\forall t_1 : (T(t_1,p) \supset T(t_1,HFp))$

Using standard quantification theory etc. it is also easy to see that the rules of inference all preserve truth by any T-operator in a B-logical structure. Summing up these observations, we have the following result:

> *Theorem.* If a tense-logical statement p is provable in K_t, then $T(t,p)$ (i.e. p is true at t) for any t in any B-logical structure $(TIME,<,T)$.

K_t makes less assumptions on the 'structure of time' than any other tense-logical system; that is, no restriction on the before-after-relation is required in the corresponding B-logic. This is why the system K_t is said to be minimal.

If we add the axiom

(A6) $\quad FFp \supset Fp$

we get a new tense-logical system corresponding to a transitive before-after relation, i.e.

(B1) $\quad (t_1 < t_2 \wedge t_2 < t_3) \supset t_1 < t_3$

If to K_t we add the axioms (A6) and

(A7) $FPp \supset (Pp \vee p \vee Fp)$

we get the system known as K_b. The subscript 'b' indicates that this system allows for branching time. Provable theorems in K_b are true for any T-operator with (T1-3) and

(B2) $(t_1 < t_2 \wedge t_3 < t_2) \supset (t_1 < t_3 \vee t_1 = t_3 \vee t_3 < t_1)$

(B2) can be called 'backwards linearity'.

A.N. Prior [1967, p.205 ff.] has demonstrated that the following statement is provable in K_t if (A7) is accepted as an axiom:

(A7x) $(Pp \wedge Pq) \supset (P(p \wedge q) \vee P(p \wedge Pq) \vee P(Pp \wedge q))$

(A7x) is less elegant than (A7), but at the intuitive level the former more directly expresses the idea of backwards linearity in a branching time model than does the latter. It is in fact also easy to see that (A7) is provable from K_t with (A7x). Thus there is a free choice between (A7) and (A7x), if one wishes to enlarge K_t into K_b.

We shall present a version of Prior's proof in order to give an example of the kind of very powerful reasoning which can be carried out in tense logic. So, we are going to prove the following meta-theorem:

In K_t enlarged with (A7), (A7x) is a theorem.

Similarly, in K_t enlarged with (A7x), (A7) is a theorem. - Prior proves the following lemmas:

Lemma 1. In a tense logical system with the axioms (A1)-(A5) and (A7) and the rules (RMP), (RG), and (RH)

$H(p \supset (Hp \supset q)) \vee H(Hq \supset p))$

is provable, where p and q are arbitrary well formed formulas.

Proof:

The proof is carried out by reductio ad absurdum i.e.

(1)	$\sim(H(p \supset (Hp \supset q)) \vee H(Hq \supset p)))$	(assumption)
(2)	$\sim H(p \supset (Hp \supset q))$	(from 1)
(3)	$\sim H(Hq \supset p))$	(from 1)
(4)	$P(p \wedge Hp \wedge \sim q)$	(from 2)
(5)	$P(Hq \wedge \sim p)$	(from 3)
(6)	$HFP(Hq \wedge \sim p)$	(from 5 and A4)
(7)	$P(p \wedge Hp \wedge \sim q \wedge FP(Hq \wedge \sim p))$	(from 4 and 6)
(8)	$P((p \wedge Hp \wedge \sim q \wedge P(Hq \wedge \sim p)) \vee$ $(p \wedge Hp \wedge \sim q \wedge Hq \wedge \sim p) \vee$ $(p \wedge Hp \wedge \sim q \wedge F(Hq \wedge \sim p)))$	(from 7 and A7)

But (8) is clearly impossible since all the components in the disjunction are impossible.

Lemma 2. In a tense logical system with the axioms (A1)-(A5) and (A7) and the rules (RMP), (RG), and (RH)

$$(H(p \supset q) \wedge H(p \supset Hq) \wedge H(Pp \supset q) \wedge Pp) \supset Hq$$

is provable, where p and q are arbitrary well formed formulas.

Proof:

By substitution in Lemma 1 we find

$$H(q \supset (Hq \supset \sim p)) \vee H(H\sim p \supset q))$$

Therefore, the problem can be split into two cases:
In the first case $H(q \supset (Hq \supset \sim p))$ is assumed and in the second $H(H\sim p \supset q))$ is assumed.

1) In the first case we can argue in the following way:

(1)	$H(p \supset q)$	(assumption)
(2)	$H(p \supset Hq)$	(assumption)
(3)	$H(Pp \supset q)$	(assumption)
(4)	Pp	(assumption)
(5)	$H(q \supset (Hq \supset \sim p))$	(assumption)
(6)	$H(p \supset (Hq \supset \sim p))$	(from 1 and 5)
(7)	$H(p \supset \sim p)$	(from 2 and 6)
(8)	$\sim P(p \wedge p)$	(from 7)
(9)	$\sim Pp$	(from 8) - contradicts (4)

This means that the assumptions in the antecedent rule out the first case.

2) In the second case, in which we can argue in the following way:

(1)	$H(p \supset q)$	(assumption)
(2)	$H(p \supset Hq)$	(assumption)
(3)	$H(Pp \supset q)$	(assumption)
(4)	Pp	(assumption)
(5)	$H(H{\sim}p \supset q))$	(assumption)
(6)	$H({\sim}q \supset Pp))$	(from 5)
(7)	$H({\sim}q \supset q))$	(from 6 and 3)
(8)	${\sim}P({\sim}q \wedge {\sim}q))$	(from 7)
(9)	Hq	(from 8)

Q.E.D.

Now, (A7x) can be proved from lemma 2 in the following way:

$$(H(p \supset q) \wedge H(p \supset Hq) \wedge H(Pp \supset q) \wedge Pp) \supset Hq$$
$$(H(p \supset q) \wedge H(p \supset Hq) \wedge H(Pp \supset q)\ ) \supset (Pp \supset Hq)$$
$${\sim}(Pp \supset Hq) \supset {\sim}(H(p \supset q) \wedge H(p \supset Hq) \wedge H(Pp \supset q)\ )$$
$$(Pp \wedge P{\sim}q) \supset (P(p \wedge {\sim}q) \vee P(p \wedge P{\sim}q) \vee P(Pp \wedge {\sim}q))$$

From this (A7x) can be obtained by substitution.

Moreover, K_t together with the axiom

(A8) $\quad PFp \supset (Pp \vee p \vee Fp)$

makes it possible by a proof similar to the above proof to deduce

(A8x) $\quad (Fp \wedge Fq) \supset (F(p \wedge q) \vee F(p \wedge Fq) \vee F(Fp \wedge q))$

The axiom (A8) corresponds to the requirement of forward linearity for the temporal ordering i.e.

(B3) $\quad (t_2 < t_1 \wedge t_2 < t_3) \supset (t_1 < t_3 \vee t_1 = t_3 \vee t_3 < t_1)$

We can also to K_t - or any of the suggested enlarged systems - add the axioms corresponding to non-ending time

(A9) $Gp \supset Fp$
(A10) $Hp \supset Pp$

i.e.

(B4) $\forall t_1 \exists t_2 : t_1 < t_2$
(B5) $\forall t_1 \exists t_2 : t_2 < t_1$

and dense time

(A11) $Fp \supset FFp$

i.e.

(B6) $\forall t_1 \forall t_2 \exists t_3 : t_1 < t_2 \supset (t_1 < t_3 \wedge t_3 < t_2)$

K_t together with all of the axioms (A7)-(A11) yields Prior's linear tense logic K_l, for which all the Hamblin implications can be proved.

In tense logics like K_l, based on just two primitive operators P and F, we can by definition introduce a number of new operators, for instance

$Ap \equiv (p \wedge Gp \wedge Hp)$ (- *always p*)
$Ip \equiv (p \vee Fp \vee Pp)$ (- *sometime p*)

However, Hans Kamp [1968] has demonstrated that some temporal operators expressible in terms of a T-operator cannot be defined in this way. One of these operators can be verbalised as 'is going to be uninterruptedly the case for some time' [Burgess 1984, p.117]; if we symbolise this operator as 'X', the relevant definition is

$$T(t,Xp) \equiv_{def} \exists t_2,t_3: (t < t_2 < t_3 \wedge \forall t_1: t_2 < t_1 < t_3 \supset T(t_1,p))$$

Kamp, however, managed to prove that this operator, and indeed, every temporal operator in a linear, dense, non-ending instant-logic, can be defined in terms of his two operators U and S, *until* and *since*, provided that time is assumed to be continuous [Burgess 1984, p.117]. Kamp's two operators can be defined in the following way (where Upq may be read as '*p until q*', and Spq can be read as '*p since q*'):

$$T(t,Upq) \equiv_{def}$$
$$\exists t_2: (t < t_2 \wedge T(t_2,q) \wedge \forall t_1: t < t_1 < t_2 \supset (T(t_1,p) \wedge T(t_1,\sim q)))$$

$$T(t,Spq) \equiv_{def}$$
$$\exists t_2: (t_2 < t \wedge T(t_2,q) \wedge \forall t_1: t_2 < t_1 < t \supset (T(t_1,p) \wedge T(t_1,\sim q)))$$

Almost from the very beginning of his development of tense logic, Prior [1967 p.111] was aware of problems concerning limitations to the expressive power of tense logic. But his approach to a solution to this problem was very different from that of Hans Kamp. Inspired by some observations due to Peter Geach, Prior pointed out that we can in fact define U and S in terms of the tense-operators P and F, if we allow ourselves

(i) the use of propositional quantifiers, and
(ii) the assumption that at each instant there is some proposition true at that instant only.

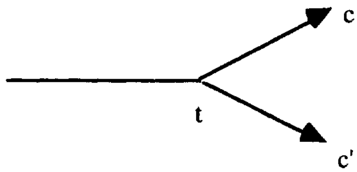
A tense-logical system based on these conditions has in fact very interesting and far-reaching implications. We shall study this issue in the next chapter.

TEMPO-MODAL SYSTEMS

Above, we have discussed purely tense-logical systems. Matters obviously become more complex, when besides the two primitive tense operators a primitive modal operator is introduced - as required in the cases of Prior's so-called Ockhamistic, respectively Peircean system.

In order to describe the semantics for these tempo-modal systems Prior [1967, p. 126 ff.] needs a notion of temporal 'routes' or 'temporal branches' i.e. maximally ordered (i.e. linear) subsets in $(TIME,<)$. We prefer the term 'chronicle'. The set of all such chronicles will be called C. We shall also need the concept of a chronicle-section (c,t), where $c \in C$ and $t \in c$ and a relation, $\approx$ on the set of chronicle-sections according to which

$(c,t) \approx (c',t)$ means that the two sections are identical up to t i.e. $\{t' \in c \mid t' \leq t\} = \{t' \in c' \mid t' \leq t\}$.



An operator *Ock* is an Ockhamistic valuation operator in a given Ockhamistic structure, if for any temporal instant *t* in any chronicle *c* and any tense-logical statement *p*, *Ock(t,c,p)* is a meta-statement which can be read 'p is true at t in the chronicle *c*'

(a) $Ock(t,c, p \wedge q)$ iff both $Ock(t,c,p)$ and $Ock(t,c,q)$
(b) $Ock(t,c,\sim p)$ iff not $Ock(t,c,p)$
(c) $Ock(t,c,Fp)$ iff $Ock(t',c,p)$ for some $t' \in c$ with $t < t'$
(d) $Ock(t,c,Pp)$ iff $Ock(t',c,p)$ for some $t' \in c$ with $t' < t$
(e) $Ock(t,c,Np)$ iff $Ock(t,c',p)$
for all (c',t) with $(c,t) \approx (c',t)$

If these conditions hold $(TIME,<,C,\approx,Ock)$ is said to be an *Ockhamistic structure*. - A formula *p* is said to be Ockham-valid if and only if *Ock(t,c,p)* for any *t* and *c* (with $t \in c$) and any Ockhamistic structure.

It may be doubted whether Prior's Ockhamistic system is in fact an adequate representation of the tense logical ideas propagated by William of Ockham. According to Ockham God knows the contingent future, so it seems that he would accept an idea of absolute truth, also when regarding a statement *Fq* about the contingent future - and not only what Prior has called "prima-facie assignments" [1967, p.126] like *Ock(t,c,Fq)*. That is, such a proposition can be made true 'by fiat' simply by constructing a concrete structure which satisfies it. But Ockham would accept that Fq could be true at t without being relativised to any chronicle. And that actually brings us back to a two-place *T*-operator, like the ones we have previously discussed. In the next part of the book we shall show that it is

possible to establish a system which seems to be a bit closer to Ockham's original ideas. On the other hand, it should be noted that the question concerning the notion of truth is mainly philosophical. Prior's Ockhamistic system appears to comprehend at least all the theorems which should be accepted according to Ockham's original ideas. Let us, for instance, consider one tense logical formula:

$$q \supset HFq$$

It is obvious from the above definitions that $Ock(t,c,q \supset HFq)$ for any t and any c with $t \in c$. Therefore $q \supset HFq$ is a theorem in Prior's Ockhamistic system.

Likewise $Pq \supset NPq$ (where F does not occur in q) is obviously a theorem, whereas the formula $PFq \supset NPFq$ is not a theorem in the system. This difference corresponds exactly to the difference between proper past and pseudo-past (see chapter 1.9).

It has proved quite difficult to find a satisfactory axiomatisation of Prior's Ockhamistic system. One prominent attempt was made by Robert P. McArthur [1976, p. 47]. He introduced a primitive operator L, for which he stated the following axioms:

(L1) $L(p \supset q) \supset (Lp \supset Lq)$
(L3) $Lp \supset LLp$
(LG) $Lp \supset Gp$
(LP) $p \supset LPp$, where p contains no occurrences of F

and the rule

(RL) If $\vdash p$, then $\vdash Lp$.

L is, of course, intuitively related to 'necessity', but McArthur's L-operator is clearly more than just a modal operator: from (LG) it is obvious that L is also temporal. It is natural to understand L as equivalent to NG, where N is a pure necessity operator. Conceived in this way, all the axioms in the system are

certainly Ockham-valid. But as far as we know, the question of completeness for Prior's Ockhamistic system is still open.

Now, let us turn to the other tempo-modal system that Prior studied carefully, the so-called Peirce system. In this system four different operators, F, G, f, and g, regarding the future can be considered. These operators can be translated into an Ockhamistic formulation in the following way:

$$F \rightarrow NF$$
$$f \rightarrow MF$$
$$G \rightarrow NG$$
$$g \rightarrow MG$$

This process of translation is well-defined, but it should be noted that there is no Peircean expression which translates into the Ockhamistic F. The great achievement of the Ockhamistic system could arguably be said to be its property of making a genuine distinction between the following three types of statement:

(i) Necessarily, Mr. Smith will commit suicide.
(ii) Possibly, Mr. Smith will commit suicide.
(iii) Mr. Smith will commit suicide.

However, in the Peirce-system the type of future statement seen in (iii) will have to be interpreted as meaning either (i) or (ii). There is no 'plain future' in this system. Of course, that is not a consequence of sloppiness on Peirce's side, but rather it is a deliberate and philosophically motivated choice, as explained in chapter 2.2. Therefore, the Ockhamistic system cannot a priori be preferred on philosophical grounds; but on linguistic grounds at least, it seems clear that (iii) should be distinguished from (i) and (ii).

Given the above translation rules, truth and validity within the Peirce system can clearly be defined in terms of truth and validity in Ockhamistic structures:

A formula p is said to be Peirce-true at a time t in $(TIME,<,C,\approx,Peirce)$ if and only if the translation of p into the Ockhamistic language is true at t with $t \in c$.

A formula is Peirce-valid iff its translation is Ockham-valid.

Let us consider the formula: $q \supset HFq$. When this formula is translated into the Ockhamistic language, we get the formula:

$$q \supset HNFq$$

It is obvious from the above definitions that $Ock(t,c,q \supset HNFq)$ can be made false for some t and some c with $t \in c$ in some structure. Therefore, the formula $q \supset HFq$, is not a theorem in Prior's Peircean system.

Now, the truth-operator in the Peircean system does not have to be defined in terms of the Ockhamistic operator. It would have been possible to present it quite independently. But since we want to compare the two systems, the above definitions are very useful. We can immediately verify the most interesting feature of Prior's definition of Peircean truth:

$Peirce(t,Fp)$ if and only if
for all (c',t) with $(c,t) \approx (c',t)$
$Peirce(t',p)$ for some $t' \in c'$ with $t < t'$

This appears to be in very good accordance with the ideas of C. S. Peirce, since he as we have seen in chapter 2.2 rejected the very idea that statements regarding the contingent future could be true.

2.9. FOUR GRADES OF TENSE-LOGICAL INVOLVEMENT

In order to construct a tempo-modal logic, which is intuitively satisfactory, we may proceed semantically; it is not demanded that a full axiomatic system together with proofs of soundness and completeness be given. Needless to say, such results would certainly be desirable, where they can be achieved. However, in the general project of formal semantics for natural languages it seems to be commonly accepted that from some point of complexity, we must necessarily depart from deductive systems in favour of model-theory [Dowty et al. 1979, p. 50 ff.]. That is, a general and satisfactory formal semantics for natural language probably cannot be finitely and completely axiomatised. On the other hand, these observations do not necessarily apply to the restricted case of tense logic.

Thus, in the discussion of issues in philosophical logic in relation to everyday language we can in principle confine ourselves to the semantics of the systems. This is exactly what we intend to do when dealing with the problem of finding an indeterministic tense logic, which is intuitively satisfactory. Nevertheless, the tension between a proof-theoretical approach to tense logic and a semantical approach should not be exaggerated. As we shall see in this chapter, A.N. Prior has shown that for tense logic the two approaches can (and should) be embedded in an approach of a higher order.

In chapter 2.4 we briefly mentioned that tense logic corresponds to McTaggart's A-series conception, which sees time in terms of past, present, and future, whereas an earlier-later calculus corresponds to his B-series conception, which sees time as a set of objectively existing instants. Prior clearly considered the A-conception to be the fundamental one:

> Time is not an object, but whatever is real exists and acts in time... But this earlier-later calculus is only a convenient but indirect way of expressing truths that are not really about 'events' but about things ... [TR p. 2-3]

Prior introduced four grades of 'tense logical involvement'. The first grade defines tenses entirely in terms of objective instants and an earlier-later relation. For instance, a sentence such as *Fp*, 'it will be the case that *p*', is defined as a short-hand for 'there exists some instant *t* which is later than now, and *p* is true at t', and similarly for the past tense; these definitions are, of course, the same ones as those we already stated in chapter 2.8, namely

(DF) $T(t,Fp) \equiv_{def} \exists t_1 : t<t_1 \wedge T(t_1,p)$
(DP) $T(t,Pp) \equiv_{def} \exists t_1 : t_1<t \wedge T(t_1,p)$

Tenses, then, can be considered as mere meta-linguistic abbreviations, so this is the lowest grade of tense logical involvement. Prior succinctly described the first grade as follows:

> ...there is a nice economy about it ... it reduces the minimal tense logic to a by-product of the introduction of four definitions into an ordinary first-order theory, and richer [tense logical] systems to by-products of conditions imposed on a relation in that theory. [Prior 1968, p. 118]

In the first grade, tense operators are simply a handy way of summarizing the properties of the before-after relations, which constitute the B-theory of McTaggart. Hence, in the first grade B-theory concepts are seen to be determining for a proper understanding of time and reality; tenses are deemed to have no independent epistemological status. The basic idea is a definition of truth relative to temporal instants (this definition is in fact already incorporated into the notion of a B-logical structure defined in chapter 2.8):

(T1) $T(t,p \wedge q) \equiv (T(t,p) \wedge T(t,q))$
(T2) $T(t,\sim p) \equiv \sim T(t,p)$

In addition, there may be some specified properties of the before-after relation, like for instance transitivity:

(B1) $(t_1 < t_2 \wedge t_2 < t_3) \supset t_1 < t_3$

In this way, instants acquire an independent ontological status. As we have seen, Prior rejected the idea of temporal instants as something primitive and objective.

In the second grade of tense logical involvement, tenses are not reduced into B-series notions. Rather, they are treated on a par with the earlier-later relation. Specifically, a bare proposition p is treated as a syntactically full-fledged proposition, on a par with what Rescher and Urquhart [1971] called 'chronologically definite' propositions such as $T(t,p)$ ('it is true at time t that p'). The point of the second grade is that a bare proposition with no explicit temporal reference is not to be viewed as an incomplete proposition. One consequence of this is that an expression such as $T(t,T(t',p))$ is also well-formed, and of the same type as $T(t,p)$ and p. Prior showed how such a system leads to a number of theses, which relates tense logic to the earlier-later calculus and vice versa [Prior 1968, p. 119]. The following crucial rule of inference makes this relation within the second grade especially obvious:

(RT) If $\vdash p$, then $\vdash T(t,p)$ for any t and any truth-operator T.

He also stated the following basic assumptions regarding the truth-operator:

(TX1) $(\forall t\colon T(t,p)) \supset p$
(TX2) $(\forall t_1\colon T(t_1,p)) \supset T(t_2, \forall t_3\colon T(t_3,p))$
(TX3) $T(t_1,p) \supset T(t_2,T(t_1,p))$

The philosophical implication of this second grade of tense logical involvement is that one must regard the basic A- and B-theory concepts as being on the same conceptual level. Neither set of concepts is conditioned by the other.

The B-theory is sometimes considered as the semantics of the corresponding A-theory. This is not surprising if we again consider the first-grade formulation of Fp, 'it will be the case that p', as a short-hand for 'there exists some instant t which is later than now, and p is true at t' (cf. (DF)).

This is tantamount to stating a truth condition for Fp. On this view of the relationship between the A- and B-theories, it may be a bit puzzling that p and $T(t,p)$ can be treated as being on the same logical level - the former apparently belonging to the logical language (or object language) and the latter to the semantics (or meta-language). In Prior's opinion, however, this is not at all surprising. In a paper on some problems of self-reference he stated:

> In other words, a language can contain its own semantics, that is to say its own theory of meaning, provided that this semantics contains the law that for any sentence x, x means that x is true. [Prior 1976a, p. 141]

It seems that this statement is exemplified exactly by the relation of the logic of tenses (the A-theory) to the logic of earlier and later (the B-theory), provided that we are willing to take the step of the second grade: syntactically conflating 'bare' p with $T(t,p)$. The relation becomes even clearer in the third grade, a system which has crucial implications for the status of the indication of time. Prior introduced the third grade in the following way:

> What I shall call the third grade of tense logical involvement consists in treating the instant-variables a, b, c, etc. as representing propositions. [Prior 1968, p. 122-23]

Such instant-propositions describe the world uniquely at any given instant, and are for this reason also called world-state propositions. Like Prior we shall use a, b, c ... as instant-propositions instead of t_1, t_2, ... In fact, Prior assumed that such propositions *are* what ought to be meant by 'instants':

> A world-state proposition in the tense-logical sense is simply an index of an instant; indeed, I would like to say that it is an instant, in the only sense in which 'instants' are not highly fictitious entities. [Prior 1967, p.188-89]

The traditional distinction between the description of the content and the indication of time for an event is thereby dissolved. From the properties of the logical language which embodies the third grade of tense logical involvement Prior also showed that *T(a,p)* can be defined in terms of a primitive necessity-operator. Then tense logic, and indeed, all of temporal logic can be developed from the purely 'modal notions' of past, present, future, and necessity.

In our opinion this idea of treating instants as some kind of world propositions was one of his most interesting constructions. We believe that the full strength of this view has not yet been displayed. It is very likely that this notion will turn out to be very useful in the part of computer science called natural language understanding [Hasle 1991].

The fourth grade consists in a tense logical definition of the necessity-operator such that the only primitive operators in the theory are the two tense logical ones: *P* and *F*. Prior himself favoured this fourth grade. It appears that his reasons for wanting to reduce modality to tenses were mainly metaphysical, since it has to do with his rejection of the concept of the (one) true (but still unknown) future. If one accepts the fourth grade of tense-logical involvement, it will turn out that something like the Peirce solution will be natural, and that we have to reject solutions which involve crucially the idea of a true or simple future - like the Ockhamistic theory.

A tense-logical approach to the concept of time involves a commitment to the third or the fourth grade. We ourselves prefer a tense-logical approach, essentially for the same reasons as Prior. However, systems based on the third grade are obviously more general than systems based on the fourth grade. If the fourth grade is accepted, interesting systems such as the Ockhamistic have to be ruled out. Therefore, we are inclined to consider the third grade to be the desirable basis for a conceptually adequate logic of time. In the following we shall present some of Prior's most important results with respect to the third grade - that is, the theorems in question are valid, if the third grade is accepted. Our presentation will differ somewhat from Prior's, though. Before proceeding it may be noted that

[Hasle 91] gives an exposition which follows Prior's own presentation more closely.

THE LOGIC OF INSTANT-PROPOSITIONS

With the standard set of well formed formulae (wff) of propositional tense logic we assume K_t, i.e. the axioms

(A1) p, where p is a tautology of the propositional calculus
(A2) $G(p \supset q) \supset (Gp \supset Gq)$
(A3) $H(p \supset q) \supset (Hp \supset Hq)$
(A4) $p \supset GPp$
(A5) $p \supset HFp$

and the rules of inference

(RMP) If $\vdash p$ and $\vdash p \supset q$, then $\vdash q$.
(RG) If $\vdash p$, then $\vdash Gp$.
(RH) If $\vdash p$, then $\vdash Hp$.

where '$\vdash$' means 'it is provable in the system that'. It is sometimes useful to mention the system explicitly, as in $K_t \vdash p$. In chapter 2.8 we demonstrated that if $K_t \vdash p$ then $T(t,p)$ holds for any t, and any T-operator which satisfies (T1-2) and the definitions (DP), (DF), (DG) and (DH). In other words, K_t is sound. We shall now argue that if the system is 'interpreted' as in Prior's third grade, it is also complete. More precisely, if $T(t,p)$ holds for any t and for any T satisfying (T1-2), (DP), (DF), (DG) and (DH) then p is provable in K_t, provided that we adopt the assumptions on which the third grade is based.

We are not going to demonstrate completeness in a traditional mathematical way, but we intend to show that a result very similar to completeness can be obtained in the context of Prior's third grade. In order to do that we need a set of instant propositions $\{a, b, c, \ldots\}$. We shall define an instant proposition as

a maximal consistent set of K_t-wff's. We extend the notion of well formed formulae in the following way, calling the enlarged system Pr_t:

(1) Any K_t-wff is a Pr_t-wff.
(2) Any instant proposition a is a Pr_t-wff.
(3) If α and β are Pr_t-wff, and a is an instant proposition, then $\sim\alpha$, $\alpha\wedge\beta$, $\forall a{:}\alpha$, $P\alpha$, and $F\alpha$ are all Pr_t-wff's.
(4) There are no other Pr_t-wff's.

In addition, we assume the standard definitions of propositional and predicate logic, including the definition of $\exists a{:}\ \alpha$ as $\sim\forall a{:}\sim\alpha$. In the following, 'p' stands for an arbitrary Pr_t-wff, whereas 'a' stands for an arbitrary instant proposition.

The axioms of Pr_t are the axioms of K_t together with the axiom

(I1) $\exists a{:}\ a$

and the rule:

(RI) For any instant proposition a and any wff p:
Exactly one of $\vdash a \supset p$ and $\vdash a \supset \sim p$

together with the rules included in Prior's quantification theory [Prior 1955, p.76 ff.]:

(Π1) If $\vdash \phi(x)\supset\beta$, then $\vdash (\forall x{:}\phi(x))\supset\beta$.
(Π2) If $\vdash \alpha\supset\phi(x)$, then $\vdash \alpha\supset\forall x{:}\phi(x)$, for x not free in α.

From (Π1-2) it is easy to deduce [Prior 1955, p. 82] that

(Σ1) If $\vdash \phi(x)\supset\beta$ then $\vdash \exists x{:}\phi(x)\supset\beta$, for x not free in β.
(Σ2) If $\vdash \alpha\supset\phi(x)$ then $\vdash \alpha\supset\exists x{:}\phi(x)$.

It should be noted that (RI) is natural in the light of what it means to be a maximal consistent set. Intuitively, an instant proposition a may be viewed as the conjunction of the elements

in the maximal consistent set. (I1) is also rather natural since it simply states that some instant proposition holds now.

It is not difficult to prove that the system Pr_t defined in this way is sound assuming (T1-2) and

- quantification as in (Π1-2) within the scope of T;
- $T(t,a)$ defines a one to one correspondence between times and instant propositions;

for the T-operators in question.

In order to obtain a result similar to 'ordinary completeness', we need a language in which we can express a notion corresponding to Pr_t-provability, i.e. a meta-language relative to the Pr_t-language. For this reason we again extend the notion of well-formed formulae and call the resulting language *$Prior_t$*:

(1) Any Pr_t-wff is a $Prior_t$-wff.
(2) If α and β are $Prior_t$-wffs, then $\sim\alpha$, $\alpha \wedge \beta$, $L\alpha$, and $\forall a{:}\alpha$ are all $Prior_t$-wff's.
(3) There are no other $Prior_t$-wff's.

In addition, we assume the standard definitions from propositional and predicate modal logic, especially the definition of M as $\sim L \sim$. The axiomatic system of $Prior_t$ consists of Pr_t and the axioms

(L1) $L(p \supset q) \supset (Lp \supset Lq)$
(L2) $Lp \supset p$
(L3) $Lp \supset LLp$
(I2) $\sim L \sim a$
(I3) $L(a \supset p) \vee L(a \supset \sim p)$
(BF) $L(\forall a{:}\ \phi(a)) \equiv \forall a{:}\ L(\phi(a))$
(LG) $Lp \supset Gp$
(LH) $Lp \supset Hp$

along with the rule

(RL) If $\vdash p$, then $\vdash Lp$.

It is obvious that (RG) and (RH) follow from (RL), (LG), and (LH).

It is also worth noting that Lp intuitively may be read as 'provable in Pr_t' (for any Pr_t-wff p). When read in this way it seems reasonable that L satisfies (RL) and the axioms above, provided that these are restricted to Pr_t-wffs. (L1) must hold for any notion of provability. (I2) holds, since if $\sim a$ is provable, then a cannot be consistent. (I3) is in fact a consequence of (RI) together with the consistency and the maximality of a.

(BF) is known as the Barcan formula after Ruth C. Barcan [1946], who was able to demonstrate it for modal logics which satisfy a few basic conditions. If Lp is understood as 'provable in Pr_t', then (BF) is rather natural since it may be read as stating what it means to prove $\forall a: \phi(a)$.

Now we want to construct a T-operator based on the full logic of instant propositions i.e. $Prior_t$. That is, we wish show how an entire earlier-later calculus can be developed - one might say boot-strapped - from definitions in the tense-logical theory.

Let Ω denote the set of instant propositions. For arbitrary elements a and b in Ω we introduce the following definitions:

(DB) $\quad a < b \equiv_{def} L(a \supset Fb)$

corresponding to 'the instant a is earlier than the instant b', and

(DT) $\quad T(a,p) \equiv_{def} L(a \supset p)$

corresponding to 'it is true at time a that p'.

Using these assumptions and definitions we can prove the theorems (T1-2), as well as (DG) and (DH). In turn, this means that $(\Omega,<,T)$ is a B-logical structure (with T defined as above).

The proofs can be carried out in the following way:

(T1.1) $\quad T(a,p \wedge q) \supset (T(a,p) \wedge T(a,q))$

Proof:

(1)	$T(a,p \wedge q)$	(assumption)
(2)	$L(a \supset (p \wedge q))$	(1, using DT)
(3)	$L((a \supset p) \wedge (a \supset q))$	(2)

(4) $L(a \supset p) \wedge L(a \supset q)$ (3)
(5) $T(a, p) \wedge T(a,q)$ (4, using DT)
Q.E.D.

(T1.2) $(T(a,p) \wedge T(a,q)) \supset T(a,p \wedge q)$
Proof:
(1) $T(a, p) \wedge T(a,q)$ (assumption)
(2) $L(a \supset p) \wedge L(a \supset q)$ (1, using DT)
(3) $L((a \supset p) \wedge (a \supset q))$ (2)
(4) $L(a \supset (p \wedge q))$ (3)
(5) $T(a,p \wedge q)$ (4, using DT)
Q.E.D.

Obviously, (T1) follows from (T1.1) and (T1.2).

(T2.1) $T(a,\sim p) \supset \sim T(a,p)$
Proof:
This is proved by reductio ad absurdum.
(1) $T(a,\sim p)$ (assumption)
(2) $T(a,p)$ (assumption)
(3) $L(a \supset \sim p)$ (1)
(4) $L(p \supset \sim a)$ (3, using L1)
(5) $L(a \supset p)$ (2)
(6) $L\sim a$ (4 & 5; Contradicts I2)
Q.E.D.

(T2.2) $\sim T(a,p) \supset T(a,\sim p)$
Proof:
(1) $\sim T(a,p)$ (assumption)
(2) $L(a \supset \sim p)$ (1, using I3)
Q.E.D.

Obviously, (T2) follows from (T2.1) and (T2.2).

(DL.1) $\sim L\sim p \supset \exists b: T(b,p)$
Proof:
(1) $\sim L\sim p$ (assumption)
(2) $L(\exists b: b)$ (using I1 and RL)

(3)	$\sim L\sim(\exists b: b \wedge p)$	(1 and 2, using L1)
(4)	$\exists b: \sim L\sim(b \wedge p)$	(3, using BF)
(5)	$\exists b: \sim L(b \supset \neg p)$	(4)
(6)	$\exists b: L(b \supset p)$	(5, using I3)
(7)	$\exists b: T(b,p)$	(6, using DT)

Q.E.D.

(DL.2) $Lp \supset \forall b: T(b,p)$

Proof:

(1)	Lp	(assumption)
(2)	$p \supset (b \supset p)$	(A1)
(3)	$L(b \supset p)$	(1 and 2, using L1)
(4)	$T(b,p)$	(3, using DT)
(5)	$Lp \supset T(b,p)$	(1 and 4)
(6)	$Lp \supset \forall b: T(b,p)$	(Π2)

Q.E.D.

It follows from (DL.1) and (DL.2) that

(DL) $\forall a: T(a,p) \equiv Lp$

In order to prove the remaining theorems, we need the following lemma about the ordering relation:

(DB.1) $a < b \supset L(b \supset Pa)$

Proof:

This proved by reductio ad absurdum:

(1)	$a<b$	(assumption)
(2)	$\sim L(b \supset Pa)$	(assumption)
(3)	$L(b \supset \sim Pa)$	(2, using I3)
(4)	$L(a \supset Fb)$	(1 by DB)
(5)	$L(a \supset FH\sim a)$	(3 and 4, by LG, L3 and L1)
(6)	$L(a \supset \sim a)$	(5 by A5)
(7)	$L\sim a$	(6, contradiction)

Q.E.D.

Similarly, it can be proved that

(DB.2) $L(b \supset Pa) \supset a < b$

This means that

(DB.3) $a < b \equiv L(b \supset Pa)$

The remaining task is to prove (DG) and (DH) for the structure $(\Omega, T, <)$ defined here. This can be done in the following way:

(DG.1) $T(a,Gp) \supset (a<b \supset T(b,p))$
Proof:
This is proved by reductio ad absurdum:

(1)	$T(a,Gp)$	(assumption)
(2)	$a<b$	(assumption)
(3)	$\sim T(b,p)$	(assumption)
(4)	$L(b \supset \sim p)$	(3, using DT and I3)
(5)	$L(Gp \supset G\sim b)$	(4 by LG)
(6)	$L(a \supset Gp)$	(1, using DT)
(7)	$L(a \supset G\sim b)$	(5 and 6)
(8)	$L(a \supset \sim Fb)$	(7)
(9)	$L(a \supset Fb)$	(2)
(10)	$L(a \supset (Fb \wedge \sim Fb))$	(8 and 9)
(11)	$L\sim a$	(10).

(11) contradicts I2. - Q.E.D.

(DG.2) $T(a,Gp) \supset \forall b:(a<b \supset T(b,p))$
Proof:
This follows immediately from the (Π2) and the fact that (DG.1) is proved for an arbitrary b.

(DG.3) $\forall b:(a<b \supset T(b,p)) \supset L(Pa \supset p)$
Proof:
This is proved by reductio ad absurdum:

(1)	$\forall b:(a<b \supset T(b,p))$	(assumption)
(2)	$\sim L(Pa \supset p)$	(assumption)
(3)	$\sim L\sim(Pa \wedge \sim p)$	(2)

(4)	$\exists b: T(b, Pa \wedge \sim p)$	(3 and DL.1)
(5)	$\exists b: T(b,Pa) \wedge T(b,\sim p)$	(4, using T1)
(6)	$\exists b: a<b \wedge T(b,\sim p)$	(5, using DB.3)
(7)	$\exists b: T(b,p) \wedge T(b,\sim p)$	(6 and 1)
(8)	$\exists b: T(b,\sim b)$	(7)
(9)	$\exists b: L\sim b$	(8)

(9) contradicts I2. - Q.E.D.

(DG.4) $\forall b:(a<b \supset T(b,p)) \supset T(a,Gp)$

Proof:

(1)	$(\forall b: a<b \supset T(b,p))$	(assumption)
(2)	$L(Pa \supset p)$	(1 and DG.3)
(3)	$L(GPa \supset Gp)$	(2 and LG)
(4)	$L(a \supset GPa)$	(A4)
(5)	$L(a \supset Gp)$	(3 and 4)
(6)	$T(a,Gp)$	(5)

Q.E.D.

(DG) $T(a,Gp) \equiv \forall b:(a<b \supset T(b,p))$

Proof:

From DG.2 and DG.4.

(DH) $T(a,Hp) \equiv \forall b:(b<a \supset T(b,p))$

Proof:

From (DG) by analogy.

We have now proved that the T we have defined is a suitable T-operator.

As we have seen Lp may be read 'p is provable in Pr_t', given the restriction to Pr_t-wffs. - With this reading of L the theorem (DL) means that p is provable if (and only if) $T(a,p)$ holds for every instant proposition a. If we know that $T(a,p)$ holds for any a in *TIME* in any B-logical structure $(TIME,<,T)$, then p is provable in Pr_t. This leads to the important result that Pr_t is complete relative to the semantics of $(TIME,<,T)$, with no restrictions on the before-after relation $<$.

A number of other interesting theorems may be proved, such as the following ones:

(TX1) $(\forall a: T(a,p)) \supset p$

Proof:

(1)	$L(a \supset p) \supset (a \supset p)$	(L2)
(2)	$(\forall a: L(a \supset p)) \supset (a \supset p)$	(1, Π1)
(3)	$a \supset ((\forall a: L(a \supset p)) \supset p)$	(2)
(4)	$(\exists a: a) \supset ((\forall a: L(a \supset p)) \supset p)$	(3, Π2)
(5)	$(\forall a: L(a \supset p)) \supset p$	(4, I1)
(6)	$(\forall a: T(a,p)) \supset p$	(5)

Q.E.D.

(TX2) $(\forall a: T(a,p)) \supset T(b, \forall c: T(c,p))$

Proof:

(1)	$L(a \supset p) \supset (b \supset L(a \supset p))$	
(2)	$L(a \supset p) \supset L(b \supset L(a \supset p))$	(1, L1, L3)
(3)	$\forall a: L(a \supset p) \supset \forall a: L(b \supset L(a \supset p))$	(2)
(4)	$\forall a: L(a \supset p) \supset L(b \supset \forall a: L(a \supset p))$	(3 and BF)
(5)	$\forall a: T(a,p) \supset T(b, \forall a: T(a,p))$	
(6)	$\forall a: T(a,p) \supset T(b, \forall c: T(c,p))$	

Q.E.D.

(TX3) $T(a,p) \supset T(b,T(a,p))$

Proof:

(1)	$L(a \supset p) \supset (b \supset L(a \supset p))$	
(2)	$L(a \supset p) \supset L(b \supset L(a \supset p))$	(1, L1, L3)
(3)	$T(a,p) \supset T(b,T(a,p))$	

Q.E.D.

In this way formulae of the *T*-calculus are mixed with wffs from Pr_t. In $Prior_t$, everything is included in one single language comprising the *T*-calculus as well as ordinary tense logic. This extended language is simply Pr_t with the addition of the logic of instant propositions - since the elements of the *T*-calculus are introduced by definitions based on Pr_t. This way of seeing things is far from the 'main-stream' tradition within formal logic, where proof theory (in this case the axiomatics of Pr_t) is kept

strictly separated from semantics (in this case the *T*-calculus). But as Prior pointed out there is nothing *semantically* wrong with it, if the *T*-calculus is given an interpretation within tense logic. He also pointed out that such an interpretation could be 'metalogically useful', since in many cases $T(a,p)$ turns out to be easier to prove than the 'bare' tense-logical formula p itself [1967, p.89].

Prior has thus shown that we can in fact interpret B-logic within A-logic, namely in a given modal context in which we can interpret instants as propositions and quantify over them. In this sense B-logical semantics is absorbed within an entirely A-logical axiomatics. In Prior's own words, this means "to treat the first order theory of the earlier-later relation as a mere by-product of tense logic" [1968, p.160].

2.10. METRIC TENSE LOGIC

In the discussion of problems like 'the sea-fight tomorrow' we need more than modal logics and tense-logical systems like K_t and K_l. We also need *metric* tense logic, in which numerical durations are taken into consideration. In metric tense logic it is assumed that some metrical systems for duration (including a relevant time unit) are given. Of course, we have already used metric tense logic in earlier discussions, precisely because this is what is required for cases like 'the sea-fight tomorrow'. In this chapter we shall examine this subject more systematically. Prior himself dealt with the problems of metric tense logic several times. We shall now study his basic ideas and present his version of the so-called minimal system for metric tense logic [Prior 1968, pp. 88-97]. We shall call the system *MT*.

The language of MT is based on a set of propositional variables: $p, q, r\ldots$, and the following definition of well-formed formulae:

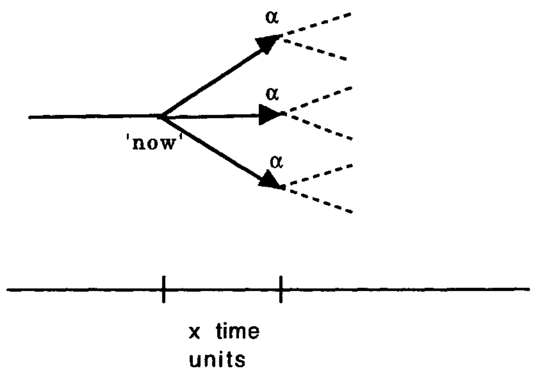
(1) Propositional variables are wff.
(2) If α and β are wff, and x is a positive number, then $\sim\alpha$, $\alpha\supset\beta$, $\alpha\wedge\beta$, $\alpha \vee \beta$, $\forall x{:}\alpha$, $\exists x{:}\ \alpha$, $P(x)\alpha$, and $F(x)\alpha$ are all wff.
(3) There are no other wff.

So the essentially new element in metric tense logic are the expressions $P(x)\alpha$ and $F(x)\alpha$, stating respectively 'x time units ago it was the case that α' and 'in x time units it will be the case that α'. In the following x and y stand for arbitrary positive numbers. N. Rescher [1966] has suggested the use of not only positive, but also negative numbers along with the definition $P(x)\alpha \equiv_{def} F(-x)\alpha$. Prior, on the other hand, argued that things can become very complicated, if we want such a definition in its full generality [1967 p.98]. For this reason he suggested that no negative numbers should occur in metric tense logic. Prior did discuss the use of the number zero in the forms $P(0)\alpha$ and $F(0)\alpha$, but towards the end of his work he decided to leave such

possibilities out. - We shall define

$$G(x)\alpha \equiv_{def} {\sim}F(x){\sim}\alpha \text{ and } H(x)\alpha \equiv_{def} {\sim}P(x){\sim}\alpha.$$

A few words on the semantical intuition involved in these definitions might be profitable. In relation to a branching time model, $G(x)\alpha$ is true iff α is true in x time units from 'now' in any future branch:



The 'quantification' implicit in the G-operator is over the set of all branches rooted in the 'now'. On the other hand it is not required that α after x time units should be true 'forever' on any branch. We mention this explicitly, because we are otherwise used to read G as 'it is always going to be the case that'.

In a model based on linear time, there would be no difference between $G(x)\alpha$ and $F(x)\alpha$, for in such a model ${\sim}F(x)\alpha \equiv F(x){\sim}\alpha$. However, since we are presently concerned with a minimal metric tense logic, we shall make no assumptions on the structure of time. Similar observations go for $H(x)\alpha$ and $P(x)\alpha$.

The axioms of the non-modal part of MT are the following:

(MT1) $G(x)(p \supset q) \supset (G(x)p \supset G(x)q)$
(MT2) $F(x)H(x)p \supset p$
(MT3) $F(y+x)p \supset F(y)F(x)p$

In addition to these axioms Prior also stated some axioms for the past-operator. However, these axioms can be omitted if we introduce the so called 'mirror-image-rule':

> (RM) The 'mirror image' of any theorem (in which all occurrences of P are replaced by F and vice versa) is also a theorem.

For instance,

(MT2') $P(x)G(x)p \supset p$

is the mirror-image of (MT2). In the following, references in proofs to axioms and theorems may also refer to their mirror-image 'versions'. The other rules of MT are the following:

(RMP)	If $\vdash p$ and $\vdash p \supset q$, then $\vdash q$.
(RF)	If $\vdash p$, then $\vdash G(x)p$ (for any x).
(RP)	If $\vdash p$, then $\vdash H(x)p$ (for any x).
(Π1)	If $\vdash \phi(x) \supset \beta$, then $\vdash \forall x{:}\phi(x) \supset \beta$.
(Π2)	If $\vdash \alpha \supset \phi(x)$, then $\vdash \alpha \supset \forall x{:}\phi(x)$, for x not free in α.
(Σ1)	If $\vdash \phi(x) \supset \beta$, then $\vdash \exists x{:}\phi(x) \supset \beta$, for x not free in β.
(Σ2)	If $\vdash \alpha \supset \phi(x)$, then $\vdash \alpha \supset \exists x{:}\phi(x)$.

As in chapter 2.9, we are using Prior's theory of quantification, which presupposes that quantification does not take place over empty sets. Since the quantification in MT is over the set of positive numbers, this is not in practice a restriction. In MT it is possible to prove a number of interesting theorems like

(MT4) $H(x)(p \supset q) \supset (P(x)p \supset P(x)q)$
(MT5) $p \supset G(x)P(x)p$
(MT6) $P(x)G(x)p \supset p$

Prior actually suggested two more axioms than (MT1-3):

(MT7) $\forall x{:}G(y)G(x)p \supset G(y)\forall x{:}G(x)p$
(MT8) $\forall x{:}G(y)H(x)p \supset G(y)\forall x{:}H(x)p$

However, these formulae can be proved in the following way:

(1)	$G(y)q \supset G(y)q$	
(2)	$\forall x{:}G(y)q \supset G(y)q$	(1 and Π1)
(3)	$H(y)(\forall x{:}G(y)q \supset G(y)q)$	(2 and RP)
(4)	$P(y)\forall x{:}G(y)q \supset P(y)G(y)q$	(3 and MT4)
(5)	$P(y)\forall x{:}G(y)q \supset q$	(4 and MT6)
(6)	$P(y)\forall x{:}G(y)q \supset \forall x{:}q$	(5 and Π2)
(7)	$G(y)(P(y)\forall x{:}G(y)q \supset \forall x{:}q)$	(6 and RF)
(8)	$G(y)P(y)\forall x{:}G(y)q \supset G(y)\forall x{:}q$	(7 and MT1)
(9)	$\forall x{:}G(y)q \supset G(y)\forall x{:}q$	(8 and MT5)

In order to get (MT7) and (MT8) we should replace q by $G(x)p$ and $H(x)p$ respectively. Observe that (MT7-8) are very close to the Barcan Formula(e).

On the basis of the metric tense logic MT we can build a non-metric tense logic. We introduce the definitions

(DUF) $Fp \equiv_{def} \exists x{:}F(x)p$
(DUG) $Gp \equiv_{def} \forall x{:}G(x)p$
(DUP) $Pp \equiv_{def} \exists x{:}P(x)p$
(DUH) $Hp \equiv_{def} \forall x{:}H(x)p$

(We are presently not assuming any special structure of time.) Note that in order to make these definitions plausible we have to assume that x is not free in p. As pointed out by Prior [1967, p.95] the reason is that two propositions like

$F(q \wedge F(x)p)$ and $\exists x{:}\ F(x)(q \wedge F(x)p)$

are obviously not equivalent.) - From the above definitions it is possible to prove the axioms of K_t as theorems of MT. Because of the 'mirror-image-rule' it is sufficient to prove (A2) and (A4).

(A2)	$G(q \supset p) \supset (Gq \supset Gp)$	
Proof:		
(1)	$G(q \supset p)$	(assumption)
(2)	$\forall x{:}G(x)(q \supset p)$	(1)
(3)	$\forall x{:}\ (G(x)q \supset G(x)p)$	(2, using MT1)
(4)	$\forall x{:}G(x)q \supset \forall x{:}G(x)p$	(3)
(5)	$Gq \supset Gp$	(4)
Q.E.D.		

(A4)	$q \supset HFq$	
Proof:		
(1)	$F(x)q \supset F(x)q$	
(2)	$F(x)q \supset \exists x{:}\ F(x)q$	(1 by Σ2)
(3)	$F(x)q \supset Fq$	(2)
(4)	$H(x)F(x)q \supset H(x)Fq$	(3 by RP)
(5)	$q \supset H(x)Fq$	(4 and MT5)
(6)	$q \supset \forall x{:}\ H(x)Fq$	(5, using Π2)
(7)	$q \supset HFq$	(6)
Q.E.D.		

In some systems with more axioms than MT the propositions $F(x)q$ and $G(x)q$ will be equivalent - for instance, in systems enlarged with an axiom for forward linearity, as we have already suggested earlier. However, the difference in systems like MT between $F(x)$ and $G(x)$ is interesting. It turns out that in such systems $G(x)$ comes very close to what we have presented as the Peircean notion of 'in x time units it is going to be that', whereas $F(x)$ corresponds to 'in x time units it is possibly going to be'. - It should also be noted that it is possible to define new non-metric tense-operators corresponding to the following four expressions: $\forall x{:}\ F(x)p$, $\forall x{:}\ P(x)p$, $\exists x{:}\ G(x)p$, and $\exists x{:}\ H(x)p$. So MT could in fact give rise to a richer tense logic than K_t.

INSTANT-PROPOSITIONS AND METRIC TENSE-LOGIC

MT can be extended so that the extended system, MT*, includes instants-propositions $a, b, c...$ as well as a modal operator L for which the Barcan formulae

(BF1) $\forall x{:}L\phi(x) \supset L(\forall x{:}\ \phi(x))$
(BF2) $\exists x{:}L\phi(x) \supset L(\exists x{:}\ \phi(x))$

hold along with the axioms

(LHX) $Lq \supset H(x)q$
(LGX) $Lq \supset G(x)q$

Note that MT* involves quantification over positive numbers as well as quantification over instant propositions. Barcan's formulae are assumed for both kinds of quantification. We also use the same quantifier symbols, since it is obvious in each case which kind of quantification is at play.

It is known from modal logic that the Barcan formulae are provable in any 'L-calculus' satisfying a few basic conditions. However, in the same way as in chapter 2.9 we may intuitively assume L to be at least as strong as 'provability' in MT. For these reasons the results demonstrated in the following with respect to MT* could also serve as an argument for the kind of completeness mentioned in chapter 2.9.

It is obvious that the following theorems can be proved from the axioms mentioned above

(LH) $Lq \supset Hq$
(LG) $Lq \supset Gq$

The assumptions (I1)-(I3) are also added to the system. This means that we now have a system stronger than the one we developed in chapter 2.8-9. Hence, the proofs of (T1-2) regarding the 'truth-operator' T can also be carried out in the present system. - In addition, we define

(DB1) $before(a,b,x) \equiv_{def} L(a \supset F(x)b)$

We can now prove a number of further theorems:

(T8) $before(a,b,x) \equiv L(b \supset P(x)a)$

Proof: This is proved by reductio ad absurdum:

(1)	$L(a \supset F(x)b)$	(assumption)
(2)	$\sim L(b \supset P(x)a)$	(assumption)
(3)	$L(b \supset \sim P(x)a)$	(2, using I3)
(4)	$L(F(x)b \supset F(x){\sim}P(x)a)$	(3 and MT4)
(5)	$L(a \supset F(x){\sim}P(x)a)$	(1 and 4)
(6)	$L(a \supset F(x)H(x){\sim}a)$	(5)
(7)	$L(a \supset \sim a)$	(6 and MT2)
(8)	$L{\sim}a$	(7)

(8) is contradicting I1!- The opposite implication is similar. Q.E.D.

(T9) $a<b \supset \exists x: before(a,b,x)$

Proof: This is proved by reductio ad absurdum:

(1)	$a<b$	(assumption)
(2)	$L(a \supset \exists x{:}F(x)b)$	(1, DB, and DUF)
(3)	$\sim(\exists x: before(a,b,x))$	(assumption)
(4)	$\forall x: \sim L(a \supset F(x)b)$	(3, using DB1)
(5)	$\forall x: L(a \supset \sim F(x)b)$	(4, using I3)
(6)	$L(a \supset \forall x: \sim F(x)b)$	(5 and BF1)
(7)	$L(a \supset \sim a)$	(2 and 6)
(8)	$L{\sim}a$	(6; contradicts I2)

Q.E.D.

(T10) $\exists x: before(a,b,x) \supset a<b$

Proof:

(1)	$\exists x: before(a,b,x)$	(assumption)
(2)	$\exists x: L(a \supset F(x)b)$	(1, using DB1)
(3)	$L(\exists x: (a \supset F(x)b))$	(2 and BF2)
(4)	$L(a \supset \exists x: F(x)b)$	(3)
(5)	$L(a \supset Fb)$	(4, using DMF)
(6)	$a < b$	(5, using DB)

Q.E.D.

It follows from (T9) and (T10) that

(T11) $a<b \equiv \exists x: before(a,b,x)$

The theorems corresponding to the definitions of $F(x)q$ and $P(x)q$ in any T-calculus can now be proved:

(T12) $\exists b: (before(a,b,x) \wedge T(b,p)) \supset T(a,F(x)p)$

Proof:

(1)	$L(a \supset F(x)b)$	(assumption)
(2)	$L(b \supset p)$	(assumption)
(3)	$L(F(x)b \supset F(x)p)$	(2)
(4)	$L(a \supset F(x)p)$	(1 and 3)

Q.E.D.

(T13) $L(P(x)a \supset {\sim}p) \supset L(a \supset {\sim}F(x)p)$

Proof:

(1)	$L(P(x)a \supset {\sim}p)$	(assumption)
(2)	$L(G(x)P(x)a \supset G(x){\sim}p)$	(1)
(3)	$L(a \supset G(x){\sim}p)$	(2 and MT5)
(4)	$L(a \supset {\sim}F(x)p)$	(3)

Q.E.D.

(T14) $T(a,F(x)p) \supset (\exists b: before(a,b,x) \wedge T(b,p))$

Proof:

(1)	$T(a,F(x)p)$	(assumption)
(2)	$L(a \supset F(x)p)$	(1)
(3)	${\sim}L(a \supset {\sim}F(x)p)$	(2)
(4)	${\sim}L(P(x)a \supset {\sim}p)$	(3 and T13)
(5)	${\sim}L{\sim}(P(x)a \wedge p)$	(4)
(6)	$\exists b: T(b,P(x)a \wedge p)$	(5 and DL)
(7)	$\exists b: (T(b,P(x)a) \wedge T(b,p))$	(6)
(8)	$\exists b: (before(a,b,x) \wedge T(b,p))$	(7)

Q.E.D.

It follows immediately from (T12) and (T14) that

(T15) $T(a,F(x)p) \equiv (\exists\, b\text{: } before(a,b,x) \wedge T(b,p))$.

Moreover, by using the definition of *before(a,b,x)* (DB1) and the 'mirror-image rule' RM on (B19), we obtain

(T16) $T(a,P(x)p) \equiv (\exists\, b\text{: } before(b,a,x) \wedge T(b,p))$

Prior suggested that *MT* corresponds to any *T*-calculus for which we have (T1-2), (T15), (T16), and

(T17) $T(a,\exists x{:}q) \equiv \exists x{:}T(a,q)$
(T18) $before(a,b,x{+}y) \supset \exists c{:}(before(a,c,x) \wedge before(c,b,y))$

These theses can also be proved in the instant-calculus. It can be done in the following way:

(T17.1) $T(a,\exists x{:}q) \supset \exists x{:}T(a,q)$
Proof: This is proved by reductio ad absurdum.

(1)	$T(a,\exists x{:}q)$	(assumption)
(2)	$\forall x{:}{\sim}T(a,q)$	(assumption)
(3)	$\forall x{:}L(a \supset {\sim}q)$	(2)
(4)	$L(\forall x{:}(a \supset {\sim}q))$	(3 and BF1)
(5)	$L(a \supset \forall x{:}{\sim}q)$	(4)
(6)	$L(a \supset {\sim}(\exists x{:}q))$	(5, contradicting 1)

Q.E.D.

(T17.2) $\exists x{:}T(a,q) \supset T(a,\exists x{:}q)$
Proof:

(1)	$\exists x{:}T(a,q)$	(assumption)
(2)	$\exists x{:}L(a \supset q)$	(1)
(3)	$L(\exists x{:}(a \supset q))$	(2 and BF2)
(4)	$L(a \supset \exists x{:}q)$	(3)

Q.E.D.

(T18) *before(a,b,x+y)* $\supset$ $\exists$*c:(before(a,c,x)* $\wedge$ *before(c,b,y))*

Proof:

(1) *before(a,b,x+y)*

(2) *T(a,F(x+y)b)*

(3) *T(a,F(x)F(y)b)*

(4) $\exists$*c:(before(a,c,x)* $\wedge$ *T(c,F(y)b))*

(5) $\exists$*c:(before(a,c,x)* $\wedge$ *before(b,c,y))*

Q.E.D.

The MT axioms are valid in each *T*-calculus in which (T1-2) and (T15-18) hold. Such a *T*-calculus can be said to be minimal, since no further conditions on the before-relation have to be assumed. Given a set of instant-propositions for which (I1-3) hold, and a modal operator *L* for which (BF1-2) and (LHX + LGX) hold, we have also demonstrated that it is possible to define a *T*-calculus within MT. In that sense MT corresponds to a minimal *T*-calculus. Given the existence of instant propositions, any theorem that can be proved in MT will also be valid in any *T*-calculus, and vice versa.

3.1. Two Paradigms of Temporal Logic

> *Not only do we measure the movement by the time, but also the time by the movement, because they define each other. The time marks the movement, since it is its number; and the movement the time.*
> [Aristotle, Physics, IV 220 b]

Since Antiquity two images of time have been discussed: the flow of the river and the line made up of stationary points. The tension between the two pictures of time, the dynamic and the static view, has for instance been expressed by the Aristotelian idea of time as the number of motion with respect to earlier and later - an idea, which comprises both pictures. On the one hand time is linked to motion, i.e. changes in the world, and on the other hand time can be conceived as a stationary order of events represented by numbers.

The basic set of concepts for the dynamic understanding of time are *past*, *present*, and *future*. After McTaggart's analysis of time, these concepts are called the A-concepts. They are well suited for describing the flow of time, since the present time will become past, i.e. flow into past. The basic set of concepts for the stationary understanding of time are *before*, *simultaneously*, and *after*. Following McTaggart, these are called the B-concepts, and they seem especially apt for describing the permanent and temporal order of events.

Philosophers and others still discuss intensively which of the two conceptions is the more fundamental one for the philosophical description of time. The situation can well be characterised as a debate between two Kuhnian paradigms - the ideas embodied by the well-established B-theory, which were for some centuries predominant in philosophical and scientific theories of time, and the rising A-theory, which in the 1950s received a fresh impetus due to the advent of Prior's tense logic. Still, many researchers do not want to embrace the A-conception. The main reason for this reluctance is the perception that the A-theory does not have the 'precedence' or priority, which can apparently

be attributed to the B-theory. Some arguments often put forward in favour of the B-theory are these:

1) A-concepts can only be defined in relation to B-concepts.
2) B-concepts cannot be reduced into A-concepts.
3) The future is just as real as the past.
4) A-concepts are not objective, but depend upon emotions.
5) B-concepts are objective.
6) The physical theories about time support the B-theory.

In the following we shall ourselves come down quite openly on one side in the conflict: we shall argue that there is indeed a stronger case for the A-conception than for the B-conception. Surely the B-concepts do play a rôle in thought and language; but they should in our opinion be seen as secondary to or derived from the A-concepts. In our argumentation, we shall go into detail with the theses (1)-(6). Towards the end of the chapter we shall be concerned with McTaggart's paradox and its relevance for the debate; we shall also outline the general contours of the two paradigms.

REDUCTION OF A-CONCEPTS

Many B-theorists have maintained that every statement couched in A-concepts can be reformulated into a statement in terms of B-concepts - or at least that its truth conditions can and should be defined by B-concepts. Take for instance the A-statement 'It will rain in London', and assume it to be uttered at 1:51 p.m. on the 10th of November in the year 1994. According to B-theorists this statement is completely equivalent with the B-statement 'It rains in London at a moment of time after 1:51 p.m. on the 10th of November in the year 1994'.

It should be noted that it is only possible to reduce the A-statement if the time of utterance is known. Already for this reason the suggested procedure for reducing the A-statement is not satisfactory. Indeed, it is not truly possible to express past,

present and future in terms of the B-theory. How, for instance, could it be possible to express 'E has happened' in such a B-language? One might try to go for something like 'E is before the utterance of this very statement'. However, such a formulation is really based on a concealed reference to the present. In other words, that which should be explained away (tenses) is really presupposed - the definition is circular.

In addition, there are examples of A-statements that cannot be reduced in that way without demonstrable loss of meaning. Consider, for example, this statement, uttered at 1 p.m. on a given date: 'Fortunately, my consultation with the dentist is over'. According to the B-theory the statement should be reduced into this: 'Fortunately, the consultation with the dentist is before 1 p.m. (on the given date)'. This B-statement, however, exhibits no semantic equivalence at all with the A-statement which is uttered at 1 p.m. While it is possible for a person by means of the A-statement to express the reason for his or her joy and relief, the B-statement merely states that the temporal arrangement between two instants is fortunate: 'Fortunately, t_1 is before t_2!'

In the B-reduction of the A-statement valuable semantic content pertinent to the reason for one's joy and relief - the fact that the pain *is over* - has been lost. Similarly, the B-theoretical reduction of an A-statement about the future would fail to capture such semantic content which could give one reason to experience fear, expectation or excitement. On this background it must be reasonable to reject the B-theoretical procedure of reduction.

REDUCTION OF B-CONCEPTS

It is quite common among B-theorists to maintain that B-concepts cannot be reduced into A-concepts. If one would try to do that, the argument goes, it would be necessary to operate with new and extra concepts like 'more past than' and 'more future than'. Such an idea apparently stems from the perception that

in A-theory, one can merely express that an event is future, but not that one event is in a nearer future than some other event - and analogously for the past. In other words, they assume that in A-theory a temporal ordering among events cannot be expressed in general. This statement, however, is plainly wrong. Of course, it is immediately disproved if we take metric tense logic; but even if we take tense logic in its 'pure' form, it is possible to reduce B-concepts into A-concepts without any loss of meaning. Take the statement "The event E_1 will take place prior to the event E_2'. This B-statement can be reduced into the following A-statement: '(Sometime) in the future, it true that E_1 is present and E_2 is future'. In general, B-statements of the form 'The event E_1 is prior to the event E_2' can be reduced into A-statements of the form: The following is either past, present or future (true): 'E_2 is present and E_1 is past'. Naturally, when giving such a definition in terms of tense logic one must ensure that this relation between events actually satisfies all demands imposed on the before/after-relation of the B-theory.

The reduction of the B-theoretical concept of an instant into A-concepts can only be carried out by employing Prior's definition of instants as a special kind of propositions. How this can be done formally we have already seen in chapter 2.9 and 2.10. However, Robin Le Poidevin [1991, p. 36 ff] has argued that Prior's propositional theory of instants is in tension with another basic tenet of Prior's, namely what Poidevin has called the anti-realist construal of past and future tensed statements. He has also maintained that the theory is based upon an incoherent view of propositions, namely the idea that different tokens of the same tensed type (e.g. 'Socrates is sitting') uttered at different times express the same proposition. In both cases Poidevin's criticism is based on the assumption that according to Prior's view tensed propositions have "present fact as their truth-conditions" [1991, p. 37]. In our opinion, Poidevin's analysis is sound and interesting on its own terms, but we believe that it based on a wrong assumption. In Prior's theory the notion of truth-conditions is not basic. It is in fact - as we have argued in part 2 - a *derived idea*. The semantical concept of being true at an instant is defined in terms of present truth. But as Richard

Swinburne has pointed out 'there is more to be known about the world than you can know by knowing the truth-values of sentences at their time of utterance. You need to know which ones are true now, which of the ones which are were, or will be true when uttered are true now. And for such truth timeless truth conditions cannot be given' [1990, p. 121]. It is an essential element of Prior's theory that the very common assumption of truth as something timeless has to be rejected.

THE REALITY AND THE POSSIBLE FUTURE

Some B-theorists have argued that the future is just as real as the past. They have claimed that statements about the future are true or false today, exactly in the same manner as statements about the past. Naturally this realism with respect to the future is necessary if one advocates a symmetrical concept of time - as a number of B-theorists do. With this symmetrical concept it is possible to be safeguarded against a certain line of A-theoretical arguments, namely arguments based on the assumption of truth-value gaps. For example, the idea that statements about the future (seen in contrast to statements about the past) have no truth-value today calls for a truth-value gap.

But as we have seen it is possible to formulate a tense logic (i.e. an A-theory) without having to give up the notion of the reality of the future, whilst preserving the natural asymmetry between past and future (the Ockham system). Some of these theories can be traced back to a number of medieval considerations regarding human freedom and divine foreknowledge.

While the assumption about the reality of the future can very well be consistent with both B-theories and A-theories, the case is different when looking at the assumptions about alternative future possibilities. Tentatively, that assumption might be expressed as the following maxim: 'Whereas it is impossible to change the past, it is possible to change the future'. But that formulation oversimplifies the matter. First, it excludes the reality of the future, and second, it is really self-contradictory to

say that one can change *the future*. The principle in question should rather be put in this way: 'Whereas no (real) alternative possibilities of the past are available, alternative possibilities of the future are (at least sometimes) available'. An essential feature in our daily lives is reflected in this formulation. It is a fundamental part of our experience of reality that for instance today, one can choose to travel to Copenhagen tomorrow, and one can also choose to stay at home. Furthermore it is clearly also a part of our experience that one today cannot choose to travel to Copenhagen yesterday. If you were not in Copenhagen yesterday, you have actually lost any possibility you may have had of going there yesterday - and you shall have to try to bear the loss. On the other hand, in some sense a certain kind of alternative possibilities of the past are in fact available for us today: one can choose to 'make it true yesterday' that one would arrive in Copenhagen within two days - this may be seen as a side-effect of choosing today to go to Copenhagen tomorrow (provided that this choice is also effectuated). However, this kind of influence is certainly felt as somewhat spurious, and it is a task for tense logic (i.e. A-logic) to determine the difference between 'genuine' and 'spurious' events of the past.

In our opinion, such considerations build a crucial part of an argumentation in favour of the A-theory. It is in fact possible to describe the asymmetry between past and future in terms of A-theory, and in such a manner as to leave open the room for genuine choice (with respect to the future). Moreover, it is possible by reference to the loss of possibilities to define the contents of our perception of the passing of time - a notion which does not fit into the basic framework of the B-theory. We believe that our experience of some freedom of choice - even though it is limited - as well as the passing of time are not illusions, but that these phenomena are properties of reality; and on that premise we have every reason to claim that A-theory reflects this reality better than does B-theory.

THE OBJECTIVITY OF THE A-CONCEPTS

It is sometimes argued that A-concepts are not objective, but on the contrary purely subjective concepts which are highly dependent on our consciousness or mind. And it is certainly true that the A-concepts account well for significant parts of subjective human experience - they are indeed *also* meant to do so. Nevertheless, it is very problematic to assert that A-concepts are entirely relative to individual human minds; for then it becomes very difficult to explain the kind of intersubjectivity, which forms the basis for the practical agreement concerning the Now. This agreement in everyday experience is strong evidence to the intersubjectivity of the A-concepts, and we emphasise that these concepts are in any case not 'private'. Within a group of observers it will not be difficult to establish some agreement regarding the A-concepts. Now what about the B-theoretical claim that B-concepts are objective? A-theorists have no quarrel with that contention, for the obvious reason that A-theory sees B-concepts as being derived from the intersubjective (objective) A-concepts.

In an interesting way the controversy between A-theory and B-theory reflects why and how time was relegated from logic for centuries. It is true that the B-theory incorporates some notion of time into logic, but essentially it is based on the long-cherished idea of tenseless propositions, into which it also strives to reduce tensed propositions. The idea of logic as a science concerned with tenseless propositions was really the major obstacle to the (re)introduction of time into logic. It must be said, therefore, that B-theory only admits time into logic in the most restricted sense possible. To the contrary, A-theory in its fullest consequence actually denies the existence of tense-less facts at all, holding that in principle all facts must be said to be tense formed. The A-theorist will maintain that our immediate perception is given in a tensed-formed way. The memory of a perception is obviously described by means of the past tense, while hope, expectation and choice are described by means of the future tense. Our communication, indeed our social lives with each other, are dependent upon the use of modal and tense-logi-

cal concepts. Therefore the temporal description of reality is inseparable from the assumption of the possibility of human experience and communication.

TIME AND EXPERIENCE

Many B-theorists maintain that the B-concepts (in contrast to the A-concepts) can be experienced directly. But it quickly turns out that this position is very difficult to support. When a B-theorist claims to be able to experience directly that for instance 'E_1 is before E_2', the A-theorists can refer to the fairly obvious fact that in reality this experience has been gained from at least two 'A-experiences', namely the experiences of the respective events. That is, there is no direct experience corresponding to 'E_1 is before E_2'; the experience expressed by such a statement presupposes the experience of E_1 and E_2, respectively. It seems to be clear that in principle all experience must be 'now-experience'. This does not necessarily mean that all statements of experience must refer to events happening now, but only that experience is actually being gained now. For instance, we may have the 'now-experience' that E_2 is happening and the 'now-experience' that E_1 has happened.

The 'nowness' of experience notwithstanding, it is desirable - also from the point of view of an A-theorist - to reformulate as much as possible of the experience statements into earlier-later statements like 'E_1 is before E_2'. In this way it is possible to achieve a formulation of the experience in question such that the truth-value does not vary with time. The A-theory in no way has to forego expressive advantages, which may for some cases be attached to B-theoretical formulations; A-theory merely makes it clear where the B-formulated experience originally comes from - that is, how we should understand the statement 'E_1 is before E_2'. It is worth noting that not all 'now-experience' refers directly to the present. When for instance one sees (the light from) a star, one may be well aware of the fact that the emission of the light is a past event, even though the ex-

perience of the light is a present one. Strictly speaking, one cannot experience the pastness of the emission of the light, but this is a limitation which would also occur in any B-theoretical epistemology.

There is, in fact, nothing to support the view that it is possible for us to observe B-relations between events directly. We find it very hard to see how it can be possible to observe that one event is later than another, without presupposing that this observation is composed of some A-observations - i.e. now-experiences of the respective events. It seems that an immediate observation of B-relations could only be attributed to God, as indeed it is in Thomas Aquinas' philosophy. According to Thomas, all events are present to God at once. Thus God has a timeless knowledge about events and their mutual relations, which means that God knows and understands B-relations directly. One might perhaps say that the B-theory belongs in a description of reality-as-it-is-to-God. B-theory ignores specifically human limitations and conditions. When, however, the aim is to discuss temporal relations in reality-as-it-is-to-us, we have no doubt that the A-theory is the obvious solution. Moreover, to the extent that our experience of reality is actually true and not an illusion, it seems that A-concepts are indispensable. In fact, as pointed out by N. Lawrence [1978, p. 24] it is very hard for us to imagine any kind of temporal notion not related to human experience.

A- AND B-CONCEPTS IN PHYSICS

We have ourselves come out quite openly as supporters of the A-theory. Where human experience, communication and language are involved, B-theory can be criticised on many counts. But A-theory, as we have emphasised, is not meant to be concerned with these matters *as opposed to* a mind-independent physical reality. To some degree, it is actually a defence of the reality of some basic human experiences. Therefore, evidence from hard science such as Physics must be taken into account -

as a minimum, it must be shown that A-theory is not in conflict with empirical science.

Now many B-theorists in fact maintain that evidence especially from Physics directly supports their theory, as opposed to the A-theory. This is supposed to be of particular importance, since it is assumed that physics has a very special rôle to play in the study of time. This view has been very clearly stated by Hans Reichenbach:

> There is no other way to solve the problem of time than the way through physics. More than any other science, physics has been concerned with the nature of time. If time is objective, the physicist must have discovered that fact, if there is Becoming the physicist must know it; but if time is merely subjective and Being is timeless, the physicist must have been able to ignore time in his construction of reality and describe the world without the help of time. Parmenides' claim that time is an illusion, Kant's claim that time is subjective, and Bergson's and Heraclitus' claim that flux is everything, are all insufficiently grounded theories. [Reichenbach, 1956, p. 16]

One argument fielded by B-theorists is the contention that within physics time is treated as a parameter associated with a relation, which is to a very large extent similar to the before/after-relation. This is true, but there is a fairly simple reply to it: the time parameter within physics must be understood as a theoretical construct, which conceptually should really be traced back to the A-concepts.

Another argument is that B-concepts fit nicely with the special theory of relativity, specifically, with the relativity of simultaneity. According to Minkowski's rendition of relativity, time is understood in terms of geometry. That is, time *exists* in the same sense as space, in an atemporal way. This can be illustrated by statements like the following by H. Weyl:

> The objective world simply *is*, it does not *happen*.. Only to the gaze of my consciousness, crawling upward along the

> life line of my body, does a section of this world come to life as a fleeting image in space which continuously changes in time. [Weyl, 1949, p. 116]

According to this interpretation, it seems that relativity is in conflict with fundamental A-theoretical concepts about becoming and happening, i.e. 'the passage of time'. However, as we have seen in chapter 2.7, the tense-logical position does not contradict any part of the empirical basis of the theory. Not all questions regarding relativistic tense logic have been satisfactorily answered, but the A-theory can by no means be rejected out of hand by appealing to the special theory of relativity.

Indeed, within the natural sciences there is ample proof of assumptions and ideas which are more in harmony with the A-theory than with the B-theory. The theories of prediction and beforehand calculations illustrate this fact. Hans Reichenbach has argued that

> The concept of 'becoming' acquires significance in physics: the present, which separates the future from the past, is the moment at which that which was undetermined becomes determined, and 'becoming' has the same meaning as 'becoming determined'. [Grünbaum 1973, p. 320]

In this way Reichenbach has pointed out that there is a crucial difference between past and future, which the physicist has to take into serious consideration. The difference is that there are future facts which cannot possibly be predicted, whereas in principle any past fact can be recorded. This makes it possible to establish an epistemological basis for a tense-logical approach within the physical sciences. In particular, this seems to be important with respect to 'quantum logic', which is the conceptual foundation of quantum physics. C. F. von Weizsäcker has stated this in the following way:

> The most general presupposition of experience is time. Its structure, as expressed by the words present, past and future is analysed in a logic of temporal propositions (tense-

> logic) which provides the conceptual frame for the quantum logic and for the theory of objective probability. [Weizsäcker 1971, p. 236]

MCTAGGART'S PARADOX

It was McTaggart who explicitly identified the dichotomy between two major conceptions of time and labelled them 'A' and 'B', respectively. McTaggart himself arrived at the conclusion that A-concepts are more fundamental than B-concepts. He did not, however, use this analysis as an argument in favour of A-theory. On the contrary, he used it for a refutation of the reality of time! McTaggart argued that A-concepts give rise to a contradiction - which has become known as 'McTaggart's Paradox'. Due to this putative contradiction within the fundamental conceptualisation of time, he went on to claim that time is not real.

The core of McTaggart's argument is that the notions of 'past', 'present' and 'future' are predicates applicable to events. The three predicates are supposed to be mutually exclusive - any concrete event happens just once (even though a *type* of event may be repeated). On the other hand, any of the three predicates can be applied to any event. In a book on history, it makes sense to speak of 'the death of Queen Anne' as a past event - call it e_1 - but in a document written in the lifetime of Queen Anne, it could well make sense to speak about her death as a future event. Apparently this gives rise to an inconsistency, since how can e_1 be both past and future - and present as well, by a similar argument? The answer must be that there is another event e_2, relative to which for instance e_1 has been present and future, and is going to be past. Now, the same kind of apparent inconsistency can be established with respect to e_2, and the problem can only be solved by introducing a new event e_3, for which a new apparent inconsistency will arise etc. - which seems to mean that we have to go ad infinitum in order to solve the inconsistency. The consequence appears to be that the inconsistency can never be solved.

Prior, however, pointed out a basic flaw in McTaggarts argument: the contradictions arise from an attempt at forcing the A-series notions into a B-series framework [1967, p. 6]. Events may be described in terms of Prior's instant-propositions, of which it also holds that they 'happen', i.e. are true, exactly once. The condition that the three predicates are mutually exclusive can be formulated as:

$$a \supset (\sim Pa \wedge \sim Fa)$$
$$Pa \supset (\sim a \wedge \sim Fa)$$
$$Fa \supset (\sim a \wedge \sim Pa)$$

The fact that any event can be past, present, and future, can be expressed in the following way, where the I-operator stands for 'the present':

$$Ia \supset (PFa \wedge FPa)$$
$$Pa \supset (PIa \wedge PFa)$$
$$Fa \supset (FPa \wedge FIa).$$

But no contradiction follows from these 6 theses. It is thus revealed that McTaggart's paradox is in no way a cogent argument against the A-series notions, let alone the reality of time.

THE TWO PARADIGMS

Both the A-theory and the B-theory are internally consistent theories. They can both profitably be used for describing a range of temporal phenomena, and indeed, from a formal point of view each of the theories can be 'absorbed' within the other, under certain premises. So why would philosophers (and we, too) present them as competing paradigms? What is at stake here is a question of two different ways of understanding reality, and consequently also two different languages for description. One might refer to Henri Bergson who discussed the use of space

metaphors for the description and analysis of temporal phenomena. Such a language is clearly B-like. According to Bergson this language is obviously unsatisfactory. He denied that time can be adequately represented by space. In this way we can only deal with "time flown" and not with "time flowing". [Bergson 1950, p. 221]. However, Bergson did not say very much about how to express more precisely "time flowing", that is, he did not suggest any A-like language.

The two theories oppose each other as two general frameworks, as two different answers to the question of how we should fully understand the temporal relations in the world. Hence, we find it natural to compare them with Kuhnian paradigms. The advocates of the A-paradigm and the B-paradigm, respectively, form two scientific communities with a discussion going on between them. Neither group can present logically cogent arguments. Of course, considerations of logical properties such as expressive power, elegance etc. are not irrelevant - and the same thing obviously goes for 'circumstantial evidence' from empirical sciences. But essentially, the argumentation offered is of a metaphysical nature.

From our own A-theoretical position, we think that the following two issues should be weighted highly in the debate between the paradigms: firstly, it seems that the processes of perception, observation, and cognition can only be described satisfactorily by means of the A-concepts; secondly, the temporal asymmetry between past and future, and the passage of time can only be described satisfactorily by means of the A-theory. Both of these points, together with the importance of communication, also suggest that A-concepts are closely related to natural language.

3.2. INDETERMINISTIC TENSE LOGIC

It is sometimes argued that classical physics establishes a convincing case for determinism, and against human freedom of choice. In its simplest - and original - form, this argument is based on the assumption that all the individual elements of the human brain and body interact according to Newtonian mechanics. This was clearly the assumption in the famous argument for the idea of *L'Homme-Machine*, 'The Man Machine', by La Mettrie (1709-51). According to John Cohen, La Mettrie "seems to have been the first to state the problem of the mind in terms of physics" [Cohen 1966, p. 70]. In fact, there was also a clearly temporal aspect to La Mettrie's idea:

> La Mettrie was no doubt encouraged to make his grand extrapolation by the ingenious successes of contemporary horologists. [Cohen 1966, p. 70]

That very same aspect of temporality has also been a basis for criticising La Mettrie's 'Horloge Model', since this model exhibits a "paradoxical timelessness, that is, its insensitivity to duration, which is so vital a feature to human experience" [1966, p. 70]. Such a criticism could in fact be carried out with reference to the philosophy of Henri Bergson who presented a very interesting analysis of the problem of human freedom in relation to the concept of time:

> Freedom is ... a fact, and among the facts which we observe there is none clearer. All the difficulties of the problem ... arise from the desire to endow duration with the same attributes as extensity, to interpret a succession by a simultaneity, and to express the idea of freedom in a language into which it is obviously untranslatable. [Bergson 1950, p. 221]

According to Bergson the idea of human freedom is in fact indefinable. He argued that "we can analyse a thing but not a process; we can break up extensity, but not duration" [Bergson

1950, p. 219]. This seems to be completely correct if all we have got are the conceptualisations which constitute a B-series conception of time. In the following we shall, however, assume that an A-series conception of time is possible. We shall concentrate on an argument concerned with the rôle of human communication when confronted with the idea of 'the man machine'.

According to Newtonian mechanics all past and future states of a closed system are implicit in the present state. If man is such a system, then all future decisions of a human being can in principle be predicted by an observer, who is fully informed of every relevant aspect of the present state. Therefore, there seems to be no freedom of choice, and there seems to be no way to argue that the human feeling of freedom is more than a mental illusion.

The basic idea in the above argument is that the state of any individual is in principle a conjunction of statements

$$q_1 \wedge q_2 \wedge \ldots \wedge q_N$$

all being true 'now' and therefore also now-unpreventable (i.e. necessary). Moreover, for any possible act α, which the person in question may perform in the future, the laws of classical physics make it necessary that either the person will or will not perform that act (for any given amount of time). Let A be the statement 'this person performs α', so that $F(1)A$ stands for 'this person is going to perform α tomorrow' (taking 'tomorrow as our example). It then follows from the assumptions discussed so far that either (1) or (2) must be true:

(1) $N((q_1 \wedge q_2 \wedge \ldots \wedge q_N) \supset F(1)A)$
(2) $N((q_1 \wedge q_2 \wedge \ldots \wedge q_N) \supset F(1){\sim}A)$

This means that it is already now given whether the person is going to perform α or not. Hence, the person has no freedom of choice, and it cannot consistently be maintained that it is possible that he will perform α, but also possible that he will not perform α. That is, the following formula is ruled out:

(3) $MF(1)A \wedge MF(1){\sim}A$

It is well known today that the premises of this argument do not hold in general. After all classical mechanics has been replaced by quantum mechanics, for which the deterministic ideal does not hold at the microscopic level. Nevertheless, various 'modernised' versions of the argument can be constructed, or it might simply be maintained that Newtonian mechanics still holds for the relevant parts of the brain and body system. However, Donald M. MacKay [1971,1973,1974] has shown that even if everything were mechanistic, the classical argument has to be rejected, when the possibility of communication is taken into consideration. We shall present the main line of MacKay's argument:

Let AG be an agent and let P be a predictor. Assume that P is fully informed of AG's state at the time t_1. Making use of this knowledge, P predicts what AG will do at some later time t_2. That is, P states the prediction $F(1)A$ (where t_1 corresponds to the present instant, and t_2 is an instant one time unit later). Now, is $F(1)A$ true at t_1? MacKay had his own definition of truth, which deserves to be mentioned here. He was by training a physicist, and his analysis of the epistemology of prediction led him to the following notion of truth:

> I do, as a matter of fact, prefer to reserve the word 'true' for propositions that anyone and everyone would be correct to believe and in error to disbelieve. [1974, p. 108]

In this sense the prediction $F(1)A$ is not true at t_1. For if the prediction is communicated to AG at t_1, the state of AG's mind (or brain and body system) will be changed, and the prediction will not be valid in general. The very premises of the prediction are changed, if it is communicated to AG. This means that the logical structure of the necessity based on the classical laws is not (1), but rather

(4) $N((q_1 \wedge q_2 \wedge \ldots \wedge q_N) \supset ({\sim}CF(1)A \supset F(1)A))$,

where C is an operator meaning 'it is communicated to AG that ...'. So P might predict AG's future decision, but since he has to assume that the prediction is not communicated to AG - $\sim CF(1)A$ - there does not exist an unconditional prediction of AG's future actions. Thus, even on the (really very strong) assumption that human behaviour is completely determined by classical mechanical laws, P can deduce no more than

(5) $N(\sim CF(1)A \supset F(1)A)$.

Along the same lines J.W.N. Watkins [1971] has pointed out that there is a *ceteris paribus* clause involved in predictions of future decisions of cognitive agents. According to Watkins, predictions can be valid only on the assumption that they are not communicated to the agents in question, exactly as stated in (4). In this way, even silent predictions concerning cognitive agents are conditional. This meets an obvious - although rather superficial - rejoinder to MacKay's argument, namely that AG's actions are still deterministic, if P simply chooses not to communicate his prediction to AG. Watkins makes it clear that even though one may choose to keep silent, one's predictions are still conditioned by exactly that choice! Therefore, it must be concluded that the possibility of silent prediction of AG's future decision does not imply that AG is unfree, since there is no way to demonstrate that a predicted decision is necessary. There is no fully determinate specification of AG's future decision, which he would be correct to accept as inevitable, and would be unable to falsify, if it was communicated to him. For these reasons it is not possible to maintain $NF(1)A \vee NF(1)\sim A$ even within the classical framework. Therefore, freedom of choice as expressed in (3) can still be consistently upheld.

MacKay has claimed that even if an agent's brain and body system was as mechanical as a clockwork, it cannot be proved that such a system is unfree, as long as it can receive information and react correspondingly. This seems to raise a question about the scope of MacKay's argument. Thus Landsberg and Evans [1970] have argued that even a computer could be free according to the argument. That is in a sense correct. MacKay's

argument certainly does not refute or rule out that computers can be free. On the other hand, MacKay has neither argued nor proved such an idea. His argument does not pretend to prove that human beings (or possibly comparable systems) *do* have a free will, but rather it is concerned with consistency. He has shown that the existence of a statement A for which (3) holds is consistent with fundamental deterministic assumptions - by showing how (1-2) must be made conditional. The question about the *reality of freedom of choice* has to be discussed with reference to other lines of reasoning, for instance metaphysical or theological. MacKay himself maintained that it is not computers, nor brains, but conscious agents who may or may not be free. According to him, the crucial property of free agents is their capability of believing - correctly or incorrectly - what they are told [1974, p. 111].

MacKay's definition of truth as well as the observations so far imply that a 'silent' prediction like *F(1)A* can not be classified as true, even if A in fact turns out to be true after one time unit. It is interesting that MacKay's notion of truth for future-tense propositions is very similar to Peirce's position with respect to the contingent future - a position also adopted by Prior. MacKay did not develop any proper logical system corresponding to his ideas. But as we shall see in the following Prior certainly did that for a very similar set of ideas.

Prior like MacKay had a strong commitment to what he called 'a belief in real freedom'. In his opinion, one of the most important differences between the past and the future is that

> ...once something has become past, it is, as it were, out of our reach - once a thing has happened, nothing we can do can make it not to have happened. But the future is to some extent, even though it is only a very small extent, something we can make for ourselves... [SFTT, p. 2].

Prior wanted to develop an indeterministic tense-logic. As he developed Kripke's ideas from 1958 (cf. chapter 2.5), Prior related his belief in real freedom to the concept of branching time:

> Genuine determinism would be the belief that there is only one possible future, and to express this you really do need to go beyond K_t and add a postulate for nonbranching of the future. [Prior 1969, p. 329]

The postulate he has in mind is this one:

(6) $PFq \supset (q \vee Pq \vee Fq)$

i.e. "Whatever has been 'on the cards' either is the case or has been the case or is 'on the cards' still" [Prior 1969, p. 329].

As John P. Burgess [1978, p. 157] later explained, Prior would agree that the determinist sees time as a line, and the indeterminist sees it as a system of 'forking paths'. As we have seen he found it highly important to examine the notion of branching in detail and formally elaborated two models for it, namely those by Ockham and Peirce, respectively [Prior 1968, p. 122 ff.].

Prior himself adopted the so-called Peirce-solution to the problem of the contingent future, according to which the following holds:

> ... from the fact that there is a sea-battle going on it does not follow that there was going to be one, though it does follow that there will have been one. [Prior 1957a, p. 95]

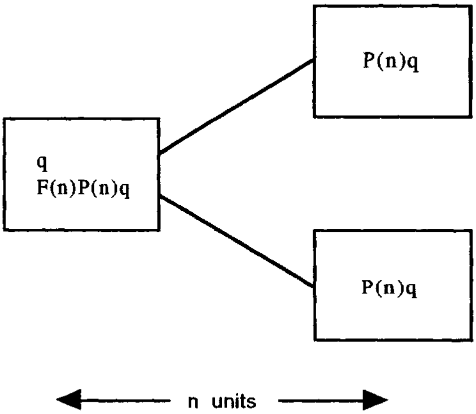
Let q be the statement 'there is a sea-battle going on'. Then it is a thesis that

(7) $q \supset F(n)P(n)q$

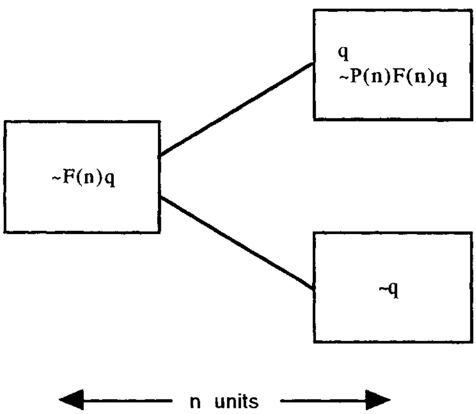
The validity of this thesis is illustrated by the diagram below: if q is true today, then $P(n)q$ will be true in any possible situation after n days. Therefore $F(n)P(n)q$ must be true today. However, the mirror image of (7):

(8) $q \supset P(n)F(n)q$

does not hold, if *F(n)q* is understood in the strong way, i.e. as "it is bound to be the case after *n* time units that *q*" [Prior 1969, p. 329].



The counter-argument against viewing (8) as a thesis is that it is possible to imagine that *q* is true at some time, but that *F(n)q* has not been true *n* days ago, for which reason *P(n)F(n)q* is not true now. The following diagram describes for instance situations, where the truth of *q* is brought about by, say, pure coincidence, or by some act of free choice:



Similarly, Prior maintained that although

(9) $q \supset GPq$

is a thesis, its mirror image

(10) $q \supset HFq$

is not valid [SFTT, p. 2], if F is understood in the strong way. (Prior's considerations here are based on the Peircean notions discussed in chapter 2.8. However, it should be mentioned that (10) is true in other branching time systems with different definitions, notably K_b and the Ockhamistic theory.)

In Prior's opinion, since the truth of future contingents cannot be known now, there cannot be any true statements about future contingents. On this view, the statement 'there will be a sea-battle tomorrow' cannot be true today, since there is no unique future but rather a number of different possible futures (unless, of course, that sentence happens to be deterministically entailed by facts of today).

The basic question concerns the interpretation of expressions regarding the future: Can it be maintained with conceptual and logical consistency of some event E that 'E will happen', this being taken as different from 'E could happen', and 'E will necessarily happen'? In chapter 2.8, we pointed out that the Ockham-system makes a genuine distinction between three types of statement, repeated here for ease of reference:

(i) Necessarily, Mr. Smith will commit suicide.
(ii) Possibly, Mr. Smith will commit suicide.
(iii) Mr. Smith will commit suicide.

Of course, this means that Ockham would answer the question positively. However, Prior like Peirce took the stance that 'E will happen' cannot make sense, unless it is interpreted as equivalent to one of the two other types of statement. The

difference to the Ockhamistic (as well as the general medieval) solution clearly has to do with the definition of truth. Prior suggested the following condition of truth with respect to future statements:

> ... nothing can be said to be truly 'going-to-happen' (futurum) until it is so 'present in its causes' as to be beyond stopping; until that happens neither 'It will be the case that p' nor 'It will not be the case that p' is strictly speaking true. [1968, p. 38]

In other words we have the following principle (P):

> (P) The proposition $F(n)p$ is true now if and only if there exist now facts which make it true (i.e., will make p true in due course).

This definition is very similar to the one MacKay suggested, and it is also quite essential in Peirce's philosophy. What may appear more surprising is Prior's conviction that St. Thomas Aquinas also accepted these ideas. To be true, Prior argued against Thomas' view that God's knowledge is in some way beyond time, but otherwise he consented to most of what Thomas had said about tense-logical reasoning. According to Prior's interpretation of Thomas' philosophy, Thomas would even agree on the rejection of (10).

(P) implies that the proposition $F(n)p$ can only be true, if it is in principle possible to verify it from facts known at the time of utterance. Future tense propositions which cannot be verified are false or not well-formed. William of Ockham, of course, could not have accepted an idea of truth corresponding to (P). Ockham held that it is possible for God to know something, even though that which He knows can in no way - short of divine revelation - be verified by us. With special regard to propositions about the future, Ockham claimed that God has a complete knowledge: "It must be held beyond question that God knows with certainty all future contingents, i.e., He knows with certainty which part of the contradiction is true and which is

false" [1969, p. 48]. (Here, Ockham obviously had in mind the embedding of contingent sentences into contradictions of the form $F(n)p \vee F(n){\sim}p$.)

On this analysis, a future contingent can be true now. The Ockhamist definition of truth with respect to future statements can be formulated in the following way:

> (O) The proposition $F(n)p$ is true now if and only if God knows that p will be in n days.

In Ockham's logic, any proposition is known by God if and only if it is true. Therefore, the above definition can also be expressed in the following way:

> (O') The proposition $F(n)p$ is true now if and only if p will be true in n days.

We have already indicated how the formal details corresponding to this definition can be worked out in an Ockhamistic structure. If (O') is accepted, it must be admitted that we are not in general able to establish whether a proposition is true or false, in spite of the fact that it is assumed to have a definite truth-value at any time - even if it is contingent. We cannot be sure that already now, there exist facts that make $F(n)p$ true. Hence the truth value of $F(n)p$ might be unknown - or even unknowable - to us. Thus future contingents can be true now, although we cannot know with certainty that they are true.

In the Ockhamistic framework $F(n)p$ is either true or false. Let us assume that $F(n)p$ is in fact true. Even so, if $F(n)p$ is about the contingent future a rational person does not have to accept its truth; he could deny it without compromising his rationality. On the other hand, while a person would certainly in the assumed case be correct in believing that $F(n)p$ is true, he would be mistaken in regarding $F(n)p$ as false.

Ockham's theory of the future is realistic, since the truth value of $F(n)p$ is well-defined, even if $F(n)p$ is about the contingent future. Furthermore, there exist true propositions about all

future events and states-of-affairs. In our opinion (O') is a more natural definition of truth for future statements than is (P). In fact, we think that few would be ready to accept the full consequences of (P). If (P) is accepted, it follows that any proposition about future contingents is false. As mentioned earlier, this also means that 'plain' future statements such as 'Mr. Smith will commit suicide' must be regarded as (a) inherently false, or (b) ill-formed (or elliptical, omitting a required modal expression). Relating this to everyday life, for instance guesses will in general be false: if we are playing the pools and we believe that we are going to win, this belief will according to (P) be false, even if it turns out that we do indeed win. On this basis, it will be almost impossible to state the difference between a true and a false prophet! In order to state such a difference, we need a truth-definition like (O').

It should also be noted that 'excluded middle',

(11) $F(n)p \vee F(n){\sim}p$

is not a thesis according to the Peirce-system. His theory makes it conceivable that the proposition 'In n days it will be the case that p', and the proposition 'In n days it will be the case that not p', are both false. The understanding of the concept of the future within the Peirce theory is realistic to the extent that it regards the truth value of the proposition $F(n)p$ as a meaningful concept. If $F(n)p$ is interpreted as either $NF(n)p$ or $MF(n)p$, it is either true or false; but otherwise, it is inherently false. The latter observation makes it clear that the theory is not realistic in the sense that we can form true propositions about future contingents. In general, bare $F(n)p$ and $F(n){\sim}p$ are both false according to the Peirce theory.

We think we have exemplified by now that this theory is not congenial with logical and linguistic intuitions evident in everyday communication. Following this investigation, one might conclude that the Ockham theory is a fairly accurate representation of our intuitions concerning valid temporal reasoning (with regard to the problems with which it deals; many other aspects of temporal reasoning, for instance

durations, have not been dealt with here). In our opinion, Prior's Ockhamist theory is indeed satisfactory for most cases. But as demonstrated by Hirokazu Nishimura [1979], there are some rare examples in which the Ockham theory is not sufficient. We shall state such an example.

Let $(TIME,<,C,\approx,Ock)$ be an Ockhamistic structure, where C is the set of all maximally ordered (i.e. linear) subsets in $(TIME,<)$. Let us for the sake of simplicity assume that *TIME* is discrete and each chronicle isomorphic to the set of integers. We may think of the elements in *TIME* as possible days.

Consider these two statements:

> A_1: 'Inevitably, if today there is life on earth, then either this is the last day (of life on earth), or the last day will come.'
> Semantically: $Ock(t,c,\ q \supset (G{\sim}q \vee F(Hq \wedge q \wedge G{\sim}q)))$ for any t, c (q standing for the statement 'there is life on earth').
> - A_1 implies that life cannot go on forever, so that on any possible day, it is true either that life has become extinct, or that this is the last day of life on earth, or that life will later become extinct.
>
> A_2: 'At any possible day on which there is life on earth, it is possible that there will be life on earth the following day.'
> Semantically: $Ock(t,c,\ q \supset MF(1,q))$ for any t, c.
> - A_2 implies that there is hope (and possibilities) for tomorrow as long as there is life!

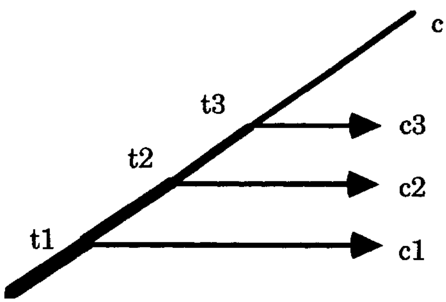
We assert that a person might hold both A_1 and A_2 without contradicting himself. But we shall show that the assumption that A_1 and A_2 can both be true is in conflict with the Ockhamistic theory. - Assume that A_1 and A_2 are both true and let t_0 be some possible day in *TIME* on some chronicle c_1. Because of A_1, this means that there is another possible day t_1 for which $Ock(t_1,c_1,\ Hq \wedge q \wedge G{\sim}q)$. Because of A_2, it follows that

$$Ock(t_1,c_1,q \supset MF(1,q)).$$

In consequence, $Ock(t_1,c_1,MF(1,q))$.

This means that there is a possible day t'_2 immediately after t_1, on some chronicle c_2, where $(t'_2, c_2) \approx (t_1, c_1)$ and $Ock(t'_2,c_2,q)$. Because of A_1 there is a possible day t_2, for which $Ock(t_2,c_2,q \wedge G{\sim}q)$ and $Ock(t,c_2,q)$ for all t between t'_2 and t_2. Now because of A_2 it follows that there is a possible day t'_3, immediately after t_2, on some chronicle c_3 where $(t'_3, c_3) \approx (t_2, c_2)$ and $Ock(t'_3,c_3,q)$. Because of A_1 there is a possible day t_3, for which $Ock(t_3,c_3,q \wedge G{\sim}q)$ and $Ock(t,c_3,q)$ for all t between t'_3 and t_3. This procedure can be carried out ad infinitum. - Obviously, the series of possible days, c: $t_1 < t_2 < t_3 < \dots$ following the part of c_1 before t_1, defines a maximally ordered subset in $(TIME,<)$, i.e. a chronicle. It is easy to see that $Ock(t,c,q)$ for all t. But this is clearly a violation of A_1. Q.E.D.

The construction procedure can be illustrated by the following figure:



If, on the other hand, we assume another tense logic different from the Ockhamistic system, in which the construction of the chronicle c from the series of c_1, c_2, ... is forbidden, then the conjunction of A_1 and A_2 might be accepted without any inconsistency. In such a tense logic the set of possible chronicles cannot be closed under the kind of construction just mentioned. We must assume that not all linear subsets in $(TIME,<)$ are possible chronicles. An Ochamistic system revised in this manner has an interesting affinity to Leibniz' philosophy. For this reason shall call such a modified Ockhamistic system *the Leibniz system*. We shall deal with the formalities of this system in the next chapter.

3.3. LEIBNIZIAN TENSE LOGIC

In this chapter we shall present an indeterministic tense logic similar to Prior's Ockhamistic theory, but modified in the light of the observations made by Hirokazu Nishimura (mentioned in the preceding chapter). Since the system is based on a kind of temporal reasoning very similar to Leibniz' philosophy, we shall call the system 'Leibnizian tempo-modal logic', LT for short. The definition of well-formed formulae within LT is:

(1) Propositional variables are wff.
(2) If α and β are wff and x is a positive number, then $\sim\alpha$, $\alpha \supset \beta$, $\alpha \wedge \beta$, $\alpha \vee \beta$, $\forall x{:}\alpha$, $\exists x{:}\ \alpha$, $N\alpha$, $P\alpha$, and $F\alpha$ are all wff.
(3) There are no other wff.

The axioms of LT are:

(A1) A, where A is a tautology of the propositional calculus
(A2) $G(A \supset B) \supset (GA \supset GB)$
(A3) $H(A \supset B) \supset (HA \supset HB)$
(A4) $A \supset HFA$
(A5) $A \supset GPA$
(A6) $FFA \supset FA$
(A7) $FPA \supset (PA \vee A \vee FA)$
(A8) $PFA \supset (PA \vee A \vee FA)$
(A9) $GA \supset FA$
(A10) $HA \supset PA$
(A11) $FA \supset FFA$
(A12) $NGA \supset GNA$
(A13) $PA \supset NPA$, where A contains no occurrences of F

(N1) $N(A \supset B) \supset (NA \supset NB)$
(N2) $NA \supset A$
(N3) $NA \supset NNA$
(N4) $MNA \supset NA$, where $M \equiv_{def} \sim N \sim$

(N1)-(N4) are the S5 axioms for N. The rules of inference are the same as for the Ockhamistic system, i.e.

(RMP) If $\vdash A$ and $\vdash A \supset B$, then $\vdash B$.

(RG) If $\vdash A$, then $\vdash GA$.

(RH) If $\vdash A$, then $\vdash HA$.

(RN) If $\vdash A$ then $\vdash NA$.

(Π1) If $\vdash \phi(x) \supset \beta$ then $\vdash \forall x{:}\phi(x) \supset \beta$.

(Π2) If $\vdash \alpha \supset \phi(x)$ then $\vdash \alpha \supset \forall x{:}\phi(x)$, for x not free in α.

where '$\vdash p$' means 'p is provable'.

The earlier-later logic (B-logic) we have in mind can be presented by the following definitions.

Definition: A Leibnizian structure is a quadruple $(TIME, <, \approx, T)$, where TIME is a non-empty set with two relations $<$ and $\approx$ such that

(B1) $(t_1 < t_2 \wedge t_2 < t_3) \supset t_1 < t_3$
(B2) $(t_1 < t_2 \wedge t_3 < t_2) \supset (t_1 < t_3 \vee t_1 = t_3 \vee t_3 < t_1)$
(B3) $(t_2 < t_1 \wedge t_2 < t_3) \supset (t_1 < t_3 \vee t_1 = t_3 \vee t_3 < t_1)$
(B4) $\forall t_1 \exists t_2 : t_1 < t_2$
(B5) $\forall t_1 \exists t_2 : t_2 < t_1$
(B6) $\forall t_1 \forall t_2 \exists t_3 : t_1 < t_2 \supset (t_1 < t_3 \wedge t_3 < t_2)$
(B7) $t \approx t$
(B8) $t_1 \approx t_2 \supset t_2 \approx t_1$
(B9) $(t_1 \approx t_2 \wedge t_2 \approx t_3) \supset t_1 \approx t_3$
(B10) $(t_1 \approx t_2 \wedge t_3 < t_2) \supset \exists t_4{:}\ (t_3 \approx t_4 \wedge t_4 < t_1)$

and an operator T such that

(T1) $T(t,A \wedge B) \equiv (T(t,A) \wedge T(t,B))$
(T2) $T(t,{\sim}A) \equiv {\sim}T(t,A)$
(T3) $\forall x{:}T(t,A) \equiv T(t, \forall x{:}A)$ where x is foreign to t
(T4) $T(t,FA) \equiv \exists t_1{:}\ (t < t_1 \wedge T(t_1,A))$
(T5) $T(t,PA) \equiv \exists t_1{:}\ (t_1 < t \wedge T(t_1,A))$
(T6) $T(t,NA) \equiv \forall t_1{:}\ (t_1 \approx t \supset T(t_1,A))$

(T7) $\forall t_1 \forall t_2: (t_1 \approx t_2 \wedge T(t_1,PA)) \supset T(t_2,PA)$,
where A contains no occurrences of F.

You may read $T(t,A)$ as 'A is true at t', but bear in mind that we are not introducing a separate semantics following usual model-theoretic procedures. Rather, we are enlarging the logical language such the extended system 'contains the semantics of the original system', following Prior's ideas in this respect.

Definition: A statement is Leibniz-valid if and only if $T(t,A)$ for every Leibnizian structure $(TIME,<,\approx,T)$ and every t in $TIME$.

It is easy to see that the axioms (A1) - (A13) and (N1) - (N4) are all Leibniz-valid. Since the inference rules carry the Leibniz-validity over, it follows that the following theorem holds:

Theorem 1: If A is provable in LT, then A is Leibniz-valid.

This theorem expresses the soundness of LT. Using Prior's idea of instant propositions we shall also argue that LT is complete relative to Leibniz-validity. First we shall enlarge the logical language in the manner already shown in chapter 2.9. The enlarged system must include instant propositions (i.e. maximal consistent sets from LT) and a modal operator L, for which we assume

(BF) $L(\forall a: \phi(a)) \equiv \forall a: L(\phi(a))$
(I1) $\exists a: a$
(I2) $\sim L \sim a$
(I3) $L(a \supset p) \vee L(a \supset \sim p)$
(L1) $L(p \supset q) \supset (Lp \supset Lq)$
(L2) $Lp \supset p$
(L3) $Lp \supset LLp$
(LG) $Lp \supset Gp$
(LH) $Lp \supset Hp$
(LN) $Lp \supset Np$

Intuitively, we may think of L as an operator corresponding to 'provability within LT'.

Using the following definitions, we can now demonstrate that the set of instant propositions forms a Leibnizian structure:

$a < b \equiv_{def} L(a \supset Fb)$
$a \approx b \equiv_{def} L(a \supset Mb)$
$T(a,A) \equiv_{def} L(a \supset A)$

Note that (A1) - (A11) are simply the axioms for K_l. Consequently the properties (B1) - (B6) and (TL1) - (TL5) which correspond to the semantics of K_l follow from the completeness of K_l. The remaining properties of the structure can be proved in the following way:

Theorem 2: The relation $\approx$ is an equivalence relation.
Proof:
Reflexivity is trivial.

Symmetry is proved by reductio ad absurdum:

(1)	$a \approx b$	(assumption)
(2)	$\sim(b \approx a)$	(assumption)
(3)	$L(a \supset Mb)$	(from 1)
(4)	$L(a \supset \sim Mb)$	(from 2 and I3)
(5)	$L(a \supset \sim a)$	(from 3 and 4)
(6)	$L\sim a$	(from 5). Contradicts (I2).

Transitivity is proved in a straightforward way:

(1)	$a \approx b$	(assumption)
(2)	$b \approx c$	(assumption)
(3)	$L(a \supset Mb)$	(from 1)
(4)	$L(b \supset Mc)$	(from 2)
(5)	$L(Mb \supset Mc)$	(from 4, L1, L3 LN)
(6)	$L(a \supset Mc)$	(from 3 and 5)
(7)	$a \approx c$	(from 6)

Q.E.D.

Theorem 2 implies that (B7) - (B9) hold.

Theorem 3 : $T(a,NA) \supset (a \approx b \supset T(b,A))$
Proof:
Let us assume that in some case this implication is violated. Then we may argue as follows:

(1)	$L(a \supset NA)$	(assumption)
(2)	$L(a \supset Mb)$	(assumption)
(3)	$L(b \supset {\sim}A)$	(assumption and I3)
(4)	$L(A \supset {\sim}b)$	(from 3)
(5)	$L(NA \supset N{\sim}b)$	(from 4)
(6)	$L(a \supset N{\sim}b)$	(from 1 and 5)
(7)	$L(Mb \supset {\sim}a)$	(from 6)
(8)	$L(a \supset {\sim}a)$	(from 7)
(9)	$L{\sim}a$	(from 8)

This obviously contradicts (I1). Therefore the implication in question cannot be violated.
Q.E.D.

Theorem 4: $T(a,NA) \supset \forall b{:}\ (a \approx b \supset T(b,A))$
Proof: This follows from theorem 3.

Theorem 5: ${\sim}L{\sim}A \supset \exists b{:}\ T(b,A)$
Proof: From chapter 2.9.

Theorem 6: $\forall b{:}\ (a \approx b \supset T(b,A)) \supset L(Ma \supset A)$
Proof:
This is proved by *reductio ad absurdum*

(1)	$\forall b{:}\ (a \approx b \supset T(b,A))$	(assumption)
(2)	${\sim}L(Ma \supset A)$	(assumption)
(3)	${\sim}L{\sim}(Ma \wedge {\sim}A)$	(assumption)
(4)	$\exists b{:}\ T(b,Ma \wedge {\sim}A)$	(3, theorem 5)
(5)	$\exists b{:}\ T(b,Ma) \wedge T(b,{\sim}A)$	(from 4)
(6)	$\exists b{:}\ a \approx b \wedge T(b,{\sim}A)$	(from 5)
(7)	$\exists b{:}\ L(b \supset A) \wedge L(b \supset {\sim}A)$	(from 1 and 6)
(8)	$\exists b{:}\ L(b \supset {\sim}b)$	(from 7)
(9)	${\sim}(\forall b{:}\ {\sim}L{\sim}b)$	(8)

(9) contradicts (I2). Q.E.D.

Theorem 7: $\forall b: (a \approx b \supset T(b,A)) \supset T(a, NA)$
Proof:
The proof is straight forward:

(1)	$\forall b: (a \approx b \supset T(b,A))$	(assumption)
(2)	$L(Ma \supset A)$	(1 by theorem 6)
(3)	$L(NMa \supset NA)$	(2)
(4)	$L(Ma \supset NA)$	(3)
(5)	$L(a \supset Ma)$	(from N2)
(6)	$L(a \supset NA)$	(from 4 and 5)
(7)	$T(a, NA)$	(from 6)

Q.E.D.

Theorem 8: $\forall b: (a \approx b \supset T(b,A)) \equiv T(a, NA)$
Proof: From theorem 4 and theorem 7.

Theorem 9: $(T(a,PA) \wedge a \approx b) \supset T(b,PA)$
Proof: Follows from theorem 2, (A13) and the theorem 8.

According to theorem 9, $a \approx b$ means that any formula of the form PA (where A contains no occurrences of F and no instant propositions) which follows with L-necessity from a (i.e. $L(a \supset PA)$)also follows with L-necessity from b (and vice versa). In consequence, equivalent instants have the same genuine past.

Theorem 10: $(a<b \wedge b \approx d) \supset (\exists c: a \approx c \wedge c<d)$
Proof:

(1)	$a<b \wedge b \approx d$	(assumption)
(2)	$T(a,FMd)$	(from 1)
(3)	$T(a,MFd)$	(2 and A12)
(4)	$\exists c: a \approx c \wedge T(c,Fd)$	(from 3)
(5)	$\exists c: a \approx c \wedge c<d$	(from 4)

Q.E.D.

Thus we have demonstrated that the set of instant propositions with the relations and the T-operator defined above is in fact a Leibnizian structure. In consequence, if A is Leibniz-valid, it will also be valid in this structure, i.e. $L(a \supset A)$ for any a. If we do interpret L as 'LT-provability', it follows that A can be proved in

LT from any instant proposition. Then by (DL) from chapter 2.9 it follows that LA, i.e. 'A is provable in LT'. When this is taken together with theorem 1, we may conclude that LT is also complete, i.e. it holds that A is Leibniz-valid if and only if A is provable in LT. (We think that for practical purposes this actually does demonstrate completeness, but we are aware that a full mathematical proof requires more than what is given here.)

However, the above notion of Leibniz-validity does not exclude a cyclical model. In order to make the models non-cyclical we would need an additional assumption like

(B11) $t_1 \approx t_2 \supset \sim(t_1 < t_2)$

In the enlarged system this would correspond to the axiom

$a \supset \sim MFa$

where a stands for an instant proposition. There is, however, no obvious axiom in LT which can do exactly the same job (that is, if instant propositions are not at our disposal).

METRIC TENSE LOGIC

We shall now present a metric tense logic MLT, which is an extension of LT. The language of MLT is based on a set of propositional variables: $p, q, r\ldots$ and the following definition of well-formed formulae

(1) Propositional variables are wff.
(2) If α and β are wff and x is a positive real number, then $\sim\alpha$, $\alpha\supset\beta$, $\alpha\wedge\beta$, $\alpha\vee\beta$, $\forall x{:}\alpha$, $\exists x{:}\ \alpha$, $N\alpha$, $P(x)\alpha$, and $F(x)\alpha$ are all wff.
(3) There are no other wff.

We shall assume S5 for N and in addition the following axioms:

(LT1) $G(x)(p \supset q) \supset (G(x)p \supset G(x)q)$
(LT2) $F(x)H(x)p \supset p$
(LT3) $F(y{+}x)p \supset F(y)F(x)p$
(LT4) $H(x)(p \supset q) \supset (H(x)p \supset H(x)q)$
(LT5) $P(x)G(x)p \supset p$
(LT6) $P(y{+}x)p \supset P(y)P(x)p$
(LT7) $F(x){\sim}p \equiv {\sim}F(x)p$
(LT8) $P(x){\sim}p \equiv {\sim}P(x)p$
(LT9) $NG(x)p \supset G(x)Np$
(LT10) $P(x)p \supset NP(x)p$,
where p contains no occurrences of F.

In addition we assume standard number theory for positive numbers. - The rules of MLT are the following:

(RMP) If $\vdash p$ and $\vdash p \supset q$, then $\vdash q$.
(RF) If $\vdash p$, then $\vdash G(x)p$.
(RP) If $\vdash p$, then $\vdash H(x)p$.
(Π1) If $\vdash \phi(x) \supset \beta$ then $\vdash \forall x{:}\phi(x) \supset \beta$.
(Π2) If $\vdash \alpha \supset \phi(x)$, then $\vdash \alpha \supset \forall x{:}\phi(x)$ for x not free in α.

Note that in LT the operators $F(x)$ and $H(x)$ are equivalent and similarly for $P(x)$ and $H(x)$. Nevertheless, we use all four operators in order to make the comparison with other systems easier.

On this basis, all the axioms of K_l can obviously be proved. Assuming standard number theory it is easy to prove (A8-11). (A12) and (A13) can be proved from (LT9) and (LT10), respectively. - Hence, we can conclude that MLT is a proper extension of LT. The semantics we have in mind for MLT can be presented by the following definitions.

Definition: A metric Leibnizian structure is a quadruple *(TIME,before,≈,T)*, where *TIME* is a non-empty set with two relations < and ≈ such that

(B1') $(before(t_1,t_2,x) \wedge before(t_2,t_3,y)) \supset before(t_1,t_3,x+y)$
(B2') $(before(t_1,t_2,x) \wedge before(t_1,t_3,x)) \supset t_2 = t_3$
(B3') $(before(t_1,t_2,x) \wedge before(t_3,t_2,x)) \supset t_1 = t_3$
(B4') $\forall t_1 \forall x \exists t_2 : before(t_1,t_2,x)$
(B5') $\forall t_2 \forall x \exists t_1 : before(t_1,t_2,x)$
(B6') $t \approx t$
(B7') $t_1 \approx t_2 \supset t_2 \approx t_1$
(B8') $(t_1 \approx t_2 \wedge t_2 \approx t_3) \supset t_1 \approx t_3$
(B9') $(t_1 \approx t_2 \wedge before(t_3,t_2,x)) \supset \exists t_4: (t_3 \approx t_4 \wedge before(t_4,t_1,x))$

and an operator *T* such that

(T1) $T(t,A \wedge B) \equiv (T(t,A) \wedge T(t,B))$
(T2) $T(t,\sim A) \equiv \sim T(t,A)$
(T3) $\forall x{:}T(t,A) \equiv T(t, \forall x{:}A)$
where x is foreign to t.
(T4) $T(t,F(x)A) \equiv \exists t_1: (before(t,t_1,x) \wedge T(t_1,A))$
(T5) $T(t,P(x)A) \equiv \exists t_1: (before(t_1,t,x) \wedge T(t_1,A))$
(T6) $T(t,NA) \equiv \forall t_1: (t_1 \approx t \supset T(t_1,A))$
(T7) $(t_1 \approx t_2 \wedge T(t_1,P(x)A)) \supset T(t_2,P(x)A)$,
where A contains no occurrences of F.

Standard number theory for positive numbers is also assumed as a background for the semantical reasoning regarding metric Leibnizian structures.

We shall say that a statement *A* is *metrically Leibniz-valid* if and only if for any metric Leibnizian structure *(TIME,before,≈,T)* and any *t* in *TIME*, it holds that *T(t,A)*. Now, it is not difficult to verify that any statement which is provable in MLT is also metrically Leibniz-valid. In order to show that this is the case we have to argue that the validity in question is carried over by the MLT rules and that all the MLT axioms are

metrically Leibniz-valid. Most of these proofs are trivial or very easy. Let us consider for example (LT7). We prove that

$T(t,F(x){\sim}p \supset {\sim}F(x)p)$
Proof:
This is proved by reductio ad absurdum.
(1) $T(t,F(x){\sim}p)$ (assumption)
(2) $T(t,F(x)p)$ (assumption)
(3) $\exists t_1$: $before(t, t_1,x) \wedge T(t_1,{\sim}p)$
(4) $\exists t_2$: $before(t, t_2,x) \wedge T(t_2,p)$
(5) $\exists t_1 \exists t_2$: $before(t, t_1,x) \wedge before(t, t_2,x) \wedge T(t_2,p) \wedge T(t_1,{\sim}p)$
(6) $\exists t_1$: $before(t, t_1,x) \wedge T(t_1,p) \wedge T(t_1,{\sim}p)$
(7) $\exists t_1$: $T(t_1,p) \wedge {\sim}T(t_1,p)$ - i.e. a contradiction.
Q.E.D.

It may be concluded that if A is provable in MLT, then A is also metrically Leibniz-valid. In order to demonstrate that the converse also holds, we shall once again make use of an enlarged system. This system includes Prior's instant propositions and the modal operator L earlier mentioned in this chapter, with the addition that any two different instant propositions are non-equivalent, i.e.

$L(a \equiv b)$ if and only if $a{=}b$.

We want to prove that the set of instant propositions forms a metric Leibnizian structure, if we make use of the definitions we have used in this chapter and in chapter 2.10:

$before(a,b,x) \equiv_{def} L(a \supset F(x)b)$
$a \approx b \equiv_{def} L(a \supset Mb)$
$T(a,A) \equiv_{def} L(a \supset A)$

Most of the proofs needed for this purpose can be copied from earlier; a few of them are new, though, but fairly easy. Let us take some examples:

$\forall a \forall x \exists b$: *before(a,b,x)*
Proof:
(1) $G(x)(p \vee \sim p)$
(2) $F(x)(p \vee \sim p)$
(3) $L(a \supset F(x)(p \vee \sim p))$
(4) $\exists b$: $(L(a \supset F(x)b) \wedge L(b \supset (p \vee \sim p))$
(5) $\exists b$: *before(a,b,x)*

$(before(a,b,x) \wedge before(a,c,x)) \supset L(b \supset c)$
Proof:
This is proved by reductio ad absurdum.

(1)	$L(a \supset F(x)b)$	(assumption)
(2)	$L(a \supset F(x)c)$	(assumption)
(3)	$\sim L(b \supset c)$	(assumption)
(4)	$L(b \supset \sim c)$	
(5)	$L(F(x)b \supset F(x)\sim c)$	
(6)	$L(a \supset F(x)\sim c)$	
(7)	$L(a \supset \sim F(x)c)$	
(8)	$L(a \supset \sim a)$.	(Contradicts I2).

Q.E.D.

By a similar argument it can be proved that

$(before(a,b,x) \wedge before(a,c,x)) \supset L(c \supset b)$

In consequence we have

$(before(a,b,x) \wedge before(a,c,x)) \supset L(b \equiv c)$

Since any two instant propositions are non-equivalent, it can be concluded that

$(before(a,b,x) \wedge before(a,c,x)) \supset b=c$

With respect to the problem of cyclicity we have the same problem as in LT, and again there is an obvious solution within the enlarged system.

MLT seems to be very useful in many cases. However, counterfactual implications in natural language like

(S1) *A* is not the case, but if *A* were the case, then *B* would also be the case,
or with a more natural wording
(S2) if *A* were the case, then *B* would be the case

cannot be satisfactorily expressed in MLT. The obvious solution in a branching time logic would seem to be

$\sim A \wedge \exists x{:}\ P(x)NF(x)(A \supset B)$

which because of (LT9) turns out to imply the following formula:

$\sim A \wedge N(A \supset B)$

This corresponds to statement:

(S2) *A* is not the case, but necessarily, if *A* were the case, then *B* would also be the case.

Now, since (S2) is normally considered to be semantically stronger than (S1), we want a logic that can reflect this relation. In the next chapter we shall see how such a logic can be constructed.

3.3. LEIBNIZIAN TENSE LOGIC

In this chapter we shall present an indeterministic tense logic similar to Prior's Ockhamistic theory, but modified in the light of the observations made by Hirokazu Nishimura (mentioned in the preceding chapter). Since the system is based on a kind of temporal reasoning very similar to Leibniz' philosophy, we shall call the system 'Leibnizian tempo-modal logic', LT for short. The definition of well-formed formulae within LT is:

(1) Propositional variables are wff.
(2) If α and β are wff and x is a positive number, then $\sim\alpha$, $\alpha \supset \beta$, $\alpha \wedge \beta$, $\alpha \vee \beta$, $\forall x{:}\alpha$, $\exists x{:}\ \alpha$, $N\alpha$, $P\alpha$, and $F\alpha$ are all wff.
(3) There are no other wff.

The axioms of LT are:

(A1) A, where A is a tautology of the propositional calculus
(A2) $G(A \supset B) \supset (GA \supset GB)$
(A3) $H(A \supset B) \supset (HA \supset HB)$
(A4) $A \supset HFA$
(A5) $A \supset GPA$
(A6) $FFA \supset FA$
(A7) $FPA \supset (PA \vee A \vee FA)$
(A8) $PFA \supset (PA \vee A \vee FA)$
(A9) $GA \supset FA$
(A10) $HA \supset PA$
(A11) $FA \supset FFA$
(A12) $NGA \supset GNA$
(A13) $PA \supset NPA$, where A contains no occurrences of F

(N1) $N(A \supset B) \supset (NA \supset NB)$
(N2) $NA \supset A$
(N3) $NA \supset NNA$
(N4) $MNA \supset NA$, where $M \equiv_{def} \sim N \sim$

(N1)-(N4) are the S5 axioms for N. The rules of inference are the same as for the Ockhamistic system, i.e.

(RMP) If $\vdash A$ and $\vdash A \supset B$, then $\vdash B$.

(RG) If $\vdash A$, then $\vdash GA$.

(RH) If $\vdash A$, then $\vdash HA$.

(RN) If $\vdash A$ then $\vdash NA$.

(Π1) If $\vdash \phi(x) \supset \beta$ then $\vdash \forall x{:}\phi(x) \supset \beta$.

(Π2) If $\vdash \alpha \supset \phi(x)$ then $\vdash \alpha \supset \forall x{:}\phi(x)$, for x not free in α.

where '$\vdash p$' means 'p is provable'.

The earlier-later logic (B-logic) we have in mind can be presented by the following definitions.

Definition: A Leibnizian structure is a quadruple $(TIME, <, \approx, T)$, where TIME is a non-empty set with two relations $<$ and $\approx$ such that

(B1) $(t_1 < t_2 \wedge t_2 < t_3) \supset t_1 < t_3$
(B2) $(t_1 < t_2 \wedge t_3 < t_2) \supset (t_1 < t_3 \vee t_1 = t_3 \vee t_3 < t_1)$
(B3) $(t_2 < t_1 \wedge t_2 < t_3) \supset (t_1 < t_3 \vee t_1 = t_3 \vee t_3 < t_1)$
(B4) $\forall t_1 \exists t_2 : t_1 < t_2$
(B5) $\forall t_1 \exists t_2 : t_2 < t_1$
(B6) $\forall t_1 \forall t_2 \exists t_3 : t_1 < t_2 \supset (t_1 < t_3 \wedge t_3 < t_2)$
(B7) $t \approx t$
(B8) $t_1 \approx t_2 \supset t_2 \approx t_1$
(B9) $(t_1 \approx t_2 \wedge t_2 \approx t_3) \supset t_1 \approx t_3$
(B10) $(t_1 \approx t_2 \wedge t_3 < t_2) \supset \exists t_4 \text{: } (t_3 \approx t_4 \wedge t_4 < t_1)$

and an operator T such that

(T1) $T(t, A \wedge B) \equiv (T(t,A) \wedge T(t,B))$
(T2) $T(t, \sim A) \equiv \sim T(t,A)$
(T3) $\forall x{:}T(t,A) \equiv T(t, \forall x{:}A)$ where x is foreign to t
(T4) $T(t,FA) \equiv \exists t_1 \text{: } (t < t_1 \wedge T(t_1,A))$
(T5) $T(t,PA) \equiv \exists t_1 \text{: } (t_1 < t \wedge T(t_1,A))$
(T6) $T(t,NA) \equiv \forall t_1 \text{: } (t_1 \approx t \supset T(t_1,A))$

(T7) $\forall t_1 \forall t_2: (t_1 \approx t_2 \wedge T(t_1,PA)) \supset T(t_2,PA)$,
where A contains no occurrences of F.

You may read $T(t,A)$ as 'A is true at t', but bear in mind that we are not introducing a separate semantics following usual model-theoretic procedures. Rather, we are enlarging the logical language such the extended system 'contains the semantics of the original system', following Prior's ideas in this respect.

Definition: A statement is Leibniz-valid if and only if $T(t,A)$ for every Leibnizian structure $(TIME,<,\approx,T)$ and every t in $TIME$.

It is easy to see that the axioms (A1) - (A13) and (N1) - (N4) are all Leibniz-valid. Since the inference rules carry the Leibniz-validity over, it follows that the following theorem holds:

Theorem 1: If A is provable in LT, then A is Leibniz-valid.

This theorem expresses the soundness of LT. Using Prior's idea of instant propositions we shall also argue that LT is complete relative to Leibniz-validity. First we shall enlarge the logical language in the manner already shown in chapter 2.9. The enlarged system must include instant propositions (i.e. maximal consistent sets from LT) and a modal operator L, for which we assume

(BF) $L(\forall a: \phi(a)) \equiv \forall a: L(\phi(a))$
(I1) $\exists a: a$
(I2) $\sim L \sim a$
(I3) $L(a \supset p) \vee L(a \supset \sim p)$
(L1) $L(p \supset q) \supset (Lp \supset Lq)$
(L2) $Lp \supset p$
(L3) $Lp \supset LLp$
(LG) $Lp \supset Gp$
(LH) $Lp \supset Hp$
(LN) $Lp \supset Np$

Intuitively, we may think of L as an operator corresponding to 'provability within LT'.

Using the following definitions, we can now demonstrate that the set of instant propositions forms a Leibnizian structure:

$a < b \equiv_{def} L(a \supset Fb)$
$a \approx b \equiv_{def} L(a \supset Mb)$
$T(a,A) \equiv_{def} L(a \supset A)$

Note that (A1) - (A11) are simply the axioms for K_l. Consequently the properties (B1) - (B6) and (TL1) - (TL5) which correspond to the semantics of K_l follow from the completeness of K_l. The remaining properties of the structure can be proved in the following way:

Theorem 2: The relation $\approx$ is an equivalence relation.
Proof:
Reflexivity is trivial.

Symmetry is proved by reductio ad absurdum:

(1)	$a \approx b$	(assumption)
(2)	$\sim(b \approx a)$	(assumption)
(3)	$L(a \supset Mb)$	(from 1)
(4)	$L(a \supset \sim Mb)$	(from 2 and I3)
(5)	$L(a \supset \sim a)$	(from 3 and 4)
(6)	$L\sim a$	(from 5). Contradicts (I2).

Transitivity is proved in a straightforward way:

(1)	$a \approx b$	(assumption)
(2)	$b \approx c$	(assumption)
(3)	$L(a \supset Mb)$	(from 1)
(4)	$L(b \supset Mc)$	(from 2)
(5)	$L(Mb \supset Mc)$	(from 4, L1, L3 LN)
(6)	$L(a \supset Mc)$	(from 3 and 5)
(7)	$a \approx c$	(from 6)

Q.E.D.

Theorem 2 implies that (B7) - (B9) hold.

Theorem 3 : $T(a,NA) \supset (a \approx b \supset T(b,A))$
Proof:
Let us assume that in some case this implication is violated. Then we may argue as follows:

(1)	$L(a \supset NA)$	(assumption)
(2)	$L(a \supset Mb)$	(assumption)
(3)	$L(b \supset \sim A)$	(assumption and I3)
(4)	$L(A \supset \sim b)$	(from 3)
(5)	$L(NA \supset N\sim b)$	(from 4)
(6)	$L(a \supset N\sim b)$	(from 1 and 5)
(7)	$L(Mb \supset \sim a)$	(from 6)
(8)	$L(a \supset \sim a)$	(from 7)
(9)	$L\sim a$	(from 8)

This obviously contradicts (I1). Therefore the implication in question cannot be violated.
Q.E.D.

Theorem 4: $T(a,NA) \supset \forall b: (a \approx b \supset T(b,A))$
Proof: This follows from theorem 3.

Theorem 5: $\sim L\sim A \supset \exists b: T(b,A)$
Proof: From chapter 2.9.

Theorem 6: $\forall b: (a \approx b \supset T(b,A)) \supset L(Ma \supset A)$
Proof:
This is proved by *reductio ad absurdum*

(1)	$\forall b: (a \approx b \supset T(b,A))$	(assumption)
(2)	$\sim L(Ma \supset A)$	(assumption)
(3)	$\sim L\sim(Ma \wedge \sim A)$	(assumption)
(4)	$\exists b: T(b,Ma \wedge \sim A)$	(3, theorem 5)
(5)	$\exists b: T(b,Ma) \wedge T(b,\sim A)$	(from 4)
(6)	$\exists b: a \approx b \wedge T(b,\sim A)$	(from 5)
(7)	$\exists b: L(b \supset A) \wedge L(b \supset \sim A)$	(from 1 and 6)
(8)	$\exists b: L(b \supset \sim b)$	(from 7)
(9)	$\sim(\forall b: \sim L\sim b)$	(8)

(9) contradicts (I2). Q.E.D.

Theorem 7: $\forall b: (a \approx b \supset T(b,A)) \supset T(a, NA)$
Proof:
The proof is straight forward:

(1)	$\forall b: (a \approx b \supset T(b,A))$	(assumption)
(2)	$L(Ma \supset A)$	(1 by theorem 6)
(3)	$L(NMa \supset NA)$	(2)
(4)	$L(Ma \supset NA)$	(3)
(5)	$L(a \supset Ma)$	(from N2)
(6)	$L(a \supset NA)$	(from 4 and 5)
(7)	$T(a,NA)$	(from 6)

Q.E.D.

Theorem 8: $\forall b: (a \approx b \supset T(b,A)) \equiv T(a, NA)$
Proof: From theorem 4 and theorem 7.

Theorem 9: $(T(a,PA) \wedge a \approx b) \supset T(b,PA)$
Proof: Follows from theorem 2, (A13) and the theorem 8.

According to theorem 9, $a{\approx}b$ means that any formula of the form PA (where A contains no occurrences of F and no instant propositions) which follows with L-necessity from a (i.e. $L(a \supset PA)$)also follows with L-necessity from b (and vice versa). In consequence, equivalent instants have the same genuine past.

Theorem 10: $(a<b \wedge b{\approx}d) \supset (\exists c: a{\approx}c \wedge c<d)$
Proof:

(1)	$a<b \wedge b{\approx}d$	(assumption)
(2)	$T(a,FMd)$	(from 1)
(3)	$T(a,MFd)$	(2 and A12)
(4)	$\exists c: a{\approx}c \wedge T(c,Fd)$	(from 3)
(5)	$\exists c: a{\approx}c \wedge c<d$	(from 4)

Q.E.D.

Thus we have demonstrated that the set of instant propositions with the relations and the T-operator defined above is in fact a Leibnizian structure. In consequence, if A is Leibniz-valid, it will also be valid in this structure, i.e. $L(a \supset A)$ for any a. If we do interpret L as 'LT-provability', it follows that A can be proved in

LT from any instant proposition. Then by (DL) from chapter 2.9 it follows that *LA*, i.e. '*A* is provable in LT'. When this is taken together with theorem 1, we may conclude that LT is also complete, i.e. it holds that *A* is Leibniz-valid if and only if *A* is provable in LT. (We think that for practical purposes this actually does demonstrate completeness, but we are aware that a full mathematical proof requires more than what is given here.)

However, the above notion of Leibniz-validity does not exclude a cyclical model. In order to make the models non-cyclical we would need an additional assumption like

(B11) $t_1 \approx t_2 \supset {\sim}(t_1 < t_2)$

In the enlarged system this would correspond to the axiom

$a \supset {\sim}MFa$

where a stands for an instant proposition. There is, however, no obvious axiom in LT which can do exactly the same job (that is, if instant propositions are not at our disposal).

METRIC TENSE LOGIC

We shall now present a metric tense logic MLT, which is an extension of LT. The language of MLT is based on a set of propositional variables: $p, q, r\ldots$ and the following definition of well-formed formulae

(1) Propositional variables are wff.
(2) If α and β are wff and x is a positive real number, then ${\sim}\alpha$, $\alpha\supset\beta$, $\alpha\wedge\beta$, $\alpha\vee\beta$, $\forall x{:}\alpha$, $\exists x{:}\ \alpha$, $N\alpha$, $P(x)\alpha$, and $F(x)\alpha$ are all wff.
(3) There are no other wff.

We shall assume S5 for N and in addition the following axioms:

(LT1) $G(x)(p \supset q) \supset (G(x)p \supset G(x)q)$
(LT2) $F(x)H(x)p \supset p$
(LT3) $F(y+x)p \supset F(y)F(x)p$
(LT4) $H(x)(p \supset q) \supset (H(x)p \supset H(x)q)$
(LT5) $P(x)G(x)p \supset p$
(LT6) $P(y+x)p \supset P(y)P(x)p$
(LT7) $F(x){\sim}p \equiv {\sim}F(x)p$
(LT8) $P(x){\sim}p \equiv {\sim}P(x)p$
(LT9) $NG(x)p \supset G(x)Np$
(LT10) $P(x)p \supset NP(x)p$,
where p contains no occurrences of F.

In addition we assume standard number theory for positive numbers. - The rules of MLT are the following:

(RMP) If $\vdash p$ and $\vdash p \supset q$, then $\vdash q$.
(RF) If $\vdash p$, then $\vdash G(x)p$.
(RP) If $\vdash p$, then $\vdash H(x)p$.
(Π1) If $\vdash \phi(x) \supset \beta$ then $\vdash \forall x{:}\phi(x) \supset \beta$.
(Π2) If $\vdash \alpha \supset \phi(x)$, then $\vdash \alpha \supset \forall x{:}\phi(x)$ for x not free in α.

Note that in LT the operators $F(x)$ and $H(x)$ are equivalent and similarly for $P(x)$ and $H(x)$. Nevertheless, we use all four operators in order to make the comparison with other systems easier.

On this basis, all the axioms of K_l can obviously be proved. Assuming standard number theory it is easy to prove (A8-11). (A12) and (A13) can be proved from (LT9) and (LT10), respectively. - Hence, we can conclude that MLT is a proper extension of LT. The semantics we have in mind for MLT can be presented by the following definitions.

Definition: A metric Leibnizian structure is a quadruple *(TIME,before,≈,T)*, where *TIME* is a non-empty set with two relations < and ≈ such that

(B1') $(before(t_1,t_2,x) \wedge before(t_2,t_3,y)) \supset before(t_1,t_3,x+y)$
(B2') $(before(t_1,t_2,x) \wedge before(t_1,t_3,x)) \supset t_2 = t_3$
(B3') $(before(t_1,t_2,x) \wedge before(t_3,t_2,x)) \supset t_1 = t_3$
(B4') $\forall t_1 \forall x \exists t_2 : before(t_1,t_2,x)$
(B5') $\forall t_2 \forall x \exists t_1 : before(t_1,t_2,x)$
(B6') $t \approx t$
(B7') $t_1 \approx t_2 \supset t_2 \approx t_1$
(B8') $(t_1 \approx t_2 \wedge t_2 \approx t_3) \supset t_1 \approx t_3$
(B9') $(t_1 \approx t_2 \wedge before(t_3,t_2,x)) \supset \exists t_4: (t_3 \approx t_4 \wedge before(t_4,t_1,x))$

and an operator T such that

(T1) $T(t,A \wedge B) \equiv (T(t,A) \wedge T(t,B))$
(T2) $T(t,\sim A) \equiv \sim T(t,A)$
(T3) $\forall x{:}T(t,A) \equiv T(t, \forall x{:}A)$
where x is foreign to t.
(T4) $T(t,F(x)A) \equiv \exists t_1: (before(t,t_1,x) \wedge T(t_1,A))$
(T5) $T(t,P(x)A) \equiv \exists t_1: (before(t_1,t,x) \wedge T(t_1,A))$
(T6) $T(t,NA) \equiv \forall t_1: (t_1 \approx t \supset T(t_1,A))$
(T7) $(t_1 \approx t_2 \wedge T(t_1,P(x)A)) \supset T(t_2,P(x)A)$,
where A contains no occurrences of F.

Standard number theory for positive numbers is also assumed as a background for the semantical reasoning regarding metric Leibnizian structures.

We shall say that a statement A is *metrically Leibniz-valid* if and only if for any metric Leibnizian structure *(TIME,before,≈,T)* and any t in *TIME*, it holds that $T(t,A)$. Now, it is not difficult to verify that any statement which is provable in MLT is also metrically Leibniz-valid. In order to show that this is the case we have to argue that the validity in question is carried over by the MLT rules and that all the MLT axioms are

metrically Leibniz-valid. Most of these proofs are trivial or very easy. Let us consider for example (LT7). We prove that

$T(t,F(x){\sim}p \supset {\sim}F(x)p)$

Proof:

This is proved by reductio ad absurdum.

(1) $T(t,F(x){\sim}p)$ (assumption)

(2) $T(t,F(x)p)$ (assumption)

(3) $\exists t_1$: $before(t, t_1,x) \wedge T(t_1,{\sim}p)$

(4) $\exists t_2$: $before(t, t_2,x) \wedge T(t_2,p)$

(5) $\exists t_1 \exists t_2$: $before(t, t_1,x) \wedge before(t, t_2,x) \wedge T(t_2,p) \wedge T(t_1,{\sim}p)$

(6) $\exists t_1$: $before(t, t_1,x) \wedge T(t_1,p) \wedge T(t_1,{\sim}p)$

(7) $\exists t_1$: $T(t_1,p) \wedge {\sim}T(t_1,p)$ - i.e. a contradiction.

Q.E.D.

It may be concluded that if A is provable in MLT, then A is also metrically Leibniz-valid. In order to demonstrate that the converse also holds, we shall once again make use of an enlarged system. This system includes Prior's instant propositions and the modal operator L earlier mentioned in this chapter, with the addition that any two different instant propositions are non-equivalent, i.e.

$L(a \equiv b)$ if and only if $a{=}b$.

We want to prove that the set of instant propositions forms a metric Leibnizian structure, if we make use of the definitions we have used in this chapter and in chapter 2.10:

$before(a,b,x) \equiv_{def} L(a \supset F(x)b)$

$a \approx b \equiv_{def} L(a \supset Mb)$

$T(a,A) \equiv_{def} L(a \supset A)$

Most of the proofs needed for this purpose can be copied from earlier; a few of them are new, though, but fairly easy. Let us take some examples:

$\forall a\, \forall x \exists b$: *before(a,b,x)*
Proof:
(1) $G(x)(p \vee {\sim}p)$
(2) $F(x)(p \vee {\sim}p)$
(3) $L(a \supset F(x)(p \vee {\sim}p))$
(4) $\exists b$: $(L(a \supset F(x)b) \wedge L(b \supset (p \vee {\sim}p))$
(5) $\exists b$: *before(a,b,x)*

$(before(a,b,x) \wedge before(a,c,x)) \supset L(b \supset c)$
Proof:
This is proved by reductio ad absurdum.

(1)	$L(a \supset F(x)b)$	(assumption)
(2)	$L(a \supset F(x)c)$	(assumption)
(3)	${\sim}L(b \supset c)$	(assumption)
(4)	$L(b \supset {\sim}c)$	
(5)	$L(F(x)b \supset F(x){\sim}c)$	
(6)	$L(a \supset F(x){\sim}c)$	
(7)	$L(a \supset {\sim}F(x)c)$	
(8)	$L(a \supset {\sim}a)$.	(Contradicts I2).

Q.E.D.

By a similar argument it can be proved that

$$(before(a,b,x) \wedge before(a,c,x)) \supset L(c \supset b)$$

In consequence we have

$$(before(a,b,x) \wedge before(a,c,x)) \supset L(b \equiv c)$$

Since any two instant propositions are non-equivalent, it can be concluded that

$$(before(a,b,x) \wedge before(a,c,x)) \supset b{=}c$$

With respect to the problem of cyclicity we have the same problem as in LT, and again there is an obvious solution within the enlarged system.

MLT seems to be very useful in many cases. However, counterfactual implications in natural language like

> (S1) *A* is not the case, but if *A* were the case, then *B* would also be the case,
> or with a more natural wording
> (S2) if *A* were the case, then *B* would be the case

cannot be satisfactorily expressed in MLT. The obvious solution in a branching time logic would seem to be

$$\sim A \wedge \exists x\colon P(x)NF(x)(A \supset B)$$

which because of (LT9) turns out to imply the following formula:

$$\sim A \wedge N(A \supset B)$$

This corresponds to statement:

> (S2) *A* is not the case, but necessarily, if *A* were the case, then *B* would also be the case.

Now, since (S2) is normally considered to be semantically stronger than (S1), we want a logic that can reflect this relation. In the next chapter we shall see how such a logic can be constructed.

3.4. TENSE LOGIC AND COUNTERFACTUAL REASONING

Temporal reasoning is intimately related to certain other kinds of reasoning. Some particularly important examples are causal, counterfactual, and diagnostic reasoning. In these kinds of reasoning we find a type of conditionals, which are crucially interwoven with temporality. (A recent Ph. D. Thesis [Crouch 1993] has shown how time and tense are in fact pervasive features of English conditionals in general.)

Since time proves to be relevant for counterfactual as well as diagnostic reasoning, we find it essential to examine the formal logical structure of such reasoning - as well as its tacit dimension, which for these cases plays a somewhat special rôle. We are going to argue that a suitable framework can be constructed in the form of a non-monotonic and tempo-modal logic, which is partly based on J.L. Mackie's suggestions regarding causality [Mackie 1974]. Moreover, the features of this framework - especially non-monotonicity - broadens the perspectives studied so far on time and tense.

Let us consider an example of everyday reasoning about time and causes, based on the following story:

> Joe wants to prepare a meal for some guests. He goes to the local shop in order to buy the ingredients, and he hires a cook, who is supposed to prepare the meal. In due time the cook arrives at Joe's place, just to find out that there is an electrical failure, which will make it impossible to have the dinner ready on time.

In the following argument between Joe and his friend Jim, we see a number of (somewhat artificial) counterfactual statements:

> *Joe:* If it had not been for the failure, we would have had dinner on time.
> *Jim:*You're wrong. If the cook left, you might still not have had dinner on time, even in the absence of the failure.

Joe: Well, I was just assuming that the cook would stay. But what I mean is that if there were no failure and the cook agreed to stay, we would have had dinner on time.
Jim: You're wrong again. If you had not been to the shop to buy the ingredients, you would not have had dinner on time, even if the cook stayed and there were no electrical failure.
Joe: I don't agree. Somebody else might have brought the ingredients. As long as the cook has got the ingredients and there is no electrical failure, we shall with necessity have dinner on time.
Jim: Well, the cook might change his mind because of all your quarrelling. Again, in that case there would not be dinner on time, even if there were no electrical failure.
Joe: Well, we might have had dinner on time, if I had ordered some ready-made dinner. The electrical failure could not have prevented the dinner in that case.
Jim: I think you're wrong again. The cook is a union man. He will bar the door to the delivery of the ready-made dinner, since the ready-made dinner company takes away jobs from proper cooks. But if the cook leaves, the ready-made dinner can be delivered.

What happens in this kind of debate is that still new elements from the scenario are introduced as relevant. At each point, it can be argued that the speaker in question is right, if his statement is evaluated from his point of view.

On the other hand, it turns out that when e.g. the first statement above is evaluated from a new and broader perspective in which a new causal factor (i.e. the cook) is taken into consideration, it is false. What is at stake here is the very notion of causation. Each of the statements can be interpreted as assertions regarding causes.

In order to account for the non-monotonicity, i.e. the apparent change in the truth-values of statements, we must introduce a new representation of causality. Whereas causality is normally conceived as a relation between a cause and its effect, the idea is to represent causality as a predicate which takes three arguments. The point is that there is always a *ceteris paribus* clause

involved in a statement regarding causality. Causal statements such as 'if it rains, the grass will become wet', or 'rain causes the grass to become wet', are asserted under the tacit assumption of 'all other things being equal'. For instance, the statement 'rain causes the grass to become wet' presupposes 'the grass has not been covered' (say, by a large tent for our garden party tonight). In general, the ceteris paribus clause determines which possible 'chronicles', i.e. possible courses of events, should be taken into consideration in order to evaluate the statement in question.

In other words, this extra argument of the causality predicate, which we shall call the 'scope', defines the set of entities relevant for the evaluation of the statement. The minimal scope can be constructed directly from the statement in question as the set of all atomic propositions (with their tenses) involved in the statement. We shall call this *the natural scope of the statement.*

Also it would be wrong to see the cause as just one single entity. The above conversation shows that it would be more correct to represent the cause as a conjunction of a number of statements.

In the following we intend to show how counterfactual implications can be introduced semantically as an extension of MLT discussed in the last chapter. The crucial features of this extension come from the idea that counterfactual statements are evaluated under ceteris paribus assumptions, and furthermore, that in any such evaluation one must assume what we have called a 'scope' for the statement in question.

First, we are going to introduce a number of fairly technical definitions, without too much motivation; then we shall come back to the dinner scenario to show how the systems work in practice.

CIMP - MODELLING COUNTERFACTUAL IMPLICATIONS

David Lewis [1973] formulated a very elegant logic of counterfactuals, which covers many important intuitions. Its main defect is, however, that possible worlds appear only as semantic indices - rather than as concrete conceptual alternatives. Instead, we shall propose a construction of possible worlds as

finite sets of propositions. These constructions will be based on Mackie causal complexes, which are arguably relevant for the evaluation of most counterfactuals. We shall describe our ideas partly with reference to a computer implementation called 'the CIMP system'. (The system is implemented in PROLOG and can be used interactively as a tool for modelling and evaluating counterfactuals, cf. [Hasle & Øhrstrøm 1992], [Øhrstrøm et al. 1992], and [Øhrstrøm and Hasle 1995].)

The central idea of CIMP is to consider conceptual alternatives as partial descriptions of a situation. Consider a case of meningitis. If we want to evaluate the conditional

> if the patient had been vaccinated, he would not have developed meningitis,

we must construct a situation in which the patient was vaccinated and exposed to the same pneumococci infection. This construction must respect known singular causal complexes and facts about the case. Furthermore, there is a causal field within which the construction takes place. To capture all this, we need formal definitions of causal statements and causal models. Moreover, we want to construct possible alternatives as possible ways in which the world might have developed, that is, as possible courses of events. This means that there will be a temporal aspect in our analysis. In CIMP, counterfactual implications are evaluated with respect to 'possible chronicles' rather than possible worlds. In a sum, CIMP is a semantical system based on the ideas in metric tense logic combined with an understanding of causality based on Mackie's ideas.

David Lewis has stressed the same relation between branching time and counterfactual reasoning in the following way:

> I suggest that the mysterious asymmetry between open future and fixed past is nothing else than the asymmetry of counterfactual dependence. The forking paths into the future - the actual one and all the rest - are the many alternative futures that would come about under various counterfactual suppositions about the present. [Lewis 1979, p. 462]

The idea seems to be that given the actual development up to the present state, it does not make sense in addition to assume another past. But even given the actual development up to the present state, we actually have the choice between a number of mutually exclusive assumptions about the future. In this way Lewis wants to see the asymmetrical structure of time as nothing but a consequence of the nature of counterfactual dependence.

We do not accept that counterfactual dependence should have conceptual priority over the very concept of time. We suggest that the asymmetry between open future and fixed past is taken for granted and used for the purpose of defining counterfactual implication.

The language of CIMP is based on a finite set of propositional expressions, called the maximal scope:

$$MSCOPE = S_{-v} \cup \ldots \cup S_{-1} \cup S_0 \cup S_1 \cup \ldots \cup S_w, \text{ where}$$
$$S_{-i} = \{P(i,q_{-i1}), P(i,q_{-i2}), \ldots, P(i,q_{-iN(-i)})\}, i=1\ldots v$$
$$S_0 = \{q_{01}, q_{02}, \ldots, q_{0N(0)}\}$$
$$S_i = \{F(i,q_{i1}), F(i,q_{i2}), \ldots, F(i,q_{iN(i)})\}, i=1\ldots w$$

and all the q_{jk} are propositional constants and the $N(j)$ are positive integers. Furthermore, we define an operator *dual* in the following way:

$$dual(P(x,\sim q_i)) = P(x,q_i)$$
$$dual(P(x,q_i)) = P(x,\sim q_i)$$
$$dual(\sim q_i) = q_i$$
$$dual(q_i) = \sim q_i$$
$$dual(F(x,\sim q_i)) = F(x,q_i)$$
$$dual(F(x,q_i)) = F(x,\sim q_i)$$

We need some additional definitions:

Definition: Two sets, S and S', are said to relate to each other if for each element A in S exactly one of A and $dual(A)$ is in S' and vice versa.

Definition: A maximal set is a set that relates to *MSCOPE*.

Obviously, related sets have the same number of elements. It is also evident that any maximal set can be written as a union of subsets: $E_{-v} \cup \ldots \cup E_{-1} \cup E_0 \cup E_1 \cup \ldots \cup E_w)$, where for each i, E_i is related to the S_i. The E_is are called *events*, and the elements of each E_i are called *basic propositions*. Note that all elements in an event have the same tense and that the events consequently can be ordered in an obvious way. Every maximal set corresponds to a proposition which is formed as the conjunction of all the propositions in the set. For this reason we shall write a maximal set as $U_a(= E_{-v} \cup \ldots \cup E_{-1} \cup E_0 \cup E_1 \cup \ldots \cup E_w)$, where a is the corresponding *maximal proposition*. - It is obvious that

> *Theorem 1:* If $B \in U_a$, then $dual(B) \notin U_a$.

This means that U_a is made consistent by its construction. It is also easy to prove that

> *Theorem 2:* If *SCOPE* is a subset of *MSCOPE* and U_a is a maximal set, then there is a unique subset of U_a which relates to *SCOPE*.

The unique subset described in the theorem we shall henceforth call $sub(SCOPE, U_a)$.

We shall make use of John Mackie's [1974] conception of singular causation. Mackie defined singular causation in the following way (the formulation here is taken from [Marsden et al. 1990, p. 66]):

> c is a cause of e when the occurrence of c is, by itself, an insufficient but necessary part of a set of conditions which, when combined, are unnecessary yet sufficient for e (in short: c is an *INUS* condition for e).

Let E (: the effect) be a basic proposition and C (: the causal complex) be a set of basic propositions. We shall also assume that the tenses in C are exactly one time unit earlier than E. We may

consider C as a conjunction of basic statements, which necessarily leads to E.

> *Definition:* An expression of the form *causal(C,E)* is said to be a causal statement if for some i, C (: the causal complex) relates to a subset of S_i and E (: the effect) relates to a member of S_{i+1}.

In CIMP, Mackie's ideas are formalised in terms of causal statements. The elements of C are INUS conditions.

> *Definition:* E is said to be causally determined by the set of U_a and the causal statements CS, if there is an element in CS *causal(C,E)* with $C \subseteq U_a$. Formally,
> $\exists Cs \in CS\!: Cs{=}causal(C,E) \wedge C \subseteq U_a$

> *Definition:* A maximal set U_a is a permissible chronicle (or history) w.r.t. a set of causal statements CS if and only if each basic proposition B which is determined by U_a and CS is a member of U_a. The set of permissible chronicles is written *perm(CS)*. We shall say that CS is consistent if *perm(CS)* is non-empty.

Note that if the basic proposition B is causally determined by a permissible chronicle U_c and the elements in CS, then *dual(B)* is not causally determined by U_c and the elements in CS.

> *Definition:* $E \in MSCOPE$ is said to be causally supported by U_a in CS, if there is a causal statement *causal(C,dual(E))* in CS, and for any such statement C is not a subset of U_a.
> Formally,
> $\exists Cs \in CS\ \exists C\!: Cs{=}causal(C,dual(E)) \wedge$
> $\forall Cs \in CS\ \forall C\!: (Cs{=}causal(C,dual(E)) \supset \sim(C \subseteq U_a))$

> *Definition:* E is said to be uniquely supported by U_a in CS, if E is causally supported by U_a and CS, but *dual(E)* is not causally supported by U_a and CS.

It is obvious that

Theorem 3: If E is supported by U_a in CS then *dual(E)* is not determined by U_a.
Theorem 4: If there is a $causal(C,E) \in CS$ and E is uniquely supported by U_a, then E is determined by U_a.

The set of permissible chronicles can be organised as a branching time system. It is important for the evaluation of counterfactuals that this system can be given an additional structure by the reference to causal models introduced in the following way:

Definition: $CM(CS,U_c)$ is a causal model, if CS is a consistent set of causal statements and U_c is a maximal set, such that if $causal(C,E) \in CS$ and C is a subset of U_c, then $E \in U_c$. - The maximal proposition c will be called *the true chronicle.*

In the following we shall refer to the causal model $CM(CS,U_c)$. It obviously follows from the above definitions that if CS is consistent, then it can give rise to at least one causal model.

We introduce the following algorithm, according to which a selected event after any given event can be constructed:

Assume that E_k is the event just before E_{k+1} in the true chronicle U_c, and that E'_k is a possible event that relates to E_k. Suppose that $E_{k+1}=\{q_j,\ldots,q_N\}$. The problem is how to construct $E'_{k+1}=\{q'_j\ldots,q'_N\}$. The algorithm can be described as follows, where q_i is an arbitrary element in E_{k+1}:
a) If q_i is causally determined by E'_k in CS, then put $q'_i=q_i$.
b) If $dual(q_i)$ is causally determined by E'_k in CS, then put $q'_i=dual(q_i)$
c) If a)-b) do not apply and $dual(q_i)$ is uniquely supported by E'_k and CS, then put $q'_i=dual(q_i)$.
d) If a)-c) do not apply, then put $q'_i=q_i$.

By this algorithm 'the future part' of a permissible chronicle can be constructed from any event. This means that the set of permissible chronicles forms a branching time system with the very special property that for each event there is a selected next event in the system, that is, 'a preferred branch'. For each tense τ there is a relation between maximal propositions, which may be written $a \approx_\tau b$, meaning that the future of U_b relative to the tense τ is constructed from U_a by the above algorithm. The latest tense in A we shall denote by $\tau(A)$.

Before we can present the semantics for the system, we need a proper definition of the full CIMP language.

From *MSCOPE* the well formed formulae (wff) of the CIMP language can be defined by the following rules:

(a) If $A \in MSCOPE$, then A is a wff.
(b) If $A \in MSCOPE$, then $dual(A)$ is a wff.
(c) If A and B are wff's and $SC \subseteq MSCOPE$, then $\sim A$, $(A \wedge B)$, $(SC: A > B)$ are wff's.
(d) Nothing else is a wff.

$(A \vee B)$ and $(A \supset B)$ are defined as usual by negations and conjunctions.

The truth operator can be presented in the following way, where we shall let the maximal propositions play the rôle of instants as originally suggested by A. N. Prior:

$T(a,P(i,q))$ iff $P(i,q) \in U_a$
$T(a,F(i,q))$ iff $F(i,q) \in U_a$
$T(a,\sim A)$ iff not $T(a,A)$.
$T(a,A \wedge B)$ iff $T(a,A)$ and $T(a,B)$
$T(a,q)$ iff $q \in U_a$

Let $Sel(CS,c,A)$ be the set of maximal propositions and $a \in perm(CS)$, where $T(a,A)$ and $c \approx_{\tau(A)} a$. $Sc(A,CS,c,SCOPE)$ is the set of propositions in $Sel(CS,c,A)$, which are maximal with respect to an ordering relation $<_{SCOPE}$, defined in the following way:

$$a <_{SCOPE} b \equiv_{def} (U_c \cap (U_a \setminus sub(SCOPE, U_a)) \subseteq U_b$$

It is easy to see that if $a <_{SCOPE} b$ then

$$(U_c \cap (U_a \setminus sub(SCOPE, U_a)) \subseteq (U_c \cap (U_b \setminus sub(SCOPE, U_b))$$

The statement $a <_{SCOPE} b$ clearly means that everything which is true according to a, but outside SCOPE, is also true according to b. We now define what it means in CIMP for the counterfactual implication *(SCOPE: a>b)* to be true relative to the set of causal statements *CS* and the true chronicle *c*:

> *Definition:* If the evaluation of B depends on the future course of events relative to τ(A), then *T(c,SCOPE: A>B)* is defined as: $\forall a \in Sc(A,CS,c,SCOPE)$: *T(a,B)*.

Three things should be noted about this definition:

(a) If no *SCOPE* is mentioned explicitly, the scope has to be constructed from the basic elements in A and B, i.e. *the natural scope for* $A > B$.

(b) If we want to draw counterfactual consequences regarding the future relative to *τ(A)*, we have to use the algorithm for constructing the selected futures.

(c) The definition does not cover cases where the counterfactual consequence B depends only on past or present events relative to *τ(A)*. Such cases are indeed difficult to express in a linguistically natural manner. One example could be

> (i) If the Germans had not lost the battle of the Marne, then they would not have transferred troops to the Russian Front (before that battle).

It would seem, however, that conceptually we are inclined to understand such cases as meaning the same thing as

> (ii) If the Germans had not transferred troops to the Russian Front, then they would not have lost the battle of the Marne.

But such a contra-position, that is, the view that (i) and (ii) are equivalent, can be questioned. On the other hand, if (i) is considered as different from (ii), it does express an odd proposition, which suggests that the past relative to the antecedent is not settled. This issue is discussed in [Pedersen et al. 1994]. However, here we choose to handle only the 'standard case', which is covered by the above definition.

CIMP differs from Lewis' classical ideas in several ways. First of all CIMP systematically takes time and tense into account, whereas Lewis [1973] makes no explicit reference to time in his definitions. Secondly, CIMP in general incorporates the notion of a scope as in *(SCOPE: A>B)*. Lewis' logic, in contrast, only deals with the logic of $A > B$ (corresponding to the cases, in which the 'natural scope' is used in CIMP). Thirdly, some of the axioms of Lewis' system are not valid in CIMP. Let us for example consider two axioms from [Lewis 1973/1986, p. 132]:

(Lewis 1) $(A > B) \supset (A \supset B)$
(Lewis 2) $(A \wedge B) \supset (A > B)$

It is not difficult to verify that (Lewis 1) holds in CIMP:

Theorem: $(A > B) \supset (A \supset B)$ is valid in CIMP
Proof:
We have to prove that $T(c,(A > B) \supset (A \supset B))$ for any c.
This is proved by reductio ad absurdum.

(1)	$\sim T(c,(A > B) \supset (A \supset B))$	(assumption)
(2)	$T(c,A > B)$	(from 1)
(3)	$\sim T(c,A \supset B)$	(from 1)
(4)	$T(c,A)$	(from 3)
(5)	$\sim T(c,B)$	(from 3)
(6)	$\forall a \in Sc(A,CS,c,SCOPE): T(a,B))$	(from 2, 5 & def.)
(7)	$c \in Sc(A,CS,c,SCOPE)$	(from 4)
(8)	$T(c,B)$	(from 6 and 7).

This contradicts (5). - Q.E.D.

This means that counterfactual implication is in fact stronger than material implication (given that the converse does not hold). - (Lewis 2), however, is not valid in CIMP. Let us as an illustration consider the causal statement:

$CS = \{causal(\{q\}, F(1, \sim r))\}$

and the true chronicle: $U_c = \{p, \sim q\} \cup \{F(1,r)\}$. This basis obviously gives rise to the following permissible chronicles:

$U_a = \{p,q\} \cup \{F(1,\sim r)\}$
$U_b = \{\sim p,q\} \cup \{F(1,\sim r)\}$
$U_d = \{\sim p,\sim q\} \cup \{F(1,r)\}$
$U_e = \{\sim p,\sim q\} \cup \{F(1,\sim r)\}$
$U_f = \{p,\sim q\} \cup \{F(1,\sim r)\}$
$U_c = \{p,\sim q\} \cup \{F(1,r)\}$

The following statement is an instance of (Lewis 2):

$((p \vee q) \wedge F(1,r)) \supset ((p \vee q) > F(1,r))$

In the CIMP evaluation of this statement it is easy to see that $T(c,(p \vee q) \wedge F(1,r))$, $b \in Sc(p \vee q, CS, c, \{p,q,F(1,r)\})$ and $\sim T(b,F(1,r))$. So, (Lewis 2) is clearly not valid in CIMP.

It should also be emphasised that the logic of counterfactual implication is non-monotonic, that is, $(A \wedge C) > B$ cannot be deduced from $A > B$. Similarly, we cannot in general deduce the proposition $A > (B \vee C)$ from $A > B$.

David Lewis has provided several important results regarding the completeness and decidability of his system(s). CIMP is not so well developed. We have no genuine axiomatics for the system. On the other hand, the obvious computability of the logic should make CIMP a proper candidate for the modelling of counterfactual reasoning in artificial intelligence and natural language understanding. We intend to illustrate this by a closer study of our 'dinner scenario'. Before doing this, it will be useful to extend the system such as to deal with modalities also. We add these definitions to the system:

$T(a,NA) \equiv_{def} \forall b\text{: } same\_past(a,b) \supset T(b,A)$
$T(a,MA) \equiv_{def} \exists b\text{: } same\_past(a,b) \wedge T(b,A)$
where *same_past(a,b)* means that
$U_{a,-v} \cup \ldots \cup U_{a,0} = U_{b,-v} \cup \ldots \cup U_{b,0}$

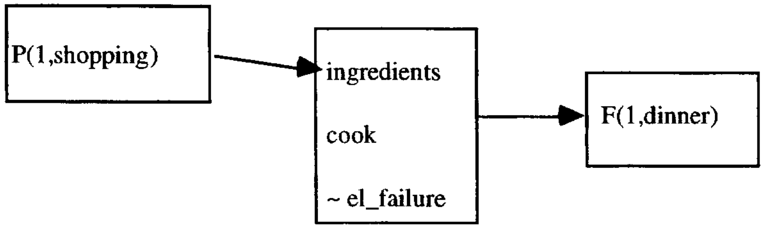
With these definitions it is obviously possible to distinguish between $A > B$, $A > MB$, and $A > NB$.

ANALYSIS OF THE DINNER EXAMPLE

Let us consider the dinner scenario example again. Clearly Joe and Jim both take the following facts for granted:

P(1,shopping) : 'Joe has been to the shop one time unit ago to buy the ingredients'
ingredients: 'The ingredients are available'
cook: 'The cook is present'
el-failure: 'The electrical failure occurs'
F(1,~ dinner): 'Dinner will not be served during the next period of time'.

The list of these 5 basic statements constitutes the true history in this example. Joe and Jim also both assume a number of causal statements, which can be presented graphically in the following way:



This model could also be presented as the following database:

causal({ingredients,cook,~el-failure},F(1,dinner)).
causal({P(1,shopping)},ingredients).

Now, how should the first statement

Joe: If it had not been for the failure, we would have had dinner on time

be evaluated within this causal model? In symbolic form the statement can be formulated as:

~ el-failure > F(1,dinner)

where *el-failure* and *F(1,dinner)* are propositional constants with the obvious meanings. Clearly, in this case only two factors are relevant. The scope of the above statement can be represented by the set *{el-failure,F(1,dinner)}*. which gives rise to three permissible chronicles:

H1. *{el-failure,F(1,dinner)}*
H2. *{~el-failure,F(1,dinner)}*
H3. *{el-failure,F(1,~dinner)}*
- (to each of these chronicles one must add the invariant set *{P(1,shopping), ingredients, cook}*. to obtain the full permissible history).

It is easy to see that the following counterfactual holds:

~el-failure > F(1,dinner)

The next statement is

Jim: You're wrong. If the cook left, you might still not have had dinner on time, even in the absence of the failure.

In symbols:

(~ cook ∧ ~ el-failure) > MF(1,~dinner)

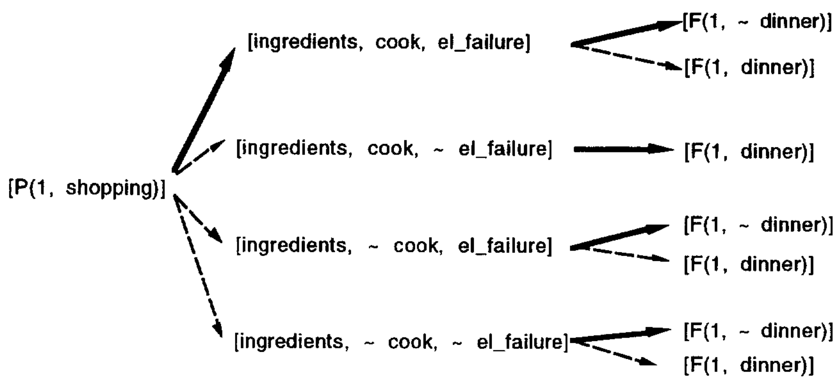
This obviously holds in the model, and indeed, the following stronger statements also holds:

> If the cook left, you would not have had dinner on time, even in the absence of the failure.

In symbols:

(~ cook ∧ ~ el-failure) > F(1,~dinner)

The truth of this counterfactual implication can be seen by inspection into this picture:



Joe's next statement in the conversation, "what I mean is that if there were no failure and the cook agreed to stay, we would have had dinner on time", is simply the counterfactual

(cook ∧ ~ el-failure) > F(1,dinner)

which clearly holds.

However, consider the following answer:

> Jim: You're wrong again. If you had not been to the shop to buy the ingredients, you would not have had dinner on time, even if the cook stayed and there were no electrical failure.

In symbols:

(cook ∧ ~ ingredients ∧ ~ el-failure ∧ ~P(1,shopping)) > F(1,~dinner))

Here the scope is changed such that in principle, 15 permissible chronicles are taken into consideration. The only chronicles corresponding to a true antecedent of the counterfactual in question are:

H1: *{P(1,~shopping), ~ingredients,cook, ~el-failure, F(1,dinner)}*
H2: *P(1,~shopping),~ingredients,cook,~el-failure, F(1,~dinner)}*

By our definitions, H2 is selected in this case. The validity of this counterfactual can be verified by inspection into the above diagram. - The next statement is

> Joe: I don't agree. Somebody else might have brought the ingredients. As long as the cook has got the ingredients and there is no electrical failure, we shall with necessity have dinner on time.

In symbols:

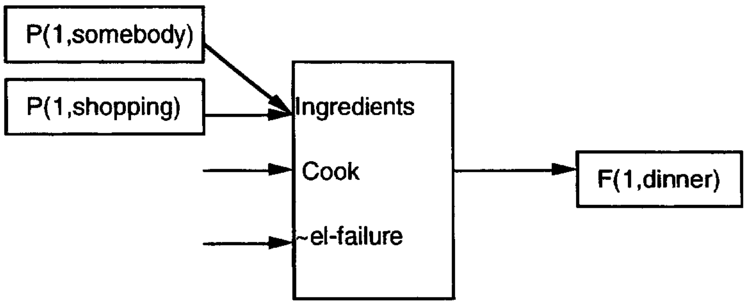
(cook ∧ ~ el-failure ∧ P(1,somebody) > NF(1,dinner)

The evaluation of this statement involves an expansion of the list of facts with the fact *~P(1,somebody)*. Obviously, this also means that the scope is expanded. The causal model is expanded with the statement:

causal({P(1,somebody)},ingredients).

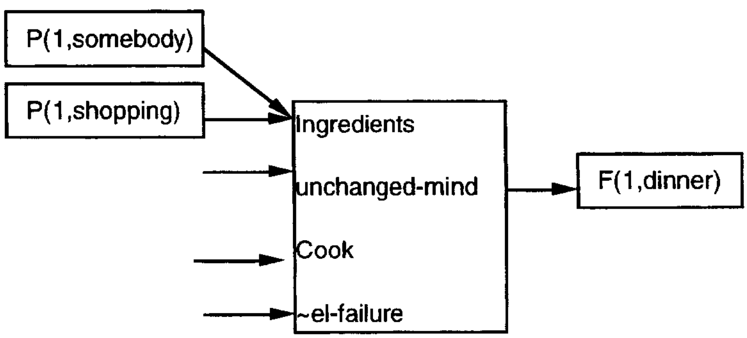
Once more, we can construct all the permissible chronicles, and it can then be verified that the statement is true (*F(1,dinner)* holds in all those chronicles in which the antecendent is true, so this consequence is true with necessity).

The new causal model can be presented graphically in the following way:



The next move in the debate is similar from a formal point of view; it involves a new model with the fact *unchanged-mind*, and an extended causality relation:

> Jim: Well, the cook might change his mind because of all your quarrelling. Again, in that case there would not be dinner on time, even if there were no electrical failure.

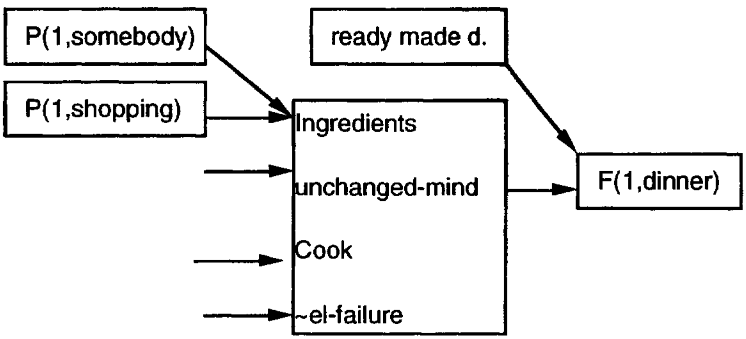


With the new scope and the new model it can easily be verified that

(~ el-failure ∧ ~ unchanged-mind) > ~ F(1,dinner)

In the next part of the conversation yet another new complex is introduced:

> Joe: Well, we might have had dinner on time, if I had ordered some ready-made dinner. The electrical failure could not have prevented the dinner in that case.



With this new complex and the new fact: *~ready-made_d.*, it can be shown that

(el-failure ∧ ready-made_d) > NF(1,dinner)

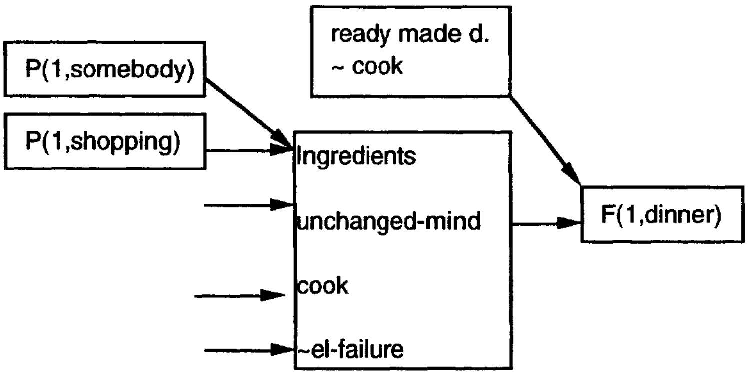
holds for the scope in question (we take 'could not...prevent' as indicating necessity).

The conversation is concluded by the statement

> Jim: I think you're wrong again. The cook is a union man. He will bar the door to the delivery of the ready-made dinner, since the ready-made dinner company takes away jobs

from proper cooks. But if the cook leaves, the ready-made dinner can be delivered.

Here, the causal model becomes even more complicated:



As the definition of an INUS-condition suggests, there can be more causal complexes for an effect. The sets of conditions of two causal complexes for some effect E may be disjoint, but they may also have a non-empty intersection. It is interesting that in the present model the proposition *cook* as well as its negation *~cook* appear in two parallel complexes. With the assumed scope and the above model the counterfactual

$$(\sim cook \wedge ready\_made\_d) > NF(1,dinner)$$

is true. - One might now fear that it holds that

$$(cook > F(1,dinner)) \wedge (\sim cook > F(1,dinner))$$

since *cook* and *~cook* are both INUS-conditions in complexes leading to the same effect *F(1,dinner)*. However, with the true history

{P(1,shopping),ingredients,cook,
el-failure,~ready_made_d, F(1,~ dinner)}

and the described causality relation, the counterfactual

~cook > F(1,dinner)

does not hold. This can be seen by considering the permissible history

{P(1,shopping),ingredients,~cook, el-failure,~ready_made_d, F(1,~ dinner)}.

PERSPECTIVES

As we have already suggested we do not consider the CIMP system to be in any sense finished. On the other hand, we believe that ideas like the ones proposed here should be included into a general account of causal and counterfactual reasoning. Moreover, we think that these ideas have far-reaching consequences. The dinner scenario is clearly a toy example, but it exhibits some patterns of reasoning which are crucial for many (partly overlapping) issues within real information systems:

• *artificial intelligence*: it is clear that CIMP implements several kinds of reasoning, and moreover, that it maintains an affinity to natural language, which also makes it a candidate for natural language understanding ([Hasle and Øhrstrøm 1992]);

• *updating databases*: the non-monotonic growth of information in databases is in fact a very general problem, having to do with how to maintain consistency. The non-monotonic features of CIMP illustrate this kind of problems, and some ways of handling them are suggested;

• *planning*: the construction of a number of futures with respect to a notion of relevance (scope) and a set of underlying causal

assumptions is essential in many planning systems (in chapter 3.7, we take up the issue of time in planning systems at a more general level);

• *diagnosis*: the fact that the techniques of CIMP are directly applicable to diagnosis has been investigated at some length, e.g. in [Pedersen et al. 1994]. Here it is shown that the CIMP-logic can be used with considerable generality for error diagnosis as well as medical diagnosis;

• *decision support and judicial systems*: the relevance in decision support is an obvious consequence of the points above. - One interesting application of CIMP shows its more specific relevance in judicial systems. Anne Rasmussen [1993] has analysed the Danish authorities' official report on the fire catastrophe on the ferry boat Scandinavian Star (which took place on the 7th of April 1990). In her analysis of the report, Rasmussen applied the CIMP notions and thus obtained a logical - as well as computable - model of the reasoning used. This kind of reasoning studied is furthermore used for determining the responsibility for what went wrong. The conclusions of the report are in the real world used both for deciding how to avoid similar events in the future, and for deciding upon verdicts in the case. Since CIMP can model such reasoning, it could be used for similar decision-making purposes (although we do not recommend that verdicts be based upon it).

Thus, we are suggesting that the CIMP-ideas are highly versatile for information systems. If that is so, the reason for this is really to be found in its temporal nature rather than in any special other merits which it might have. Conceptually, CIMP formalises and unifies a number of features of reasoning within a temporal framework - specifically, a metric branching time model. Therefore, the specialised study of this chapter in fact also holds more general implications for the concept of time and its importance.

3.5. LOGIC OF DURATIONS

The logics studied so far have mainly been based on some notion of temporal *instants* - rather than *durations*. Of course, we have also seen some exceptions to this rule, for instance John Buridan's thoughts presented in chapter 1.5, or Zeno Vendler's distinctions mentioned in chapter 1.6. The prevalence of instant-based logics has not been left unchallenged, though: from a fairly early stage of the development of formal semantics for natural languages it has been argued almost vehemently that human language and reasoning call for an interval-based rather than point-based semantics. This was emphasised and formally elaborated in [Dowty 1979], which forms a milestone in this respect. Similar discussions, albeit from rather different perspectives, have been going on within philosophical logic as well as computer science. For instance, in philosophical logic Peter Simons [1987] has carried out some careful studies of 'temporal parts' and Antony Galton has - with reference to conceptual structures in natural language - argued that temporal logic should take heed of durations, and worked out proposals for meeting this requirement [Galton 1984]. Similar considerations have been put forward within artificial intelligence research, notably by Allen [1983, 1984], Allen and Hayes [1985, 1989], and Peter Ladkin [1987]. In theoretical computer science, Ben Moszkowski [1983], Roger Hale [1987] and others have shown that at least for some purposes durational logic offers more than instant logic.

There can be no doubt that a fully adequate temporal logic must be able to account also for durations; and the idea that durations should take conceptual priority over instants is worth considering. But that does not mean that instant-based approaches must be discarded altogether. On the contrary, many aspects of temporal concepts and phenomena are best studied within such logics - if for no other reason, then simply because they are generally speaking less complex. The fact that meaningful and fruitful studies can be conducted within instant-

based frameworks should have become quite clear from previous discussions.

In fact, the logic of durations was studied even some years before Prior's rediscovery of tense logic. The first modern logician to formulate a calculus in this field was A. G. Walker [1947]. Walker was, however, not concerned with temporal logic in any general sense.

Walker considered a structure $(S,<)$, where S is a non-empty set of periods. This set is ordered by a partial ordering relation '$<$', analogous to the before-after-relation among instants. Two interesting and related aspects of this model should be mentioned right away: first, it does not seem counterintuitive to call one period 'earlier' than another one, even if they 'overlap'. Thus 'Mary opened the door before John rushed in' seems quite right, even if John begins his rushing in before Mary concludes her opening the door. Nevertheless, the '$a<b$'-relation is to be considered as 'strict' in the sense that no overlap between a and b is permitted. Second, since the ordering relation is only partial, and since the notion of overlap has already made itself manifest, it is interesting to consider also the latter relation, defined as

$$a/b \equiv_{def} \sim(a<b \vee b<a).$$

This obviously corresponds to the idea of two periods a and b overlapping each other. - Walker formulated an axiomatic system using the following two axioms:

(W1) a/a
(W2) $(a<b \wedge b/c \wedge c<d) \supset a<d$

In relation to these axioms Walker was able to construct a set-theoretic structure of triplets (A,B,C), where A, B, and C are all sets of durations such that

1) A and B are non-empty
2) the union of A, B and C is the set of all durations
3) every element in A is before every element in B

4) every element in C is overlapping some element in A as well as some element in B.

Walker demonstrated that the structure of these triplets has all the algebraic properties which we would intuitively expect the structure of temporal instants to have. For this reason it may be reasonable to view a temporal instant as such a 'secondary' construct from the logic of durations.

As we have seen C. L. Hamblin contributed significantly to the development of temporal logic in its very early period. More than two decades later than Walker, Hamblin [1972] also put forth a theory of the logic of durations. Hamblin was not aware of Walker's work when he developed his theory [Hamblin 1972, p. 331], but he achieved some similar results using a different technique. Hamblin also considered a fundamental structure consisting of a set of durations with a partial ordering relation $(S,<)$. In addition he defined the following relations for arbitrary durations, where $(a \mathfrak{I}\, b)$ may be read 'b follows immediately after a', and $(a \angle b)$ may be read 'a is contained in b'.:

$$a \mathfrak{I}\, b \equiv_{def} (a<b \wedge \sim(\exists c\colon a<c \wedge c<b))$$
$$a \angle b \equiv_{def} \forall c\colon (\, c/a \supset c/b)$$

Using the definition of $a \mathfrak{I}\, b$, Hamblin could also offer a derived notion of an instant:

Any pair of durations (a,b) uniquely defines an instant if and only if $(a \mathfrak{I}\, b)$.

We shall use expressions like $a \mathfrak{I}\, b \mathfrak{I}\, c$ for the conjunction of $a \mathfrak{I}\, b$ and $b \mathfrak{I}\, c$. Hamblin's axioms can be formulated in the following way using our notation (and omitting external universal quantification):

(Hamblin 1): $\sim(a<a)$
(Hamblin 2): $(a<b \wedge c<d) \supset (a<d \vee c<b)$
(Hamblin 3): $a<b \supset (a \mathfrak{I}\, b \vee (\exists c\colon a \mathfrak{I}\, c \mathfrak{I}\, b))$

(Hamblin 4): $(a \mathrel{\mathbb{I}} c \wedge a \mathrel{\mathbb{I}} d \wedge b \mathrel{\mathbb{I}} c) \supset b \mathrel{\mathbb{I}} d$
(Hamblin 5): $(a \mathrel{\mathbb{I}} b \mathrel{\mathbb{I}} d \wedge a \mathrel{\mathbb{I}} c \mathrel{\mathbb{I}} d) \supset b=c$
(Hamblin 6): $\exists b: a<b$
(Hamblin 7): $\exists b: b<a$
(Hamblin 8): $\exists b: (b \angle a \wedge \sim(b=a))$
(Hamblin 9): $b \angle a \supset (T(a,p) \supset T(b,p))$
(Hamblin 10): $\forall b:(b \angle a \supset (\exists c: c \angle b \wedge T(c,p)) \supset T(a,p))$

It is interesting that Hamblin 9-10 express two features, which are also known from lattice-based theories of mass terms [Link 83] and event structures [e.g. Bach 1986, Link 1987]. Specifically, (Hamblin 9) states a kind of *dissectiveness*: if some proposition p 'is true with respect to' some interval a, and b is contained in a, then p is true also with respect to b. We might also say that this expresses 'downwards inheritance'. In a dual manner, (Hamblin 10) expresses a sort of *cumulativity*. However, it is well known, at least from later literature on durations, that not all 'properties' of durations behave like this: thus for instance, an 'accomplishment' like 'Mary baked a cake' (say, from 1 p.m. to 4 p.m.) does not entail that Mary baked a cake during the sub-periods, say, from 2 p.m. to 3 p.m. (Note that even though it may be tempting to say that Mary was 'engaged in the process' also during all sub-periods, she certainly did not accomplish it during any of those). It is therefore clear that Hamblin's theory is confined to certain subsets of (properties of) durations.

During the last decade various kinds of durational logic have been studied and applied within artificial intelligence research and natural language understanding (usually under the heading 'interval semantics', which seems more popular in this scientific community). Two researchers in this field, who have contributed significantly to the development of durational logic, are James Allen and Patrick J. Hayes [1985, 1989]. Like Walker and Hamblin, Allen and Hayes have taken as their starting point the study of the structure of the partially ordered set of durations. They have suggested an axiomatic system, which we

reformulate as (AH 1-5). We shall use $\veebar$ for a generalised kind of 'exclusive disjunction', i.e.

$$(p_1 \veebar p_2) \equiv \sim(p_1 \equiv p_2).$$

A generalised definition can be given as

$$(p_1 \veebar \ldots \veebar p_N) \equiv_{def} \bigwedge_{i=1..N} p_i \equiv (\sim p_1 \wedge \ldots \wedge \sim p_{i-1} \wedge \sim p_{i+1} \wedge \ldots \wedge \sim p_N)$$

The axioms are:

(AH1) $(a \mathrel{\mathrm{I}} c \wedge a \mathrel{\mathrm{I}} d \wedge b \mathrel{\mathrm{I}} c) \supset b \mathrel{\mathrm{I}} d$
(AH2) $(a \mathrel{\mathrm{I}} b \wedge c \mathrel{\mathrm{I}} d) \supset (a \mathrel{\mathrm{I}} d \veebar \exists e{:}a \mathrel{\mathrm{I}} e \mathrel{\mathrm{I}} d \veebar \exists f{:}c \mathrel{\mathrm{I}} f \mathrel{\mathrm{I}} b)$
(AH3) $\exists b, c{:}\ b \mathrel{\mathrm{I}} a \mathrel{\mathrm{I}} c$
(AH4) $(a \mathrel{\mathrm{I}} b \mathrel{\mathrm{I}} d \wedge a \mathrel{\mathrm{I}} c \mathrel{\mathrm{I}} d) \supset b{=}c$
(AH5) $a \mathrel{\mathrm{I}} b \supset \exists e \forall c,d{:}\ (c \mathrel{\mathrm{I}} a \mathrel{\mathrm{I}} b \mathrel{\mathrm{I}} d \supset c \mathrel{\mathrm{I}} e \mathrel{\mathrm{I}} d)$

This axiomatic system obviously takes the I-relation as the primitive. However, this does not constitute any essential step away from Hamblin's system, in which the opposite implication of (Hamblin 3) can easily be proved. We therefore have as a theorem

(Hamblin 3'): $a<b \equiv (a \mathrel{\mathrm{I}} b \vee (\exists c{:}\ a \mathrel{\mathrm{I}} c \mathrel{\mathrm{I}} b))$

which may obviously be used as a definition of the <-relation in the AH-system. With this definition (Hamblin 2) is provable in the AH-system. (AH1) and (AH4) are just (Hamblin 4) and (Hamblin 5), and (Hamblin 6-7) are immediate consequences of (AH3). Because of the exclusive disjunctions in (AH2), we can derive $\sim a \mathrel{\mathrm{I}} a$, i.e. (Hamblin 1). So it seems that (Hamblin 8) is the only difference between the systems (if we disregard Hamblin's special requirements of cumulativity and dissectiveness, cf. Hamblin 9-10).

It follows from (AH4) that the e in (AH5) is uniquely determined by the durations a and b. Following Allen and Hayes, we

shall call this resulting duration the *sum* of a and b, i.e. $e=a+b$. However, we point out that this sum-operator is *not* commutative and is in effect a kind of concatenation rather than a 'usual' sum-operator.

Allen and Hayes have shown that two arbitrary durations can be related in exactly 13 different ways, which can all be expressed solely in terms of the $\parallel$ -operator and equality:

a meets b $\equiv_{def} a \parallel b$
a is met by b $\equiv_{def} b \parallel a$
a is before b $\equiv_{def} \exists c{:}\ a \parallel c \parallel b$
a is after b $\equiv_{def} \exists c{:}\ b \parallel c \parallel a$
a starts b $\equiv_{def} \exists c{:}\ b=a+c$
a is started by b $\equiv_{def} \exists c{:}\ a=b+c$
a finishes b $\equiv_{def} \exists c{:}\ b=c+a$
a is finished by b $\equiv_{def} \exists c{:}\ a=c+b$
a overlaps b $\equiv_{def} \exists c,d,e{:}\ a=c+d \wedge b=d+e$
a is overlapped by b $\equiv_{def} \exists c,d,e{:}\ (b=c+d \wedge a=d+e)$
a during b $\equiv_{def} \exists c,d{:}\ b=c+a+d$
a contains b $\equiv_{def} \exists c,d{:}\ a=c+b+d$
a equals b $\equiv_{def} a=b$

It is very illuminating to study various combinations among these 13 relations. Using Allen's and Hayes' axiomatisation, it is possible to implement a reasoning system, by means of which statements like

If a overlaps b and b is started by c, then a overlaps c;
If a finishes b and b starts c, then a during c;
If a during b and b overlaps c, then a is not met by c;

can be proved. This kind of reasoning will be important in any system, which should be able to perform or simulate commonsense reasoning involving time periods. Consider for instance this situation:

Mary was reading during the postman's visit. John finished his beer just when the postman left.

It is clear that John's and Mary's respective activities are not explicitly related by the above statements. On the other hand, it is also clear that a certain temporal relation must obtain between them. The information in this scenario can be captured in durational logic in terms of the following two statements:

> The postman's visit takes place during Mary's reading.
> The postman's visit finishes or is finished by John's drinking the beer.

In the logic of durations it can be formally demonstrated from these statements that the duration during which 'John is drinking the beer' is during, overlaps, or starts 'Mary's reading'. - The task of carrying out all such kinds of reasoning about durations is by no means simple (see also [Knight & Jixin 1992]).

We have already pointed out that for some durations, or perhaps rather, certain types of events, there are no sub-parts; for instance, if 'John opened the door' during some period a, it will not be true to say that John opened the door during any subinterval b contained in a. In this case, dissectiveness does not obtain (cf. Hamblin 9). When reasoning about durations we often come across durations without parts corresponding to for example opening a door. Allen's and Hayes' reason for excluding in general the axiom (Hamblin 8) is precisely that they want to study these so-called 'moments', which can be understood as durations without any internal structure (not to be confused with 'instants'). It appears that nothing is contained in a moment, and that two moments cannot overlap each other.

Hamblin's (as well as Allen's and Hayes') durational logic is based on a conception of durations as something similar to real intervals. A number of interesting theorems can be proved from Hamblin's axioms, but the system is not sufficient to establish that linear intuition about time on which it is obviously based. The reason for this is that there is nothing in (Hamblin 1-8) to exclude a genuine branching time model. On the other hand, if time should in fact be conceived as branching, then the

'containment'-relation $\angle$ in the above axioms will yield some very strange results, and will be rather far from the inclusion relation that Hamblin probably had in mind.

Peter Röper [1980] has developed a more fine-grained logic from very much the same intuition as Hamblin's. Röper starts from a non-empty set S of durations and a relation $\subseteq$ defined on S, which should express the 'inclusion' relation among durations. Röper defines a P-frame as a structure $(S, \subseteq, <)$ satisfying:

(A1) If $x<y$, $x'\subseteq x$ and $y'\subseteq y$, then $x'<y'$.
(A2) If for every $x'\subseteq x$ and $y'\subseteq y$ there are $x''\subseteq x'$ and $y''\subseteq y'$ such that $x''<y''$, then $x<y$.
(A3) If $x<y$ and $y<z$, then $x<z$.
(A4.1) For any x, there exists $x'\subseteq x$ and y such that $x'<y$.
(A4.2) For any x, there exists $x'\subseteq x$ and y such that $y<x'$.
(A5.1) For any x, y and z, if $x<y$ and $x<z$, then there exists $y'\subseteq y$ and $z'\subseteq z$ such that $z'<y'$ or $y'<z'$.
(A5.2) For any x, y and z, if $y<x$ and $z<x$, then there exists $y'\subseteq y$ and $z'\subseteq z$ such that $z'<y'$ or $y'<z'$.

Obviously, (A5.1) corresponds to forwards linearity, whereas (A5.2) ensures backwards linearity. On the other hand, there is nothing in Röper's system to ensure the irreflexivity of the ordering relation.

Some of the further details of Röper's system are mainly concerned with that distinction between dissective and non-dissective 'events', which we have already suggested. We shall recapitulate the main problem by considering the following two propositions:

p: 'Percival drinks a pint of bitter'
q: 'Araminta is in Oxford'.

Let us assume that both propositions are true for a duration a, and let b be an arbitrary sub-duration, i.e. $b \subseteq a$. Then a proposition such as q will also be true for the duration b. Following

Röper, we shall say that q is *persistent* (i.e. dissective). This can be symbolically expressed as:

$$(T(a,q) \wedge b \subseteq a) \supset T(b,q)$$

A persistent proposition denotes 'a property' in Allen's terminology. On the other hand, a proposition such as p may be false for some or all sub-durations. That is, it is in general conceivable that for some sub-duration b, the following formula holds:

$$T(a,p) \wedge b \subseteq a \wedge \sim T(b,p)$$

Without doubt, this is true for our present example. Suppose that Percival drank one pint of bitter, beginning at 11:30 a.m. and finishing at 11:40 a.m. Then it is false that he drank one pint of bitter during the subinterval from 11:35 to 11:36. - Allen reserves the term 'an event' for propositions of this type. The distinction between these two types of propositions is central for any attempt at establishing an adequate durational logic.

It is evident that Hamblin's theory (cf. Hamblin 9-10) is about what Allen has called properties, that is, persistent propositions. Röper, however, makes a distinction between the logic of what he has called *homogeneous* sentences and the logic of 'other sentences'. According to Röper a sentence p is homogeneous if and only if it is 1) persistent (dissective) and 2) cumulative (i.e. for any a, if p is true for all sub-durations of a, then p is true for a).

To avoid terminological confusion, let us recapitulate the major distinctions: in chapter 1.6, we briefly described Zeno Vendler's famous four-fold distinction between verb phrases. It seems fair to say, though, that the main distinction is between

(a) predicates, which are dissective and cumulative, and
(b) predicates, which describe 'non-divisible' phenomena.

The former we have called *persistent*; Allen calls them *properties*, and Röper collects them under the heading *homogeneous*. The latter are (somewhat misleadingly) called *events* by

Allen. Further refinements of these distinctions may be found in the work of Allen as well as other authors.

Röper's way of assigning truth-values to homogeneous sentences closely follows the intuitions embodied by (Hamblin 9-10). Röper introduces a valuation function V from pairs consisting of a propositional variable and a time period into the truth values $\{0,1\}$, in such a way that his $V(p,a)=1$ corresponds to Hamblin's $T(a,p)$. Röper defines a P-model based on the P-Frame $(S,\subseteq,<)$ as a structure $(S,\subseteq,<,V)$, where the V-function satisfies

(i) If $V(p,x)=1$ and $y \subseteq x$, then $V(p,y)=1$
(ii) If for every $y \subseteq x$ there is a z such that $z \subseteq y$ and $V(p,z)=1$, then $V(p,x)=1$

The truth of a wff relative to the P-model is defined as

(P1) If p is a propositional variable, then $P \models_x p$ iff $V(p,x)=1$
(P2) $P \models_x \sim A$ iff for all $y \subseteq x$, not $P \models_y A$
(P3) $P \models_x A \wedge B$ iff $P \models_x A$ and $P \models_x B$
(P4) $P \models_x A \vee B$ iff for every $y \subseteq x$, there exists a $z \subseteq y$ such that $P \models_z A$ or $P \models_z B$
(P5) $P \models_x A \supset B$ iff for every $y \subseteq x$, if $P \models_y A$, then $P \models_y B$
(P6) $P \models_x GA$ iff for every $y \subseteq x$ and all z with $y<z$, $P \models_z A$
(P7) $P \models_x HA$ iff for every $y \subseteq x$ and all $z< y$, $P \models_z A$

It is possible to derive truth conditions for F and P:

(P8) $P \models_x FA$ iff for all $y \subseteq x$, there exists $z \subseteq y$ and w with $z<w$ such that $P \models_w A$
(P9) $P \models_x PA$ iff for all $y \subseteq x$, there exists $z \subseteq y$ and w with $w<z$ such that $P \models_w A$

Röper has formulated an axiomatic system corresponding to the P-models. One of the most interesting features is the fact that the formulae

$A \supset PA$
$A \supset FA$

turn out to be P-valid (true in any P-model), and provable in the corresponding axiomatic system. Röper points out that the presence of these theorems is a "perhaps unexpected feature of the system" [p. 459]. It is interesting that John Buridan in his durational logic made a similar observation, as we have seen in chapter 1.5. He introduced a distinction between relative and absolute tenses. The above theses would also be valid in Buridan's logic, *provided* that the past and the future are understood as relative in this context. However, Röper's logic of homogeneous sentences contains no similar distinction. Nevertheless, we shall show that Buridan's idea can also be incorporated into Röper's system.

In order to construct a semantical model for the logic of non-homogeneous sentences, Röper has introduced an E-frame $(S,\subseteq,<)$ as a structure that satisfies the following conditions:

(D1) $\subseteq$ is reflexive.
(D2) $\subseteq$ is transitive.
(D3) $<$ is transitive.
(D4) For any x, y, x', y', if $x<y$, $x'\subseteq x$ and $y'\subseteq y$, then $x'<y'$.

In addition, an E-model based on the E-frame $(S,\subseteq,<)$ is defined as a structure $(S,\subseteq,<,V)$. V is a function from wff's into $\{0,1\}$, which satisfies this condition:

For any x in S, there exists a $y \subseteq x$ such that either $V(p,z)=1$ for every $z \subseteq y$, or $V(p,z)=0$ for every $z \subseteq y$.

The truth of a wff relative to this E-model is defined as follows:

(E1) If p is a propositional variable, then $E \models_x p$ iff $V(p,x)=1$
(E2) $E \models_x \sim A$ iff not $E \models_x A$

(E3) $E \models_x A \wedge B$ iff $E \models_x A$ and $E \models_x B$
(E4) $E \models_x A \vee B$ iff $E \models_x A$ or $E \models_x B$
(E5) $E \models_x A \supset B$ iff: (not $E \models_x A$) or $E \models_x B$
(E6) $E \models_x GA$ iff for every y with $x<y$, $E \models_y A$
(E7) $E \models_x HA$ iff for every y with $y<x$, $E \models_y A$

So far the semantics is exactly as it would be for an 'instant-oriented' tense logic. However, in order to express a durational equivalent of linearity and density we need two extra operators, called L and W. These operators are defined by the following semantic properties:

(E8) $E \models_x LA$ iff for every y with $y \subseteq x$, $E \models_y A$
(E9) $E \models_x WA$ iff for some y with $x \subseteq y$, $E \models_y A$

Consider now forwards and backwards linearity:

For any durations x, y, z, if $x<y$ and $x<z$, then either $y<z$, $z<y$, or there is a duration w such that $w \subseteq z$ and $w \subseteq y$.

For any durations x, y, z, if $y<x$ and $z<x$, then either $y<z$, $z<y$, or there is a duration w such that $w \subseteq z$ and $w \subseteq y$.

Röper has argued that these properties correspond to the following axioms:

(Ax1) $(Fp \wedge Fq) \supset (F(p \wedge Fq) \vee F(Fp \wedge q) \vee F(Wp \wedge Wq))$
(Ax2) $(Pp \wedge Pq) \supset (P(p \wedge Pq) \vee P(Pp \wedge q) \vee P(Wp \wedge Wq))$

Density is expressed as follows:

For any duration x there are durations y and z, such that $y<z$, $y \subseteq x$, and $z \subseteq x$.,

which leads to the axioms (where $M \equiv {\sim}L{\sim}$):

(Ax3) $Lp \supset MFp$

(Ax4) $Lp \supset MPp$

The truth conditions for F and P can easily be derived:

(E10) $E \models_x FA$ iff for some y with $x<y$, $E \models_y A$
(E11) $E \models_x PA$ iff for some y with $y<x$, $E \models_y A$

(E10) and (E11) obviously give rise to what Buridan called an absolute understanding of the tenses. The relative tenses may be expressed as follows:

(E12) $E \models_x F_{rel}A$ iff for some $z \subseteq x$ there is a y with $z<y$ such that $E \models_y A$
(E13) $E \models_x P_{rel}A$ iff for some $z \subseteq x$ there is a y with $y<z$ such that $E \models_y A$

It is easy to verify that $F_{rel} \equiv MF$ and $P_{rel} \equiv MP$. - Röper has demonstrated that the axiomatic system for a minimal non-homogeneous durational tense logic, corresponding to a logic based on (D1-4), would be the axioms of K_t with the addition of these axioms:

(MD1) $L(p \supset q) \supset (Lp \supset Lq)$
(MD2) $\sim W \sim (p \supset q) \supset (Wp \supset Wq)$
(MD3) $WLp \supset p$
(MD4) $p \supset LWp$
(MD5) $Lp \supset p$
(MD6) $p \supset Wp$
(MD7) $Lp \supset LLp$
(MD8) $WWp \supset Wp$
(MD9) $Gp \supset GGp$
(MD10) $Hp \supset HHp$
(MD11) $MGp \supset GLp$
(MD12) $MHp \supset HLp$

and the rules

(MDR1) If $\vdash p$, then $\vdash Lp$

(MDR2) If $\vdash p$, then $\vdash \sim W\sim p$

We may call this minimal system for durational logic DK_t. In chapter 2.8 we have shown that for instant logic the usual axioms for backwards and forwards linearity can in fact be proved as theorems, if we instead adopt some considerably simpler axioms. Similarly, we shall show that (Ax1-2) can be proved on the basis of DK_t and some simpler axioms of linearity:

(Ax1') $(PFp \supset (Fp \vee Pp \vee MWp)$
(Ax2') $(FPp \supset (Fp \vee Pp \vee MWp)$

In order to do that we need some lemmas.

Lemma 1. $DK_t \vdash H(L\sim W\sim p \supset (Hp \supset q)) \vee H(Hq \supset p))$

Proof:

The proof is carried out by reductio ad absurdum.

(1)	$\sim(H(L\sim W\sim p \supset (Hp \supset q)) \vee H(Hq \supset p)))$	(ass.)
(2)	$\sim H(L\sim W\sim p \supset (Hp \supset q))$	(1)
(3)	$\sim H(Hq \supset p))$	(1)
(4)	$P(L\sim W\sim p \wedge Hp \wedge \sim q)$	(2)
(5)	$P(Hq \wedge \sim p)$	(3)
(6)	$HFP(Hq \wedge \sim p)$	(5, A4)
(7)	$P(L\sim W\sim p \wedge Hp \wedge \sim q \wedge FP(Hq \wedge \sim p))$	(4,6)
(8)	$P((L\sim W\sim p \wedge Hp \wedge \sim q \wedge P(Hq \wedge \sim p)) \vee$ $(L\sim W\sim p \wedge Hp \wedge \sim q \wedge MW(Hq \wedge \sim p)) \vee$ $(L\sim W\sim p \wedge Hp \wedge \sim q \wedge F(Hq \wedge \sim p)))$	(7, A7)

But (8) is clearly impossible since all the components in the disjunction are impossible.

Lemma 2.

$DK_t \vdash (H(Wp \supset \sim W\sim q) \wedge H(p \supset Hq) \wedge H(Pp \supset q) \wedge Pp) \supset Hq$

Proof:

By substitution in Lemma 1 we find

$$H(L{\sim}W{\sim}q \supset (Hq \supset {\sim}p)) \vee H(H{\sim}p \supset q))$$

Therefore, the problem can be split into two cases: in the first case $H(L{\sim}W{\sim}q \supset (Hq \supset {\sim}p))$ is assumed, and in the second $H(H{\sim}p \supset q))$ is assumed. In the first case we make the following derivation:

(1)	$H(Wp \supset {\sim}W{\sim}q)$	(ass.)
(2)	$H(p \supset Hq)$	(ass.)
(3)	$H(Pp \supset q)$	(ass.)
(4)	Pp	(ass.)
(5)	$H(L{\sim}W{\sim}q \supset (Hq \supset {\sim}p))$	(ass.)
(6)	$HL(Wp \supset {\sim}W{\sim}q)$	(1, MD12)
(7)	$H(LWp \supset L{\sim}W{\sim}q)$	(1, MD1)
(8)	$H(LWp \supset (Hq \supset {\sim}p))$	(5,7)
(9)	$H(p \supset (Hq \supset {\sim}p))$	(8, MD5)
(10)	$H(p \supset {\sim}p)$	(2, 9)
(11)	${\sim}P(p \wedge p)$	(10)
(12)	${\sim}Pp$	(11)

(11) contradicts contradicts (4). This means that the assumptions in the antecedent rule out the first case. In the second case, we can argue in the following way:

(1)	$H(p \supset q)$	(ass.)
(2)	$H(p \supset Hq)$	(ass.)
(3)	$H(Pp \supset q)$	(ass.)
(4)	Pp	(ass.)
(5)	$H(H{\sim}p \supset q))$	(ass.)
(6)	$H({\sim}q \supset Pp))$	(5)
(7)	$H({\sim}q \supset q))$	(6 and 3)
(8)	${\sim}P({\sim}q \wedge {\sim}q))$	(7)
(9)	Hq	(8)

Q.E.D.

Now, (Ax1) can be proved from lemma 2 in the following way:

$$(H(Wp \supset \sim W\sim q) \wedge H(p \supset Hq) \wedge H(Pp \supset q) \wedge Pp) \supset Hq$$
$$(H(Wp \supset \sim W\sim q) \wedge H(p \supset Hq) \wedge H(Pp \supset q)) \supset (Pp \supset Hq)$$
$$\sim(Pp \supset Hq) \supset \sim(H(Wp \supset \sim W\sim q) \wedge H(p \supset Hq) \wedge H(Pp \supset q))$$
$$(Pp \wedge P\sim q) \supset (P(Wp \wedge W\sim q) \vee P(p \wedge P\sim q) \vee P(Pp \wedge \sim q))$$

From this (Ax1) can be obtained by substitution. In a similar way it is possible to prove (Ax2) in a system enlarged with (Ax2'). - In this way we have demonstrated that

$DK_t \cup \{Ax1'\} \vdash Ax1$, and $DK_t \cup \{Ax2'\} \vdash Ax2$

It can also be demonstrated that

$DK_t \cup \{Ax1\} \vdash Ax1'$, and $DK_t \cup \{Ax2\} \vdash Ax2'$

This means that it is in fact a matter of choice whether we want to use {Ax1,Ax2} or {Ax1',Ax2'} to express linearity. As we have seen Röper uses {Ax1,Ax2}, but Ax1' and Ax2' are obviously simpler than Ax1 and Ax2.

It is worth considering whether there is a durational parallel to Prior's idea of instant propositions. In fact, (E2) corresponds to this Priorian postulate within the third grade:

$$T(x,\sim p) \equiv \sim T(x,p)$$

and that (E3) corresponds to

$$T(x,p \wedge q) \equiv (T(x,p) \wedge T(x,q)).$$

For that reason this logic can be treated along the lines of Prior's third grade, now using 'duration propositions' instead of instant propositions. Moreover, this can in principle be done not only with a linear conception of time, but also for a logic of durations based on a branching time. On the other hand, it must be recalled that we are currently dealing only with a *non-homogeneous* durational logic.

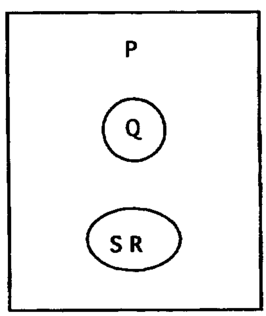
As we mentioned at the beginning of this chapter, a number of researchers have argued that representations of time based on intervals are more natural than instant-based systems. Even if the arguments for the conceptual priority of durations over mathematical instants seem convincing, it must also be admitted that the notion of durationless instants is very useful in many cases - especially for the description of change. For this reason, the idea of constructing instants from durations - as proposed by Walker and Hamblin - is of considerable interest. Such constructions may turn out to be very important, if they can be shown to give rise to a full instant-logic.

But it should also be mentioned that the project of durational logic, respectively interval semantics, has been attacked. There can be no doubt that for natural language some idea of durations is required - and hence, that conceptually one must (sometimes) speak of durations. But that certainly does not prove that the desired notions of duration could not be built within an instant-logic, a point which has been put forcefully and elegantly by P. Tichy [1985] as well as A. Galton [1990]. This issue cannot be expected to be settled soon, but regardless of the priority between these two conceptions, both approaches are useful for the general study of time.

3.6. GRAPHS FOR TIME AND MODALITY

As we have mentioned in part 2, C.S. Peirce established a calculatory technique of logical graphs. These so-called existential graphs have been studied carefully by computer scientists and others for some years. Since the beginning of the 1980's, John Sowa [1984] and others have tried to systematise a modern version of Peirce's existential graphs - and indeed, to implement it computationally. (A somewhat different but also highly interesting modernisation is Harmen van den Berg's 'Knowledge Graphs' [1993a].) It seems that the modern version of the Peircean graphs known as 'conceptual graphs' is useful within artificial intelligence in a broad sense - including interfaces to databases, deductive databases, and the like. In this chapter we shall study the Peircean ideas and some of the problems Peirce left open. We shall focus on the 'graphical' representation of tempo-modal problems.

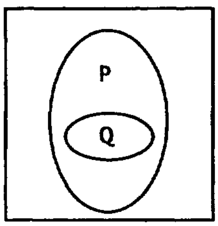
The graphs which C.S. Peirce introduced in his logic are divided into 3 classes: the Alpha, Beta, and Gamma graphs. In all of them the statements in question are written on the so-called 'sheet of assertion', SA. The most simple statement is the empty statement, which is supposed to hold according to the only axiom in the Peircean Alpha-system. Propositions on the SA may be enclosed using so-called 'cuts', which in fact correspond to negations (we also speak of 'negated contexts'). That is, the following combination of graphs means that *P* is the case, *Q* is not the case, and the conjunction of *S* and *R* is not the case.



Here the box represents the SA, whereas the curved closures symbolise the cuts, i.e. the negated contexts. In terms of the established formalism of propositional logic, the above graph is equivalent to

$$P \wedge \sim Q \wedge \sim(S \wedge R)$$

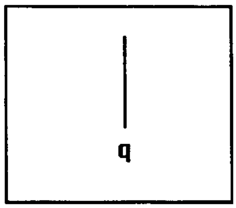
It is obvious that such conjunctional forms are rather easy to represent in Peircean graphs. Disjunctions and implications are, however, slightly more complicated. The implication $(P \supset Q)$ is represented by the following graph



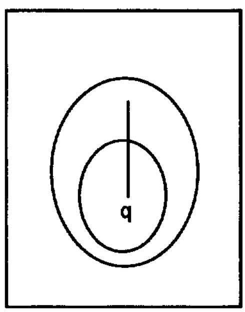
In standard formalism this graph can be expressed as $\sim(P \wedge \sim Q)$, which is exactly the definition of material implication in terms of negation and conjunction.

It is a remarkable theoretical result that Peirce's Alpha graphs correspond exactly to standard propositional calculus (cf. John Sowa [1992b]). His Beta graphs, in turn, correspond to first order predicate calculus with a 'non-empty' quantification theory (see below). With that restriction, this means that theorems which can be proved in first order logic can also be proved in terms of existential graphs. To prove a theorem corresponding to a certain graph one must transform the empty proposition on SA into the graph in question. A number of rules are available for this procedure, and they will be stated in the following. John Sowa [1992b] has argued that it is in many cases significantly easier to prove a theorem by using the graphs rather than the established logical procedures. He has substantiated this view by giving some rather convincing examples.

In the Beta graphs Peirce introduced a predicate calculus with a quantification theory formulated in terms of what he called 'lines of identity' (ligatures). These graphs are immediately designed for existential statements. The statement which is now normally formalised as *∃x:q(x)* is represented by the graph:



Universal statements have to be represented in a slightly more complicated way using two cuts (i.e. two negations) corresponding to the formula *~(∃x:~q(x))*:



Roberts [1973, p. 138] enumerates the rules for the Alpha and Beta graphs as follows:

R1. The rule of erasure. Any evenly enclosed graph and any evenly enclosed portion of a line of identity may be erased.
R2. The rule of insertion. Any graph may be scribed on any oddly enclosed area, and two lines of identity (or portions of lines) oddly enclosed on the same area may be joined.
R3. The rule of iteration. If a graph P occurs in the SA or in a nest of cuts, it may be scribed on any area not part of P, which is contained by the place of P. Consequently, (a) a

branch with a loose end may be added to any line of identity, provided that no crossing of cuts results from this addition; (b) any loose end of a ligature may be extended inwards through cuts; (c) any ligature thus extended may be joined to the corresponding ligature of an iterated instance of a graph; and (d) a cycle may be formed, by joining by inward extensions the two loose ends that are the innermost parts of a ligature.

R4. The rule of deiteration. Any graph whose occurrence could be the result of iteration may be erased. Consequently, (a) a branch with a loose end may be retracted into any line of identity, provided that no crossing of cuts occurs in the retraction; (b) any loose end of a ligature may be retracted outwards through cuts; and (c) any cyclical part of a ligature may be cut at its inmost part.

R5. The rule of the double cut. The double cut may be inserted around or removed (where it occurs) from any graph on any area. And these transformations will not be prevented by the presence of ligatures passing from outside the outer cut to inside the inner cut.

In addition to these rules there are two axioms: the empty graph (SA) and the unattached line of identity. From these two axioms it is possible to derive a number of theorems and rules using (R1-5). For instance, the graph corresponding to the following implication

$\forall x\colon q(x) \supset \exists x\colon q(x)$

turns out to be provable, as demonstrated by Roberts [1992]. The axiom of the unattached line of identity has a crucial rôle to play in this proof. This means that quantification cannot be empty in the logic of the Beta graphs (which is in fact also the case with Prior's quantification theory).

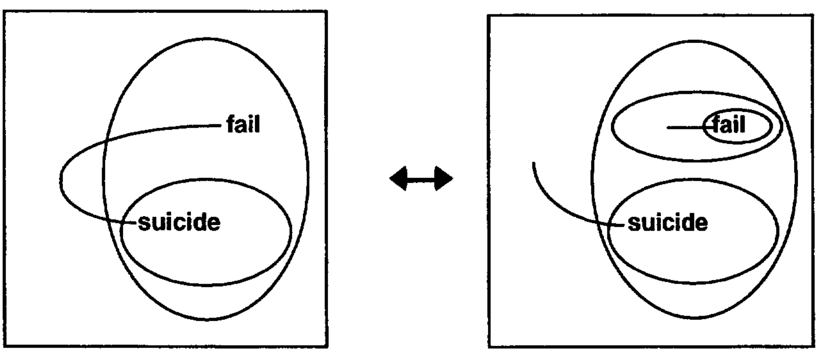
THE NEED FOR MORE THAN ALPHA AND BETA GRAPHS

The logic of Beta graphs is clearly useful in many cases. Peirce realised, however, that the Alpha and Beta graphs are not satisfactory in all cases. For instance, he considered the following two propositions (see [CP 4.546]):

(1) Some married woman will commit suicide, if her husband fails in business.
(2) Some married woman will commit suicide, if every married woman's husband fails in business.

Peirce argued that these two conditionals are equivalent if we analyse them in a merely classical and non-modal logic - i.e. in terms of Beta graphs within his own logical system. For the sake of simplicity we reformulate the problem using only predicates with one argument.

According to Peirce's rules for Beta graphs and their lines of identity, the graphs corresponding to (1) and (2) can be proved to be equivalent, i.e.



- where *fail(x)* means 'x is married to a businessman who fails in business', and *suicide(x)* means 'x commits suicide'. This equivalence can be established by the rules of transformation for Beta graphs. The two graphs respectively correspond to the following two expressions of standard predicate notation (where

quantification is understood to be over the set of women married to businessmen):

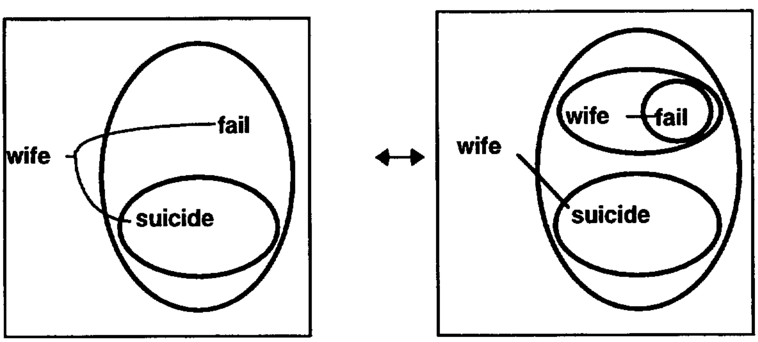
(1a) $(\exists x)(fail(x) \supset suicide(x))$
(2a) $(\exists x)((\forall y)fail(y) \supset suicide(x))$

Both of these expressions are equivalent with

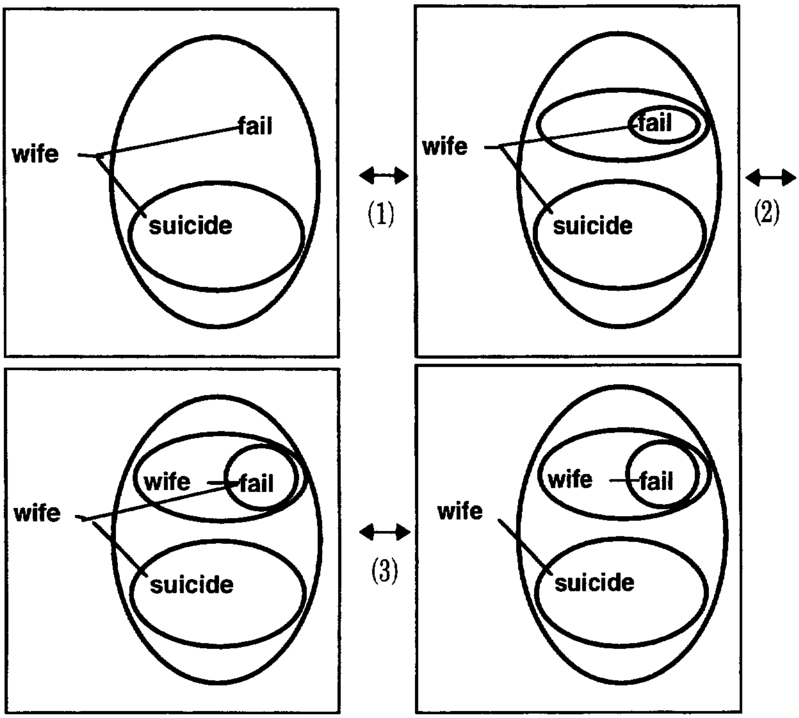
$(\exists\, x){\sim}fail(x) \vee (\exists x)\ suicide(x)$

The inference from (2a) to (1a) appears rather natural, whereas the opposite inference is clearly counterintuitive. Nevertheless, (1a) and (2a) turn out to be logically equivalent, as long as we are moving strictly within classical predicate logic, respectively the Beta graphs. Therefore, as long as we are trying to represent our case within those systems, we are obliged to accept the counterintuitive inference.

However, it may be more natural to formulate the problem in terms of three predicates, so let *wife(x)* stand for 'x is the wife of a businessman'. When the statements (1) and (2) are represented with three predicates, the graphs in question will be:



Again, these graphs can be shown to be equivalent. Essentially, their equivalence is due to the fact that the term *wife* is outside the scope of the negations. Therefore, the rules of iteration and deiteration for Beta graphs can be applied to the inner copies. The proof using Beta graphs could run as indicated below.



Step (1) in the above deduction is the introduction of a double cut in the first graph. In step (2) iteration is used, and in step (3) the rules of erasure and deiteration are used; a few other rules are also used, but the details are omitted here. - In every step the opposite operation is also allowed. The only counterintuitive step seems to be the implication from right to left in (3).

In terms of standard formalism (1) and (2) are represented by

(1b) $(\exists x)(wife(x) \wedge (fail(x) \supset suicide(x)))$
(2b) $(\exists x)(wife(x) \wedge ((\forall y)(wife(y) \supset (fail(y)) \supset suicide(x)))$

Using fundamental equivalences from first order logic, (1b) and (2b) can be written as disjunctions

(1b') $(\exists x)(wife(x) \wedge \sim fail(x)) \vee (\exists x)\ (wife(x) \wedge suicide(x)))$

(2b') $(\exists x)((wife(x) \wedge (\exists y)(wife(y) \wedge \sim fail(y)) \vee$
$(\exists x)((wife(x) \wedge suicide(x)))$

By the 'omission' of *wife(x)* in the first part of the disjunction in (2b'), it becomes evident that (1b') follows from (2b'). That is, they are both equivalent to

$(\exists x)(wife(x) \wedge \sim fail(x)) \vee (\exists x)(wife(x) \wedge suicide(x))$

Peirce stated that the equivalence of these two propositions is "the absurd result of admitting no reality but existence" [CP 4.546]. As Stephen Read [1992] has pointed out, Peirce's analysis is a strong argument against anybody inclined to assert that conditionals in natural language are always truth-functional. But the Peircean analysis is also an argument for the need of a new tempo-modal logic. Peirce formulated his own solution in the following way:

> If, however, we suppose that to say that a woman will suicide if her husband fails, means that every possible course of events would either be one in which the husband would not fail or one in which the wife will commit suicide, then, to make that false it will not be requisite for the husband actually to fail, but it will suffice that there are possible circumstances under which he would fail, while yet his wife would not commit suicide. [CP 4.546]

This means that we have to quantify over 'every possible course of events'. Prior's tense-logical notation systems provide the means for doing just that. The operator suited for the problem at hand is *G*, corresponding to 'it is always going to be the case that'. As we have seen Prior established a system designed to capture Peirce's ideas on temporal logic - appropriately called 'the Peircean solution' (see chapter 2.2 and 2.6). In the Peircean system, *G* means 'always going to be in

every course of events'. Using the operator in this way, we can express (1) and (2) as respectively

(1c) $(\exists x)G(husband\_fail(x) \supset suicide(x))$
(2c) $(\exists x)G((\forall y)husband\_fail(y) \supset suicide(x))$

(It should be mentioned that a linguistically more appropriate representation perhaps should take the form $N(p \supset Fq)$. However, (1c) and (2c) are sufficient for the conceptual considerations which are important here.)

(1c) clearly means that there is some married woman w for whom

(1d) $(\sim husband\_fail(w) \vee suicide(w))$

holds at any time in any possible future course of events. (2c) means that there is a married woman w for whom

(2d) $(\exists y) \sim husband\_fail(y) \vee suicide(w)$

holds at any time in any possible future course of events.

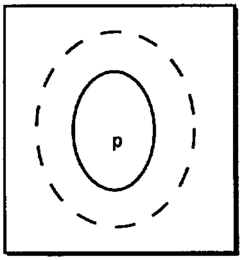
For this reason it is formally clear that (1c) entails (2c), but not conversely. And this corresponds exactly to intuition with respect to the two statements (1) and (2): the inference from (1) to (2) is valid, but Peirce was justified in maintaining that the inference from (2) to (1) must be rejected.

Generally speaking, some kind of tempo-modal logic is required for describing conditionals in natural language reasoning in a satisfactory way - a fact which has quite recently been more systematically expounded [Crouch 1993]. Peirce's considerations on the example discussed in this section clearly demonstrate that he realised this.

THE GAMMA GRAPHS

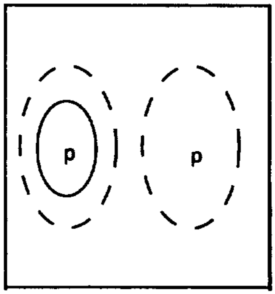
Peirce himself made some attempts at solving the problems of modality by introducing a new kind of graphs. In what he called 'The Gamma Part of Existential Graphs' [CP 4.510 ff.], he put forth some interesting suggestions regarding modal logic. Some of his considerations on this topic were linked to what is now called epistemic logic, i.e. the logic of knowledge. In the following we shall describe his ideas.

In epistemic logic, the idea is that relative to a given state of information a number of propositions are known to be true. In Peirce's graph theory, propositions describing the information in question should be written on the 'sheet of assertion' SA, using just Alpha and Beta graphs. Other propositions, however, are to be regarded as merely possible in the present state of information. Peirce represented such propositions using 'broken cuts', combined with the 'unbroken cuts' which we already know from the Alpha and Beta graphs. A broken cut should be interpreted as corresponding to 'it is possible that not ...'. This means that 'it is possible that ...' must be represented as a combination of a broken and an unbroken cut:



Now consider a contingent proposition, i.e. a proposition, which is possible, but not necessary according to the present state of information.

In this state the sheet will include at least two propositions:



Now suppose that p is contingent relative to some state of information, and that we then learn that p is in fact true. This would mean that the SA should be changed according to the new state of information. The graph corresponding to Mp ('possibly p') should be changed into the graph for 'it is known with certainty that p', i.e. $\sim M \sim p$. Obviously, this means that the graph for $M \sim p$ should be dropped, which results in a new (and simpler) graph representing the updated state of information. In this way Peirce in effect pointed out that the passage of time does not only lead to new knowledge, but also to a loss of possibility. With respect to this epistemic logic Don D. Roberts [1973, p. 85] has observed that the notions of necessity and possibility both may seem to collapse into the notion of truth. Roberts himself gave an important part of the answer to this worry by emphasising how "possibility and necessity are relative to the state of information" [CP 4.517], and that there will only be a total collapse in case of omniscience. In the context of existential graphs Peirce in fact established an equivalence between 'p is known to be true' and 'p is necessary'. In consequence, 'p is not known to be false' and 'p is possible' should also be equivalent in a Peircean logic. Therefore, the kind of modal logic which Peirce was aiming at was in fact an epistemic logic, which should be sensitive to the impact of time.

He furthermore realised that it would be useful not only to have a logic for knowledge in a strong sense, but also a logic for confidence:

> The idea of time really is involved in the very idea of an argument. But the gravest complications of logic would be involved, if we took account of time so as to distinguish between what one knows and what one has sufficient reason to be entirely confident of. [CP 4.523]

TEMPO-MODAL PREDICATE LOGIC AND EXISTENTIAL GRAPHS

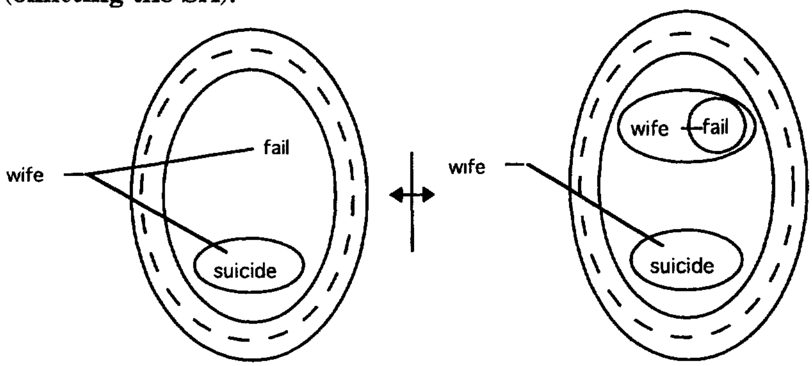
Peirce was concerned with the epistemic aspect of modality, but he also wanted to apply his logical graphs to modality in general - that is, to use them for representing any kind of modality. However, he was aware of the great complexity in which a full-fledged logic involving temporal modifications would result. This is probably the reason why Peirce's presentations of the Gamma graphs remained tentative and unfinished. In the following we intend to explain some of the problems he was facing, and suggest some ideas regarding the possible continuation of his project.

Our analysis of the problem from [CP 4.546] suggests that the two statements should in fact be understood as follows:

> (1') Some married woman will (in every possible future) commit suicide if her husband fails in business.
> (2') Some married woman will (in every possible future) commit suicide if every married woman's husband fails in business.

We intend to formulate a graph-theoretical version of the tense-logical solution. So, we have to make sure that there are proper graphical representations of (1') and (2') such that the graphs are non-equivalent. In fact, it is not difficult to create graphs corresponding to the modal expressions in (1') and (2').

Obviously, a graph with a broken cut inside an unbroken cut with q would clearly correspond to the statement 'in every possible future q'. A representation of (1') and (2') could then be (omitting the SA):



In order to treat problems like the one we have been discussing we must be able to handle graphs involving the two kinds of cuts (broken and unbroken) as well as lines of identity. In consequence, we have to establish rules for modal conceptual graphs, specifically such that (1') and (2') would be non-equivalent.

Harmen van den Berg [1993b] has shown how it is possible to formulate propositional modal rules of inference corresponding to conceptual graphs with broken as well as unbroken cuts. However, the rule of iteration has to be slightly changed, and a few extra rules have to be added in order to obtain the propositional modal logic T (sometimes called M), for which the so-called rule of necessitation and the following axioms hold:

(Nec) If p is provable, then Np is provable

$N(p \supset q) \supset (Np \supset Nq)$
$Np \supset p$

The question of a similar account for modal predicate logic is left open by Harmen van den Berg. In order to obtain a predicate modal logic we have to accept (R1-R5) for graphs only in-

volving broken cuts in subgraphs which are untouched by the operations of the rules. Since classical quantification rules should be valid in general, it follows that any loose end of a ligature may be extended inwards through broken as well as unbroken cuts (cf. R3). The resulting minimal change of the R-rules and van den Berg's rules then leads to the following rules:

R1'. The rule of erasure. Any evenly enclosed graph and any portion of a line of identity evenly enclosed by unbroken cuts may be erased.
R2'. The rule of insertion. Any graph may be scribed on any oddly enclosed area, and two lines of identity (or portions of lines) oddly enclosed by unbroken cuts on the same area may be joined.
R3'. The rule of iteration. If a graph P occurs in the SA or in a nest of unbroken cuts, it may be scribed on any area not part of P, which is contained by the place of P. Consequently, (a) a branch with a loose end may be added to any line of identity, provided that no crossing of cuts results from this addition; (b) any loose end of a ligature may be extended inwards through cuts; (c) any ligature thus extended may be joined to the corresponding ligature of an iterated instance of a graph; and (d) a cycle may be formed, by joining by inward extensions through unbroken cuts the two loose ends that are the innermost parts of a ligature.
R4'. The rule of deiteration. Any graph whose occurrence could be the result of iteration may be erased. Consequently, (a) a branch with a loose end may be retracted into any line of identity, provided that no crossing of cuts occurs in the retraction; (b) any loose end of a ligature may be retracted outwards through unbroken cuts; and (c) any cyclical part of a ligature crossing no modal cut may be cut at its inmost part.
R5'. The rule of the double cut. The double cut may be inserted around or removed from any graph on any area. And these transformations will not be prevented by the presence of ligatures passing from outside the outer cut to inside the inner cut through unbroken cuts.

R6'. The rule of modal conversion. An evenly enclosed unbroken cut may be replaced by a broken cut. An oddly enclosed broken cut may be replaced by an unbroken cut.
R7'. The necessitation rule. If the graph corresponding to p is provable, then the graph corresponding to Np is also provable.
R8'. The distribution rule. The graph corresponding to $N(p \wedge q)$ is provable if and only if the graph corresponding to $(Np \wedge Nq)$ is provable.

The rule of necessitation implies the following rule:

(Nec N): If the implication $(p \supset q)$ is provable, then the implication $(Np \supset Nq)$ is also provable.

This derived rule could also be stated directly in terms of existential graphs. If we want the modal logic S4, we have to add this rule:

The duplication rule. A combination of two cuts around a graph (broken or unbroken cuts) may be duplicated. The inverse operation is also allowed.

Using this rule we can prove the graph corresponding to the axiom:

$$Np \supset NNp$$

If we want the modal logic S5, we have to add the rule:

The generalised duplication rule. A combination of two cuts around a graph (broken or unbroken cuts) may be duplicated in random order. The inverse operation is also allowed.

With the generalised duplication rule, we can prove the graph corresponding to the axiom:

$Mp \supset NMp$

The change from (R1-5) to (R1'-R8') seems to be minimal if the system should include T. For that reason one may suspect that the system with the two Beta axioms together with (R1'-R8') is something rather close to the modal predicate logic corresponding to T. But we shall suggest that something more seems to be needed. As we shall see this idea can in fact be traced back to the Peircean analysis.

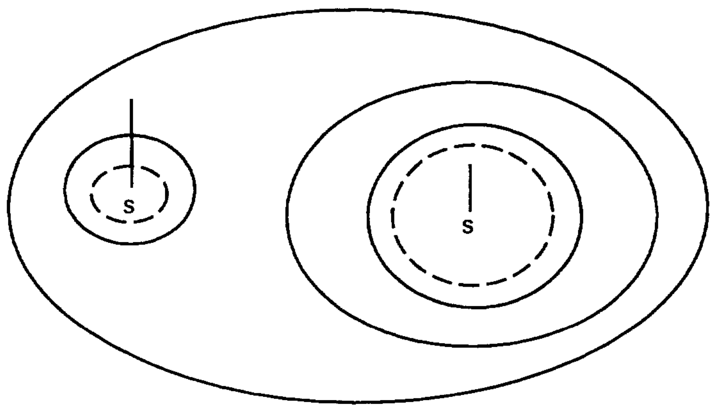
The question regarding the relation between modal operators and quantifiers is crucial for any modal predicate logic. Peirce was aware of this problem. He stated:

> Now, you will observe that there is a great difference between the two following propositions:
> First, There is some *one* married woman who under all possible conditions would commit suicide or else her husband would not have failed.
> Second, Under all possible circumstances there is some married woman or *other* who would commit suicide or else her husband would not have failed. [CP 4.546]

It is very likely that what Peirce had in mind was the insight that we cannot with complete generality derive $\exists x{:}\ Ns(x)$ from $N(\exists x{:}\ s(x))$. - that is, not without making some restrictions. This is in fact a very old wisdom which was also known to the medieval logicians. One cannot deduce 'there is a man who will live forever' from 'it will forever be true that there is a man'. However, the opposite deduction is clearly reasonable. In fact the implication

(NEX) $\exists x{:}\ Ns(x) \supset N(\exists x{:}\ s(x))$
or equivalently:
(NEX') $M(\forall x{:}\ s(x)) \supset \forall x{:}\ Ms(x)$

turns out to be provable in predicate modal logic corresponding to T. This theorem can be represented by the following graph:



Let an *N-double cut* be a broken cut inside an unbroken cut, with nothing enclosed in the outer area except for portions of lines of identity which pass from inside the inner area to outside the outer area. We can then formulate a rule for modal graphs, from which the graph version of (NEX) can be proved:

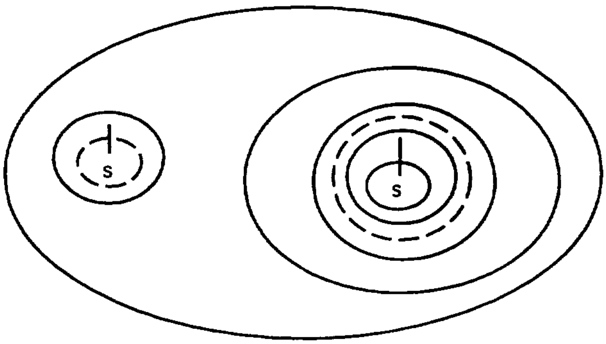
> (R-NEX) Any oddly enclosed loose end of a ligature may be extended outwards through an N-double cut.
> - Any evenly enclosed loose end of a ligature may be retracted inwards through an N-double cut.

It appears that we need a rule like (R-NEX) in order to establish the graph-theoretical equivalent to a modal predicate logic including T. We suspect that this rule can be made nicer and more general. (How that should be done in detail will depend on further logical investigations.) As far as we know, Peirce did not investigate the corresponding logical relation between the necessity operator and the universal quantifier. In chapters 2.9 and 2.10 we discussed a certain relation between

quantifiers and modal operators, namely Barcan's formula [Prior 1967, p. 137 ff.]:

> $\forall x$: $Ns(x) \supset N(\forall x$: $s(x))$
> or equivalently
> $M(\exists x$: $s(x)) \supset \exists x$: $Ms(x)$

This formula corresponds to the following graph:



The validity of Barcan's formula within a modal system would mean that for instance the following implication holds:

> if 'at some future time there will be a suiciding wife',
> then 'there is some wife who at some future time will commit suicide'.

Such an implication seems not to be acceptable in the context of Peirce's example, since it excludes the possibility that the suiciding wife could come into being at some future time. As we have seen, Barcan's formula is provable in S5, but not so in S4. Therefore S4 could be a reasonable candidate for a modal logic capable of 'solving' the Peircean example. But we have to be cautious in trying to 'embed' S4 within the Gamma graphs.

The rule of duplication for predicate modal logic must be adapted very carefully such that Barcan's formula can *not* be derived:

> R9'. The duplication rule. A combination of two cuts around a graph (broken or unbroken cuts) may be duplicated.
> These transformations will not be prevented by the presence of ligatures passing from outside the outer cut to inside the inner cut through unbroken cuts.

Thus revised, the rule of duplication will obviously not validate the Barcan formula. A generalised duplication rule, which can yield a modal graph logic as strong as S5, should be formulated in an analogous manner:

> R10'. The generalised duplication rule. A combination of two cuts around a graph (broken or unbroken cuts) may be duplicated in random order. These transformations will not be prevented by the presence of ligatures passing from outside the outer cut to inside the inner cut through unbroken cuts.

With this rule we can in fact prove the theorems of S5 including Barcan's formula. However, as we have argued the logic Peirce needed to solve his problem should not allow for the validity of Barcan's formula, since that would make it difficult to give an account of the logical aspect of 'coming into being'. In consequence it seems that the logic Peirce was looking for should be weaker than S5. We shall suggest the modal predicate logic corresponding to the system with the Beta axioms and the rules (R1'- R9' + R-NEX).

CONCEPTUAL GRAPHS AND TEMPO-MODAL LOGIC

Peirce clearly realised the need for more modal operators than just one. Peirce's central ideas were in fact amenable to systematisation in the form of tempo-modal calculi, as indeed Prior showed, but these logics have so far not been formulated in terms of Peirce's existential graphs. However, during the last decade the study of so-called conceptual graphs has been developed as a field within artificial intelligence and logic. Conceptual graphs constitute a formal logical system based on the ideas laid out in Peirce's existential graphs. The field was first established by John Sowa [1984]. There are a few differences between conceptual graphs and existential graphs, first and foremost that

(i) in conceptual graphs 'contexts' are not in general negated as in existential graphs, and
(ii) it is possible to treat variables, individuals and types in a more direct and elegant manner in conceptual graphs.

For instance, Sowa [1992a, p. 22] represents the statement 'some dog does not eat meat' in the following way:

[DOG: *x]
$\neg$[[*x] $\leftarrow$ (AGENT) $\leftarrow$ [EAT] $\rightarrow$ (PATIENT) $\rightarrow$ [MEAT]]

In the following, we shall use Sowa's notation for conceptual graphs for our discussion.

During the last years some researchers have tried to formulate various systems of temporal logic in terms of conceptual graphs [Esch and Nagle 1992], [Moulin and Coté 1992]. There is, however, still a lot to be done in this area.

In order to formulate a graph-theoretical equivalent of Prior's 'Peircean system' (and other relevant tempo-modal systems, for

that matter) we shall need at least two tense operators corresponding to past and future:

(P) -> [SITUATION: GRAPH]
(F) -> [SITUATION: GRAPH]

meaning respectively 'the situation GRAPH has been' and 'the situation GRAPH will be'. We can define graphs corresponding to Prior's H ('has always been') and G ('will always be'), i.e.

(H) -> [SITUATION: GRAPH]
(G) -> [SITUATION: GRAPH]

as

¬[(P) -> [SITUATION: ¬[GRAPH]]]
¬[(F) -> [SITUATION: ¬[GRAPH]]].

With such graphs we can also formulate fundamental tense-logical theorems like, say $HFq \supset q$, in terms of conceptual graphs.

We may need metric tense operators, which can be represented graphically in the following way:

[TIME: t] -> (P') -> [SITUATION: GRAPH]
[TIME: t] -> (F') -> [SITUATION: GRAPH]

where t is any number (or integer if time is supposed to be discrete). If (P') and (F') are taken to be primitive, we may define (P) and (F) as

[TIME: *] -> (P') -> [SITUATION: GRAPH]
[TIME: *] -> (F') -> [SITUATION: GRAPH]

It is an open problem how a full tense logic should be incorporated into the theory of conceptual graphs - in other words, how the Gamma rules corresponding to Prior's tempo-modal systems should be formulated. This problem seems to be a rather complicated one.

The analysis of the Peirce problem can also be formulated in terms of conceptual graphs. Within this theory, the crucial statements could be expressed in the following way:

(1) Some married woman will commit suicide if her husband fails in business

```
[WOMAN: *x]
[
[WOMAN: *x] -> (MARRIED) -> [MAN] <-(FAILING)
]
=>
 [
[WOMAN: *x] <- (AGNT) <- [SUICIDE]
]
```

(2) Some married woman will commit suicide if every married woman's husband fails in business

```
[WOMAN: *x]
      [
         [WOMAN: @every=*y]
         [WOMAN: *y] ->
                              (MARRIED) -> [MAN] <-
(FAILING) ]
      =>
      [
      [WOMAN: *x] <- (AGNT) <- [SUICIDE]
      ]
```

where we define the graph [WOMAN: @every=*y] as an abbreviation of ^[[WOMAN: *y] ^[[*y]]].

The graphs for (1) and (2) turn out to be logically equivalent.

This is, however, not the case for the corresponding modal statement:

(1') Some married woman will commit suicide if in every possible future her husband fails in business

```
    [WOMAN: *x]
    (N)->
    [
          [
             [WOMAN: *x] ->
                                            (MARRIED) -> [MAN] <-
(FAILING)                                   ]
          =>
          [
             [WOMAN: *x] <- (AGNT) <- [SUICIDE]
          ]
    ]
```

(2') Some married woman will commit suicide if in every possible future every married woman's husband fails in business

```
    [WOMAN: *x]
          (N)->
             [
                                            [
                                            [WOMAN: @every=*y]
                                            [WOMAN: *y] ->
                                                        (MARRIED) ->
    [MAN] <- (FAILING)
                                            ]
             ]
             =>
             [
             [WOMAN: *x] <- (AGNT) <- [SUICIDE]
             ]
          ].
```

In this case there is only one modal operator and consequently only two kinds of cut, and the rules (R1'-R9') and (R-NEX) may still give us a very convincing logic. However, if we introduce a separate future operator *F* along with the modal operator *N*, things become more complicated. And of course, if we wish to embed a tense logic within conceptual graphs, we must include not only a future-operator, but also some past-operator *P*. However, there is yet no graph-theoretical equivalent to Prior's tense logics. It is a question for further investigation how this should be done.

John Sowa [1992a, p. 26] has himself defined the tense operators in terms of PTIM (point in time). For instance,

```
relation PAST(x) is
[SITUATION: *x] →
        (PTIM) → [TIME] → (SUCC) → [TIME: #now]
```

That would allow us to refer to situational contexts. The relation between this approach and the operator approach is rather unexplored. Nevertheless, the way in which we dealt with the Gamma graphs should provide useful clues as to how we may proceed with the task of constructing tense-logical conceptual graph theory.

3.7. Temporal Logic and Computer Science

The usefulness of systems of this sort [on discrete time] does not depend on any serious metaphysical assumption that time is discrete; they are applicable in limited fields of discourse in which we are concerned only with what happens next in a sequence of discrete states, e.g. in the working of a digital computer. A. N. Prior [1967, p. 67]

The relevance of temporal logic within computer science was realised in the course of the 1970s. Temporal logic has by now become an established discipline within this science, but the first researchers to take up the connection were not acquainted with Prior's tense logic. The initial studies in the field were based on *Temporal Logic* by N. Rescher and A. Urquhart [1971]. This book was in fact dedicated to the memory of Arthur Prior, and at any rate computer scientists would in due course also begin to study Prior's own works on tense logic. The above quotation makes it clear that one decade earlier it had occurred to Prior himself that his tense logic might be useful in computer science.

One of the first computer scientists to realise the relevance of temporal logic for the purposes of computer science was Amir Pnueli. Pnueli has himself described [1994, personal communication] how he was working on problems pertinent to the logic of time, when in late 1975 or early 1976 Saul Gorn of the University of Pennsylvania made him aware of Rescher's book *Logics of Commands*. This book, however, turned out to be of little relevance for Pnueli's purposes. But on the back of the dust cover there was a reference to another book by the same author, namely Rescher's and Urquhart's *Temporal Logic*. Pnueli went on to study this book, where a firm basis for dealing with temporal logic could be found.

Pnueli's pioneering work on temporal logic within computer science, as well as subsequent work in this area, has been

concerned not only with the application of temporal logic to specific problems, but also with general theory development. (This fact is highly evident in the contributions to the first international conference on temporal logic in Bonn, 1994 [Gabbay and Ohlbach 1994]). Together with formal linguistics, computer science is today a chief contributor to the continued development in this field.

Temporal logic has become an important formalism for various purposes within computer science, ranging from fundamental theoretical issues to special types of application software. One significant example of the latter type is natural language understanding. In the context of this book we have for obvious reasons been paying special attention, directly and indirectly, to this problem domain. In natural language understanding we have to deal with the problem of giving a semantical representation of time, as it is manifest in linguistic expressions (tenses, aspect, temporal connectives and adverbs) [Hasle 1991]. An important goal of this kind of work is, of course, to give a formal account of intuitively valid inferences. Broadly speaking, this is tantamount to formalising rational common sense reasoning involving time. Moreover, Martha S. Palmer et al. [1993] have argued that the specific (i.e. context-dependent) interpretation of tense and other temporal expressions in natural language often requires common-sense reasoning. Thus, studies in natural language understanding almost irresistibly call for techniques from temporal logic, which is in fact itself a theoretical field based on a study of valid common-sense reasoning. Time and again, we have exemplified this by scrutinising such deliberations, from Antiquity to the present day. From this kind of investigation into natural language understanding we are led to more general studies regarding the representation (and manipulation) of temporal knowledge. Such studies are immediately relevant for other issues within artificial intelligence, where temporal logic is important, for example when prediction, explanation, planning or similar purposes are involved. In many expert systems, e.g. medical diagnosis systems, the representation of time is essential.

Likewise, robotical systems as a rule require that some understanding of time must be simulated.

A most interesting application of ideas from temporal reasoning can be found in planning systems. For the construction of linear as well as non-linear 'planners', the representation of time is fundamental [Shoham 1994, p. 199 ff.]. In our opinion, all such applications of temporal logic should be viewed as ultimately inspired from the study of valid common-sense reasoning as embodied in elements of natural language. This view can be supported by the careful work of R. S. Crouch and S. Pulman [1993], who have specifically demonstrated how a natural language interface to a planning system can be constructed. In this connection they argue that the task of building a natural language interface to an information system is one of *modelling the domain in question as a reasoning system*. Indeed, this way of relating natural language to reasoning and logic may be seen as a modern version of the medieval conception of logic discussed in part 1.

Time has shown, however, that the relevance of temporal logic within computer science extends far beyond natural language understanding and artificial intelligence. It can also be applied in various phases of the system development process. Usually, this process is divided into four consecutive phases: analysis of the problem at hand, design, implementation, and validation. (Sometimes validation is not counted as a separate phase; moreover, it is generally recognised that the phases are not clearly distinct. Rather, they overlap and are usually reiterated). Temporal logic has proved its worth within each of these phases. In design and implementation, it is used for specifying properties that the system in question should possess. In analysis, it plays the same rôle as in common-sense reasoning. In validation, its use is normally restricted to verification, i.e. the task of proving that the program does have the required properties (the other part of validation being various ways of empirically testing the program). Obviously, temporal logic can thus play a very general rôle in system development; it even appears that it may be a natural candidate

for integrating the various phases, or at least relating them to each other.

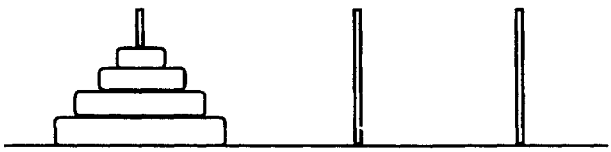
The fact that temporal logic can be used for specifying computer programs, and for reasoning about them, has been known for some years [Burstall 1974, Pnueli 1977, Manna & Wolper 1981]. In particular, temporal logic has become an important tool for the analysis of concurrrent (parallelistic) programs. The main idea is that the execution history can be described in terms of temporal logic, without necessarily referring to specific program states or times. This means that general properties of programs such as freedom of deadlock, mutual exclusion etc. can be expressed in a very nice way in terms of formulae of temporal logic.

Anyone who wants to use temporal logic has to choose between its two major paradigms, namely A- and B-logic. Of course, that choice may be seen as a choice between a syntax of tenses (operator approach) and a syntax involving reification of time (quantifier approach). With a B-logic it seems that one has to formulate a theory of time as a structure. With an A-logic, on the other hand, it is not required that any such structure be specified - not immediately, at least. On the other hand, Erik Sandewall [1992, pp. 609-610] has argued that the reified approach should be preferred for reasons of notational convenience. A similar answer has been presented by Yoav Shoham [1994, p. 234 ff.], who has suggested a so-called 'time map management' in which temporal reasoning is based on a B-logic with 'points in time'. We agree that in many cases one would like to refer to specific moments of time. However, as demonstrated in previous chapters such references can also be obtained in an A-logic, to which ideas from Prior's third grade are added.

Roger Hale [1987] has used a well known programming example known as 'The Towers of Hanoi' to illustrate some of the ideas in so-called temporal logic programming. In the following we shall make use of the same example in order to clarify how temporal logic may be applied for specification purposes.

THE TOWERS OF HANOI

Let us assume that N different rings are given. The sizes of the rings are 1, 2, ..., N. We shall use the size of each ring as its name, i.e. the smallest ring is called '1', and so forth. The rings can be placed on three pegs. At the beginning all the rings are placed on the first peg in an ordered way as indicated on the figure:



If N=4 as in the above figure, this state can be represented by the following kind of proposition:

state([1,2,3,4],[],[])

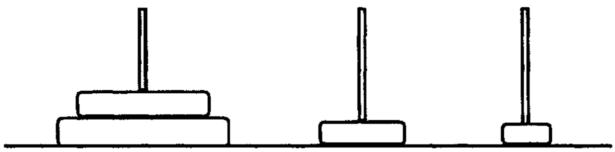
In any state the rings on each peg form a 'tower'. In general, a state can be represented by a proposition of the form state(A,B,C), which should be read as 'in the present state the first tower corresponds to the list A, the second tower corresponds to the list B, and the third tower corresponds to the list C'. As can be seen we use PROLOG-like lists to describe the states.

With this setting a game can be introduced. There is only one rule in the game, called MOVE:

MOVE:
the player is allowed to pick any ring at the top of one of the three pegs, and move it to any other peg, provided that this move does not place the ring on top of some other ring which is smaller than the one being moved.

Strictly speaking, this is hardly yet a game, since no purpose or goal has been stated; but for now, we wish to discuss some properties of the game at this general level.

From the state at the beginning we can in two steps reach for instance this state *state([3,4],[2],[1])*, graphically:



There are N^3 possible states, which may be numbered as s_1, s_2, ..., s_M, where $M=N^3$. To simplify matters, we shall currently assume that the 'first' peg (A) is the one where the rings are initially placed:

$$s_1 = state([1,\ldots,N],[],[])$$

Using these atomic state propositions as primitives, we can form a logical language in the usual way. We may also form instant propositions as maximal consistent sets. Given the underlying 'model', it is sound to assume that the states are mutually exclusive:

$$\vdash s_i \supset \sim s_j \text{, where } i \neq j$$

This means that each maximal consistent set contains exactly one 'atomic' (specifically, unnegated) state proposition. Thus, given an instant proposition a_i there is a unique state proposition s_i such that

$$\vdash a_i \supset s_i$$

We want to know whether there is a series of possible moves which can bring us from the initial state described by s_1 to some other state s_i. In order to discuss this problem, we need a description of the possible moves in any conceivable state. Such a description can be obtained in terms of metric tense logic. The

permissible moves can be specified by the following implications, which in fact form a complete inductive definition of the possible future states, 'PFS':

PFS:
$\vdash$ *(state([X|A],B,C)* $\wedge$ *legal(X,B))* $\supset$ *MF(1)state(A,[X|B],C)*
$\vdash$ *(state([X|A],B,C)* $\wedge$ *legal(X,C))* $\supset$ *MF(1)state(A,B,[X|C])*
$\vdash$ *(state(A,[X|B],C)* $\wedge$ *legal(X,C))* $\supset$ *MF(1)state(A,B,[X|C])*
$\vdash$ *(state(A,[X|B],C)* $\wedge$ *legal(X,A))* $\supset$ *MF(1)state([X|A],B,C)*
$\vdash$ *(state(A,B,[X|C])* $\wedge$ *legal(X,A))* $\supset$ *MF(1)state([X|A],B,C))*
$\vdash$ *(state(A,B,[X|C])* $\wedge$ *legal(X,B))* $\supset$ *MF(1)state(A,[X|B],C)*

where *legal* is defined by

legal(X,[]) for any *X*, and
legal(X,[Y|L]) iff *X<Y*.

The above implications in effect define an infinite number of possible developments of the game, all starting from some given $state s_1$. These developments can be described by instant propositions. For instance

$$T(a_i,MF(1)p) \equiv (\exists a_j\colon T(a_i,MF(1)a_j) \wedge T(a_j,p))$$

Intuitively, it is clear that the rings are 'ordered' in the initial state of the game, and that the one rule of the game in all cases preserves this order. When this condition is compared with the rules 'defining' *MF(1)*, it should be intuitively clear that at any instant there is a possible next state. This means that there are no deadlocks, that is, no states from which we cannot reach another state following the rule. We may in fact state this property as the axiom:

$$\vdash p \supset MF(1)P(1)p$$

Let us now discuss the notion of 'order' more formally. In every state each tower must be ordered such that the smallest ring is at the top of the tower - in general, the sequence of rings

as stated in its list-form *[X,Y,...,Z]* must be increasing. This ordering can be defined recursively:

ordered([])
ordered([A])
$ordered([A,B \mid L]) \equiv_{def} (A<B \wedge ordered([B \mid L]))$

The statement that the rings are ordered on all pegs can be defined in the following way:

$order \equiv_{def}$
$(state(A,B,C) \supset (ordered(A) \wedge ordered(B) \wedge ordered(C))$

By inspection of the implications defining MF(1), it is easily shown that

$\vdash order \supset NF(1)order$ (*where* $NF(1)p \equiv_{def} {\sim}M{\sim}F(1)p$)

Since we have the following theorem for metric tense logic

$\vdash NF(1)(p \supset NF(1)q) \supset (NF(1)p \supset NF(1)NF(1)q)$

we can in fact prove that once the order has been established, it must be preserved forever, i.e. the theorem

ORDER $\quad \vdash order \supset NG(order)$
(where $Gp \equiv_{def} {\sim}F(X){\sim}p$, for any natural number X)

Since the implications on which this result is based (PFS) formalise the one rule of the game, we have also shown that this rule is order-preserving.

We now wish to add a 'purpose' or goal to the game, namely that the game has been successfully finished when the following condition obtains:

FINISH:
The tower of Hanoi which was originally on the first peg, corresponding to the proposition s_1, has been moved to one of the other pegs. (That is, all the rings are now placed in order on one of the other pegs, and this has been achieved by following the MOVE-rule only.)

This final state of success corresponds to the proposition

$$finish \equiv_{def} (state([],[],[1,\ldots,N]) \vee state([],[1,\ldots,N],[]))$$

In a computer science terminology, we say that the *problem* has been *solved* if the game has been successfully finished; in general, our program or plan is said to solve the problem if and only if it will always lead to the game being successfully finished. Given our definitions so far, the statement that the tower problem *can* be solved is equivalent to asserting the provability of the statement

P-SOLVE $\vdash state([1,\ldots,N],[],[]) \supset MF\, finish$
(where $Fp \equiv_{def} F(X)p$, for some natural number X)

We have already in PFS specified $MF(1)p$ in terms of all possible moves in any type of situation. We now proceed to establish a definition of $F(1)$ corresponding to a plan for future actions, that is, such that $F(1)$ in effect specifies which move to make in order to 'approach' a solution. This definition of $F(1)$ should be made such that

SOLVE $\vdash state([1,\ldots,N],[],[]) \supset F\, finish$

is provable. Obviously, any demonstration of SOLVE will also be a demonstration of P-SOLVE.

PFS is, as pointed out before, in effect a specification of the future operator. However, the specification of F corresponding to the plan must exclude loops. We therefore propose a rather different definition of the future operator (in effect, a 'select'-operator), availing ourselves of two derived concepts:

- *smallest* $\equiv_{def} (s_1 \vee P(n)s_1)$, where n is even, and
- *another* $\equiv_{def} P(1)smallest$.

With a view to practical programming it should be noted that the history of moves, i.e. the sequence of states during computation, must be recorded, when these concepts are to be used in connection with a given program.

The specification now runs like this:

$$(smallest \wedge state([1\,|A],B,C)) \supset F(1)state(A,[1\,|B],C)$$
$$(smallest \wedge state(A,[1\,|B],C)) \supset F(1)state(A,B,[1\,|C]))$$
$$(smallest \wedge state(A,B,[1\,|C])) \supset F(1)state([1\,|A],B,C)$$

$$(another \wedge state([1\,|A],[X|B],C) \wedge legal(X,C)) \supset F(1)state([1\,|A],B,[X|C])$$
$$(another \wedge state([1\,|A],B,[X|C]) \wedge legal(X,B)) \supset F(1)state([1\,|A],[X|B],C)$$
$$(another \wedge state([X|A],B,[1\,|C]) \wedge legal(X,B)) \supset F(1)state(A,[X|B],[1\,|C])$$
$$(another \wedge state(A,[X|B],[1\,|C]) \wedge legal(X,A)) \supset F(1)state([X|A],B,[1\,|C])$$
$$(another \wedge state([X|A],[1\,|B],C) \wedge legal(X,C)) \supset F(1)state(A,[1\,|B],[X|C])$$
$$(another \wedge state(A,[1\,|B],[X|C]) \wedge legal(X,A)) \supset F(1)state([X|A],[1\,|B],C)$$

Intuitively, the proposition 'smallest' means that the ring labelled '1' is the next to be moved, whereas the proposition 'another' implies that a ring different from the one labelled '1' is the next to be moved. The plan inherent in this F in effect selects a specific 'goal peg' among the two empty pegs (this is directly reflected in the theorem below). There is really nothing surprising about that, but of course any of the two empty pegs will do for a 'goal peg'. The only important point is that one such must be selected from the beginning.

In order to demonstrate that SOLVE holds with the above definition of F we prove the following:

Theorem:

(a) $(smallest \wedge state([1...k],[],[]) \supset F(2^k\text{-}1, state([],[1...k],[]) \wedge another)$, where k is an odd integer

(b) $(smallest \wedge state([],[1...k],[]) \supset F(2^k\text{-}1, state([],[],[1...k]) \wedge another)$, where k is an odd integer

(c) $(smallest \wedge state([1...k],[],[]) \supset F(2^k\text{-}1, state([],[],[1...k]) \wedge another)$, where k is an even integer

(d) $(smallest \wedge state([],[],[1...k]) \supset F(2^k\text{-}1, state([],[1...k],[]) \wedge another)$, where k is an even integer

Proof:
The theorem is proved by mathematical induction. The proof is immediate for $k=1$ and $k=2$. Let us first assume that n is even, and let the inductive hypothesis be the assumption that the theorem has been proved for $k=1...n$. We must now show that the theorem also holds for the $n+1$ case. Since n is even we have

$$(smallest \wedge state([1...n,n+1],[],[]) \supset \\ F(2^n\text{-}1, state([n+1],[],[1...n]) \wedge another)$$

since the ring $n+1$ does not influence the first 2^n-1 steps. In the next step, however, this ring is moved. In consequence, we have

$$(smallest \wedge state([1...n,n+1],[],[]) \supset \\ F(2^n)(state([],[n+1],[1...n]) \wedge another)$$

By inspection into the rules for the F-plan one can assure oneself of the fact that this ring $n+1$ will not be moved anymore, and we do not have to pay further attention to it in the following steps. For this reason it follows that

$$(smallest \wedge state([1\ldots n,n+1],[],[]) \supset$$
$$F(2^n\text{-}1+2^n)(state([],[],[1\ldots n,n+1]) \wedge another)$$

i.e.

$$(smallest \wedge state([1\ldots n,n+1],[],[]) \supset$$
$$F(2^{n+1})(state([],[],[1\ldots n,n+1]) \wedge another)$$

So, if n is even the induction step has been established. The case with odd n is similar. Q.E.D.

The above theorem obviously implies SOLVE, and hence also P-SOLVE.

THEOREM PROVING AND DECISION PROCEDURES

We have by now given an example of using temporal logic for reasoning about program properties. But it might be said that the proof above gives nothing more than one can have with an 'ordinary' technique, in which mathematical induction is also used. There is, however, one important modification to such objections: consider again the theorem

SOLVE $\vdash state([1,\ldots,N],[],[]) \supset F\, finish$

Loosely speaking, this theorem states that the F-plan can lead to the desired result, provided that the initial state is as specified, and moreover, that in time it *will* indeed achieve this result. We might implement the F-plan in PROLOG, or in some algorithmic language, say Pascal. In fact, the Towers of Hanoi problem is a computer science classic; an example of a very simple PROLOG solution may be found in [Clocksin and Mellish 1984, pp. 146], and a Pascal solution may be found in [Grogono 1978, pp. 102-103]. Now call the implemented program - in whichever language - *P*: if we can prove that *P* satisfies SOLVE, we have proved that *P can and will* solve the problem. This is in contrast to non-temporal techniques. In general, non-temporal techniques only describe whether a

program solves a problem correctly, *if* the program terminates 'in the desired state'; this is called *partial correctness*. In such a framework (for instance, so-called operational semantics), one has to resort to other techniques in order to prove also that the program will indeed terminate. When both conditions:

(i) P can lead (only) to the desired final state
(ii) P will in time achieve this state

are proved, we have proved what is usually called the *total correctness* of P. The point in connection with temporal logic is that SOLVE states both conditions in one single formula, and hence, that a proof of this formula is immediately a proof of the total correctness. We shall illustrate this with one more example below. On the basis of that example we shall conclude this chapter with some fairly general observations on the use of temporal logic in computer science.

A particularly important issue when computation is involved is the question of decidability. It is desirable to have general procedures of theorem proving for the tempo-modal logics which we would like to use for specification and reasoning purposes. In computer science some results in this respects have been obtained. As an example we mention the discrete temporal logic suggested by M. Abadi and Z. Manna [1986] and further studied by H. Bestougeff and G. Ligozat [1992, pp. 267 ff.]. We also mention the branching time logic of Ben-Ari et al. [1981], for which the authors managed to establish some nice decidability and complexity results.

The temporal logic with which we described the F-plan is discrete, and it only treats one temporal 'direction', that is, it does not include any operator for the past. Many of the temporal logics studied in computer science share these limitations, especially the latter limitation. However, in reasoning systems as well as for many practical purposes we would like to have continuous logics with operators for the past as well as for the future. For most of these logics, however, we do not have any general decision procedure, but only partial procedures valid for fragments of the logics in question - like the one studied in

[Øhrstrøm & Klarlund 1986]. It would be interesting to have such results for a tense logic corresponding to Prior's third grade and the Leibnizian tempo-modal logic which we have presented in chapter 3.3 (some recent findings within nominal tense logic [Blackburn 1993] seem promising in this respect).

As an alternative to general procedures for theorem proving, one may use semantical models for the evaluation of tempo-modal statements. Most of the temporal logics we have been studying have in fact been proved to be decidable, i.e. they have *the finite model property*. This means that any formula A is provable in the logic if and only if it is valid in any frame corresponding to the logic. The computational complexities of decision procedures of such logic have been studied and important results have been found (see for instance [Sistla and Clarke 1985]). One highly interesting result was found by Hiroakira Ono and Akira Nakamura [1980]. They have considered some of the most well known tense and modal logics with the finite model property. Let L be one of these logics. We then define a function r_L such that $r_L(m)$ is the smallest number r which satisfies the following condition:

> For any formula A with m modal (or tense) operators, A is provable in L if and only if A is valid in every L-model with at most r worlds.

With respect to 'pure' tense logics Ono and Nakamura have shown that we have the inequality $m+1 \leq r(m) \leq m+3$ for K_b. Their results leave us with a higher degree of uncertainty with respect to linear tense logic, since they have only been able to deduce that the inequality $m+1 \leq r(m) \leq 2m+3$ holds for K_l. At any rate, such results are important for the implementation of evaluation procedures for tempo-modal logics.

ONE FURTHER EXAMPLE

Consider the following program, which computes the greatest common denominator (GCD) of two natural numbers A and B; when the program terminates, $A = GCD$ of the two original input values:

L0: start
L1: read A;
L2: read B;
L3: if A = B then goto L10 else goto L4;
L4: if A < B then goto L5 else goto L8;
L5: C:= A;
L6: A:= B;
L7: B:= C;
L8: A:= A-B;
L9: goto L3;
L10: write A;
L11: end.

The program is written in a very simple programming language known as flowchart language, where a label is attached to each instruction. An ordering among labels is assumed, as indicated by their numbering. L0 is called the start label, and L11 is called a terminal label (in principle, there may be several 'termination points' in programs).

Since the program is supposed to compute the GCD of two arbitrary programs, we can specify this crucial property of the program by the following definitions and conditions:

$$
\begin{aligned}
&GCD(A,B) = g \equiv_{def} \\
&\quad (A \ mod\ g = 0) \wedge (B \ mod\ g = 0) \wedge \\
&\quad \forall x \in N. ((A \ mod\ x = 0 \wedge B \ mod\ x = 0) \supset x \leq g)
\end{aligned}
$$

The input condition can be specified as follows:

$$C_I = \{GCD(A,B) = g \wedge A,B,g \in N\}$$

Strictly speaking, $g \in N$ is implied by the definition of GCD. It should also be noted that the program is written such that $g=1$ is counted as a denominator of any pair of natural numbers.

The output condition is this one:

$$C_O = \{A=g\}$$

Any program which takes some input and computes an output can be described as an input-output function. That is, if P is the program and Π is the corresponding input-output function, then Π is the semantical meaning of P. These notions are customary in so-called operational semantics, which is the least abstract kind of formal semantics for programming languages. Operational semantics for a given programming language contains general rules for systematically constructing the input-output function of any program written in that language. We here ignore these rules (in [Andersen et al., forthcoming] the full set of such rules are given for the same example).

Let X denote the set of possible inputs to P, and Y the set of possible outputs. In our current case, $X = N \times N$ and $Y = N$. We can now state quite precisely the crucial property of P, namely that it computes the greatest common denominator among two natural numbers:

$$\text{(a)} \ \forall x \in X, \forall y \in Y\text{:} \ (C_I(x) \wedge \Pi(x) = y) \supset C_O(y).$$

We can consider (a) to be the formal specification of P, where Π is the input-output function corresponding to P. Moreover, (a) is a necessary condition of the correctness of P: if we can show that C_O obtains, whenever the terminal label L11 is reached - provided that the input condition C_I is satisfied from the start - then we have shown that the program is partially correct. (For the current example this can be shown by a fairly simple inductive proof.) In general, correctness is a relation between a specification and a program. The specification states what the program should do, and a concrete program in some programming language is meant to implement this specification. If we can prove that the program in question satisfies the specification, we have proved its correctness.

However, (a) needs to be augmented in order to state that L11 will indeed be reached. We therefore propose this specification instead, in which we avail ourselves of temporal notions:

(b) $\forall x \in X, \forall y \in Y$: $((at\ L0) \wedge C_I(x)) \supset F((at\ L11) \wedge C_O(y))$.

In this formula we still refer explicitly to P by referring to its labels; we have introduced a predicate 'at' to be able to do so. However, we need not refer to the input-output function of P. A proof that P satisfies (b) is a proof of P's total correctness. - In the current case, we have to do with a deterministic program, and a linear tense logic will suffice. But in many cases, notably for indeterministic or concurrent programs, we can do better with a branching time logic. Moreover, the linear cases can also be described within this framework.

Now let P be any program with input conditions φ, output conditions ψ, 'starting point' β (where computation begins), and a set of terminal points E (where computation ends).

A quite general criterion for the correctness of P can then be stated as

(c) $\forall x \in X, \forall y \in Y$: $((at\ \beta) \wedge \varphi(x)) \supset (NF((at\ e) \wedge \psi(y)))$,
for $e \in E$

In fact, (c) does not refer to P at all. It can be used to refer to any program for which we have an identifiable 'starting point', and an identifiable set of 'termination points'. Loosely, we may read (c) as follows:

> if computation begins and the input conditions are satisfied, then for all branches we 'reach' some terminal state e such that the output conditions are satisfied (and computation ends).

Note that quantification over branches is implicit here. However, we omit the semantical details of this branching time

logic, but these can be found in Ben-Ari et al. [1981]. General and thorough overviews of how to use temporal logic in verification can be found in [Emerson 1990, Stirling 1992].

Given any program P with φ, ψ, β, and E as above, a proof that P satisfies (c) is a proof of its total correctness. Such a proof may of course avail itself of all the techniques in any tense-logical axiomatisation of this branching time logic.

When investigating the properties of a program, we are interested not only in correctness, but also in being able to reason in general about its properties. For instance, in the loop from L3 to L9 the GCD-program may perform a number of subtractions. Hypothetically speaking, this could lead to A becoming negative. The 'swap operation' performed in L5-L7 is designed to prevent this. Again, we can use temporal logic to describe this 'local' property of the program

$$((at\ L3) \wedge (A > 0)) \supset NG(A > 0)$$

Here, we have also used the observation that 'after' the L3-L9 loop, A is never changed. We could have proceeded in smaller steps, first stating a weaker 'invariant' of the loop, and then we could have deduced the above result using other logical statements about the program. But at any rate, the above formula holds and is a strong statement of one important property of the GCD-program.

Especially for reasoning about concurrent programs, temporal logic has proved to be *the* suitable tool. Let *P1* and *P2* be two concurrent processes, sharing some common resource R - say, a printer, or the CPU for that matter. In principle, *P1* can be prevented from ever getting to use R by *P2* snatching it just before *P1*, whenever *P1* requests it. If the concurrent program is to work with satisfactory results, then the following condition must be satisfied:

$requests(P,R) \supset NF(access(P,R))$,
where P is any of the processes involved.

In general, the salient features of concurrent programs can be described with temporal logic, for instance

- freedom of deadlock - e.g. if a number of processes simultaneously request the same resource, then the resource is allocated to one of them (they do not all begin to wait for each other);

- mutual exclusion - it must in some cases be prevented that more than one process is allowed 'into' a certain 'region'; for instance, *P1* and *P2* cannot both work on the same printer at the same time (the results could be imagined);

- fairness - the property briefly discussed above that any process which regularly requests access to some resource does obtain access sooner or later. This condition can be refined in many ways. A (conceptually) simple refinement would be that processes are granted access in the same order as they have requested it;

- liveness - the property that any process which has been temporarily suspended is sooner or later resumed.

This list could be prolonged significantly, and many interesting questions could be raised. The notion of concurrency is not only computationally important, but it also has conceptual - information-theoretic - implications. Just for instance, who has the privilege of giving what information when? What can and should processes be capable of predicting about each other? But we shall leave these issues here.

PERSPECTIVES

The use of temporal logic in computer science is a large and rapidly expanding field. We have but suggested its more central uses and issues, and it must be admitted that even this has been done only in a sketchy manner. Nevertheless, it should have

become evident that temporal logic is very versatile in computer science. It is useful, sometimes crucial, at all levels of computer science, which we might sum up as follows:

• theoretical computer science:
Temporal logic is useful for program verification, specification, and for reasoning about programs in general. In connection with concurrency it is crucial for such purposes. It is also the natural formalism for expressing a generalised idea of total correctness.

• programming languages and their theory:
Here, the crucial question is the development of temporal programming languages. This question is closely associated with the question of decision procedures for temporal logics (cf. Frank Leßke [1991]). Some temporal logical programming languages already exist. We mention Tokio [Fujita et al. 1986], Tempura [Moszkowski 1986], and the work of Dov Gabbay [1987]. A useful overview is given by M. A. Orgun and W. Ma in [Gabbay and Ohlbach 1994, pp. 445-479]. A concomitant but more general question is how to characterise the temporal properties of existing programming languages, and perhaps to establish temporally motivated criteria for evaluating languages. The use of temporal logic in program synthesis - the automatic generation of programs from more general specifications - is also under investigation [e.g. Emerson and Clarke 1982]. Such endeavours, if successful, would tie together specification, programming, and verification in a very fruitful way.

• applications:
The most obvious use of temporal logic in computer science is perhaps within the field of natural language understanding. A large number of fairly advanced 'information systems' also call for temporal logic, for instance planning systems, decision support systems, and diagnostical systems. All these kinds of applications may be seen as cases of artificial intelligence. Our

discussions of the CIMP system (chapter 3.4) and of conceptual graphs (chapter 3.6) are also examples of this kind.

• media systems:
In some systems, wherein aesthetical and communicative properties are especially important, time assumes a crucial rôle. For instance, in hypertext systems or multimedia the conscious control of timing and montage is crucial. Such systems are best understood as well as designed with explicit reference to time (see e.g. [Andersen and Øhrstrøm 1994]). (The general theoretical study of such 'sign production' has recently become known as 'Computer Semiotics'; see for instance [Andersen 1990], [Andersen et al., forthcoming], [Hasle 1993], [Hasle 1995].)

• system development:
As pointed out at the beginning of this chapter, temporal logic can be relevant in problem and domain analysis, as well as in design, implementation and validation. Its rôle in analysis and design is particularly related to its rôle in philosophical logic (analysis of concepts and language). Its possible rôle in implementation and validation is a direct consequence of its relation to theoretical computer science and 'programming languages'.

At the beginning of this chapter we saw that Prior himself in the 1960's anticipated the use of temporal logic in connection with computers. He also observed that there might be some practical gains from the study of tenses "in the representation of time-delay in computer circuits" [TR, p. 4]. This remark also seems to anticipate its use in program verification and even in hardware verification. At the other end of the spectrum, it is clear that the general linguistic and philosophical motivation for tense logic explains its obvious relevance for artificial intelligence and advanced information systems. Thus in general, it would be in a good Priorian spirit to have logicians provide computer science with a collection of logical systems dealing with aspects of time, tense, and modality.

What does all this come down to? Needless to say, the present authors stand in every danger of overestimating the importance of temporal logic, and a sweeping conclusion is indeed tempting: it would seem that temporal logic, or perhaps we should merely say 'temporality', is pivotal within computer science. It extends in a vertical direction, ranging from fundamental theory to applications, and in a horizontal direction, ranging from analysis to validation in concrete system development. But even if we, in deference to computer science proper, have to go for less, it is certainly no exaggeration that temporal logic has proved its practical worth in many areas within computer science.

4. CONCLUSION

The logic of time provides one of the most striking examples of a fruitful interaction between a variety of disciplines, which are normally kept apart, more or less strictly. Philosophy, logic and computer science have played the key parts in this interaction, but we can well refine that picture considerably: we have (to varying degrees) been drawing - as has the development of the logic of time - on what appears to be very diverse sources, namely:

- general philosophy
- ethical and theological considerations
- conceptual analysis
- linguistic considerations
- literary fiction
- the history of ideas
- mathematics
- physics
- computer science.

The pivotal discipline for linking together our various observations has been logic in a broad sense, that is, logic in a 'presystematic' as well as a fully symbolical form. This is in accordance with the conviction stated in the introduction that in order to study time we need to establish a common language for the discussion, and that such a language should be developed within logic. However, logic is not 'merely' a mediating language: the reason why the logic of time can take input from all those various fields, and also contribute to them, is in our conviction that logic in its broad sense is really active in all systematic human thinking. This is, however, not to be taken in a strict psychological sense, but rather as a philosophical statement - and we add that human rationality in our opinion should be seen as comprising more than logic, respectively systematic thinking (just for instance, social intuition, aesthetical sense, rhetorical skill).

In the case of the logic of time, we believe that this subject can only artificially be separated into one part belonging to the humanities and another part belonging to natural science. To be true, for practical purposes the subject may be isolated into for instance one linguistic discipline and another computer science discipline; but their mutual relevance should not be forgotten, and the enterprise of the logic of time should still be seen as a whole.

This conviction may be provocative for traditional humanists as well as traditional natural scientists. We first take the case of opposing humanists: for quite some time and in a good many places people brought up within the humanities have been told that logic is completely irrelevant in a field such as, say, literature, and outright misleading when applied within linguistics. Now such assertions raise many issues, which we shall not deal with in any detail here. But we may remind the reader of just one example. The analysis of Borges' short story 'The Garden of Forking Paths' demonstrated how certain logical ideas were anticipated in a piece of literary fiction - ideas, which are indeed formalisable as well as technically applicable. On the other hand, it also showed how logical concepts can be applied in a literary analysis: we think it would be much more difficult to discern and present the ideas and the structure of the story without those concepts. - The Borges-example may seem biased to the extent that the story in question lends itself to logical considerations in an unusual degree; and certainly, we did choose that example because it is particularly striking. But we also think that the kind of two-ways traffic exhibited in this connection is quite general: logical analysis plays a rôle in systematic thinking and is therefore, rightly, manifest also in the humanities - much more so than is normally recognised. And conversely, symbolic logics to a very high degree reflect or embody presystematic analyses of conceptual structures, philosophical problems and linguistic examples. The logic of time is a particularly evident example, but the same observation also applies to such prominent foundations of mathematical logic as Boolean Algebra and Frege's Predicate Logic - which

Frege himself called 'Begriffsschrift', meaning approximately 'conceptual writing'.

With these remarks we also anticipate an answer to sceptical natural scientists, who may actually be using temporal logic in their field, but who hold that its philosophical and historical background are really irrelevant to the purposes within the respective discipline. In fact, such a view was vividly expressed by [Ben-Ari et al. 1981], who were among the first computer scientists to systematically apply branching time systems within program verification. In discussing the linear time approach versus the branching time approach within the field, they stated that

> The difference in approaches has very little to do with the philosophical question of the structure of physical time which leads to the metaphysical problems of determinacy [sic] versus free will. Instead, it is pragmatically based on the choice of the type of programs and properties one wishes to formalise and study. [p. 164]
>
> In the end, the choice between linear and branching models cannot be made on philosophical grounds but instead should be dictated by the type of programs, execution policies and properties which one wishes to study. [p. 165]

There is a point here which is much too common-sensical to be brushed aside easily. Clearly, the philosopher, the linguist, the physicist, or the computer scientist using temporal logic may and must adapt this logic to specific purposes. In doing so, the background of temporal logic can sometimes be ignored - and nobody would call a scientist using temporal logic in this way incompetent, because he or she did not know Diodorus Cronus! Nevertheless, the conceptual background as well as still new conceptual analyses have proved to be an important source - we say, a crucial source - for innovations and progress in the field of temporal logic. Such new developments in turn will have an impact on the sciences for which temporal logic is useful. But there is more to it than that fairly utilitaristic argument. It

seems to become ever more recognised in our age that the classical division between 'hard' empirical science on one hand and more qualitative conceptual considerations on the other hand is highly mythical. In this connection we shall as our last example reflect on the notion of 'information systems'. Such systems are clearly related to artificial intelligence - for instance, the CIMP system of chapter 3.4 is an information system as well as a case of artificial intelligence. But 'information systems' may be understood as comprising a broader range of applications than 'artificial intelligence'.

The term 'information systems' refers to a certain class of computer applications, which is rapidly gaining in importance (indeed the notion is integral to the idea of an 'information society'). The very term indicates that attention is shifted away from the underlying computer architecture and towards the information content of the system. Of course, that does not mean that the underlying architecture has become unimportant - that part still ultimately defines the possibilities as well as limitations in the construction of such systems. Moreover, there is no sharp boundary between the construction of an information system and a 'classical' program development: the latter also models some kind of information process. The difference is a matter of degree: in classical program development, the central activity is the construction of an algorithm. Such an algorithm specifies step-by-step how the computer is to carry out its computation. To that extent it is fair to call the process machine-oriented. In the construction of information systems, on the other hand, such considerations only enter at a very late stage, if at all: the emphasis is clearly put on the modelling of information, and with modern development tools the algorithmic aspect comes to be of secondary importance (for instance, when fourth generation languages, expert system shells etc. are used). Therefore, an appropriate analysis of information in the domain in question is the crucial foundation of the entire construction process of an information system; to that extent it is fair to call the process information-oriented.

Logic in its broad sense provides a bridge between a presystematic analysis of information and a formalisation which can be implemented. When focusing on the relation between logic and the analysis of information in a domain, it is worth recollecting Prior's view of logic: logic in his opinion "is not primarily about language, but about the real world" [TR, p. 1] (cf. chapter 2.5). In a domain we will find real phenomena and relations between them. Logical analysis is not 'just' a language game, but an attempt at singling out crucial phenomena and to capture the relations between them. However, this endeavour - at least when it becomes systematical - presupposes that we formulate our initial ideas about the domain in language, and therefore logical analysis is mediated by language. According to Peter Geach [1970, p.187] the young Russell as well as Prior held that "ordinary language is not the logician's master, but it must be his guide".

Critics of logic have often contended that it is a study of highly artificial linguistic examples - indeed we have tried to show that such protestations were abroad already in the Renaissance and a cause of the downfall of Scholastic Logic. Such sentiments may even be shared by scientists and other professionals working in the field of information systems. Some of those may consider formal logic to be a useful language, but they may at the same time hold that the analyses current within philosophical logic are somewhat esoteric and 'out of bounds' for their purposes. To meet such objections we first point out that on Prior's - and our - conception of logic, it must always be remembered that the linguistic examples mediate a study of real phenomena and relations. This means that logic does not have to be empirically faithful to natural language in all respects, but on the other hand it does not render logic irrelevant to the study of language, for the latter also has to deal with reality in a logically reasonable way. Logical analysis of the kind we have been studying in this book is required for the construction of most information systems. We assert that logical analysis *is* information analysis. The latter, of course, comprises more than logic, for instance statistical methods, but any information analysis which should lead to a computerised

system must from some stage be logical (or translatable into a logical analysis). And that is also evident if we turn our attention away from the domain and towards computer implementation. Such obvious vehicles for the implementation of information systems as relational databases and logic programming are based on relational logic, also in a technical sense. In general, the development of ever higher level programming languages reflects how attention is increasingly directed towards modelling information and away from reflecting internal computer architecture (cf. [Andersen et al. (forthcoming)]).

Recent developments such as object-oriented programming and constraint programming emphasise this trend and its connection with logic. Thus in object-oriented programming one strives to identify objects (phenomena), their properties and the relations between them. Furthermore, the crucial notions of generalisation and specialisation are equivalent to logical implication, or set inclusion; and the notions of intension and extension crucial in object-oriented analysis are imported directly from the logical tradition. - Constraint programming also strives to identify logical properties within programs.

The fact that logic is active both in the analysis of a domain and its information content, and in relational databases, logic programming, object-oriented programming etc., explains why the latter are particularly well suited for the construction of information systems. And the fact that these programming paradigms are constantly gaining importance at the expense of classical algorithmic approaches reflects how the use of the computer is increasingly becoming a matter of information handling rather than 'brute' data transformation, for which the algorithmic approach was ideally suited. To fully understand these developments one must have knowledge of more than programming languages and their principles: one must also understand the relation between logical languages and their conceptual background, as well as their history. The development of temporal logic is a brilliant and exemplary case in point, in its historical as well as systematical aspects.

We consider temporal logic as a field worth studying in its own right, and this would be our conviction even it had no 'practical

applications' at all. We hope that this interest in the subject for its own sake has been conveyed to the reader. Nevertheless, with the case of temporal logic in computer science we have shown how concepts and formalisms originally developed for entirely analytical purposes have proved their worth within applied science. Thus, the movement from the historical background (part one), via the formal development of tense-logical calculi (part two), and into computational applications (part three) may also serve as a demonstration of a more general point, namely that the philosophical analysis of concepts, language and logic is highly relevant to the field of information systems. It seems to us that this point is particularly well exemplified with reference to time. Therefore, exactly by emphasising the internal unity of the concept of time our study may have shown how 'time is ubiquitous'.

APPENDIX

The following is a summary of the systems discussed in the chapters 2.9, 2.10, and 3.3.

1.1 NON-METRICAL A-LOGIC (TENSE LOGICS)

Given a set of propositional variables (denoted p, q, r, ...) and a non-empty subset of this set i.e. the instant variables (denoted a, b, c ...). Then the language of non-metrical A-logics can be presented by the following formation rules for well-formed formulas (wff):

(1) Propositional variables, p, q, r ... are wff
(2) If p and q are wff, then $\sim p$, $p \wedge q$, Pp, Fp, Lp are also wff's.
(3) If p is a wff, then $\forall a{:}p$ is also a wff.
(4) Nothing else is a A-wff.

Abbreviations/definitions:

$Hp \equiv_{def} \sim P \sim p$
$Gp \equiv_{def} \sim F \sim p$
$(p \supset q) \equiv_{def} \sim (p \wedge \sim q)$
$(p \vee q) \equiv_{def} \sim (\sim p \wedge \sim q)$

SYSTEM K_t

Axioms:

(A1)	p, where p is a tautology of the propositional calculus
(A2)	$G(p \supset q) \supset (Gp \supset Gq)$
(A3)	$H(p \supset q) \supset (Hp \supset Hq)$
(A4)	$p \supset HFp$
(A5)	$p \supset GPp$

Rules:

(RMP)	If $\vdash p$ and $\vdash p \supset q$, then $\vdash q$.
(RG)	If $\vdash p$, then $\vdash Gp$.
(RH)	If $\vdash p$, then $\vdash Hp$.

SYSTEM K_b

Add to K_t the axioms:

(A6)	$FFp \supset Fp$
(A7)	$FPp \supset (Pp \vee p \vee Fp)$

Some theorems in K_b:

(A6x)	$PPp \supset Pp$
(A7x)	$(Pp \wedge Pq) \supset (P(p \wedge q) \vee P(p \wedge Pq) \vee P(Pp \wedge q))$

SYSTEM K_l

Add to K_b the axioms:

(A8)	$PFp \supset (Pp \vee p \vee Fp)$
(A9)	$Gp \supset Fp$
(A10)	$Hp \supset Pp$
(A11)	$Fp \supset FFp$

Some theorems in K_l:

(A8x)	$(Fp \wedge Fq) \supset (F(p \wedge q) \vee F(p \wedge Fq) \vee F(Fp \wedge q))$
(A11x)	$Pp \supset PPp$

SYSTEM Pr_t

Definition: An instant proposition from K_t is any set of K_t-wff's, which is maximal and consistent with respect to K_t.

Well-formed formulae (wff):

(1)	Any K_t-wff is a Pr_t-wff.
(2)	Any instant proposition from K_t is a Pr_t-wff.

(3) If α and β are Pr_t-wff's, and x is an instant proposition, then $\sim\alpha$, $\alpha \wedge \beta$, $\forall x{:}a$, $P\alpha$, and $F\alpha$ and all Pr_t-wff's.

(4) There are no other Pr_t-wff's.

Abbreviation:

$(\exists a{:}p) \equiv_{def} \sim(\forall a{:}\sim p)$

Axiom:

(I1) $\exists a{:}\ a$

Rule:

(RI) For any instant proposition a and any wff p:
If not$\vdash a \supset p$, then $\vdash a \supset \sim p$

Prior's quantification rules:

(Π1) If $\vdash \phi(x) \supset \beta$ then $\vdash \forall x{:}\phi(x) \supset \beta$.
(Π2) If $\vdash \alpha \supset \phi(x)$ then $\vdash \alpha \supset \forall x{:}\phi(x)$, for x not free in α.

Deduced rules:

(Σ1) If $\vdash \phi(x) \supset \beta$, then $\vdash \exists x{:}\phi(x) \supset \beta$, for x not free in β.
(Σ2) If $\vdash \alpha \supset \phi(x)$, then $\vdash \alpha \supset \exists x{:}\phi(x)$.

THE SYSTEM, Prior_t

Well-formed formulae (wff):

(1) Any Pr_t-wff is a Prior_t-wff.

(2) If α and β are Prior_t-wff's and x is an instant proposition, then $\sim\alpha$, $\alpha \wedge \beta$, $L\alpha$, and $\forall x{:}\alpha$ are all Prior_t-wff's.

(3) There are no other Prior_t-wff's.

Axioms:

(L1) $L(p \supset q) \supset (Lp \supset Lq)$
(I2) $\sim L \sim a$
(I3) $L(a \supset p) \vee L(a \supset \sim p)$
(BF) $L(\forall a{:}\ \phi(a)) \equiv \forall a{:}\ L(\phi(a))$
(LG) $Lp \supset Gp)$
(LH) $Lp \supset Hp)$

Rule:

(RL) If $\vdash p$, then $\vdash Lp$

In the same way, we can construct the systems Prior_b and Prior_l from K_b and K_l, respectively.

1.2 Non-metrical tempo-modal logics

In the non-metrical tempo-modal logics the modal operator L is taken into account. The standard modal logics are M, S4 and S5. They can be presented as axiomatic systems.

System M

Axioms:

(L1) $L(p \supset q) \supset (Lp \supset Lq)$

(L2) $Lp \supset p$

Rule:

(RL) If $\vdash p$, then $\vdash Lp$.

System S4

Add to M the axiom:

(L3) $Lp \supset LLp$

System S5

Add to S4 the axiom:

(L4) $\sim L \sim Lp \supset Lp$

The McArthur system

Add to K_l the axioms:

(L1) $L(p \supset q) \supset (Lp \supset Lq)$

(L3) $Lp \supset LLp$

(LG) $Lp \supset Gp$

(LP) $p \supset LPp$, where p contains no occurrences of F

and the rule

(RL) If $\vdash p$, then $\vdash Lp$.

1.3. Leibnizian Tense Logic LT

Axioms of LT:

(A1)	A, where A is a tautology of the propositional calculus
(A2)	$G(A \supset B) \supset (GA \supset GB)$
(A3)	$H(A \supset B) \supset (HA \supset HB)$
(A4)	$A \supset HFA$
(A5)	$A \supset GPA$
(A6)	$FFA \supset FA$
(A7)	$FPA \supset (PA \vee A \vee FA)$
(A8)	$PFA \supset (PA \vee A \vee FA)$
(A9)	$GA \supset FA$
(A10)	$HA \supset PA$
(A11)	$FA \supset FFA$
(A12)	$NGA \supset GNA$
(A13)	$PA \supset NPA$, where A contains no occurrences of F

In addition we have the S5 axioms for N:

(N1)	$N(A \supset B) \supset (NA \supset NB)$
(N2)	$NA \supset A$
(N3)	$NA \supset NNA$
(N4)	$MNA \supset NA$, where $M \equiv_{def} \sim N \sim$

Rules in LT:

(RMP)	If $\vdash A$ and $\vdash A \supset B$, then $\vdash B$.
(RG)	If $\vdash A$, then $\vdash GA$.
(RH)	If $\vdash A$, then $\vdash HA$.
(RN)	If $\vdash A$, then $\vdash NA$.
(Π1)	If $\vdash \phi(x) \supset \beta$, then $\vdash \forall x{:}\phi(x) \supset \beta$.
(Π2)	If $\vdash \alpha \supset \phi(x)$, then $\vdash \alpha \supset \forall x{:}\phi(x)$, for x not free in α.

1.4. NON-METRICAL B-LOGIC (INSTANT LOGICS)

Let *TIME* be a non-empty set. The elements in *TIME* are called instants, dates or just times. Assume that there is defined a relation, < , on *TIME*. The expression $t_1 < t_2$ is read 't_1 is before t_2'.

The language i.e. the well-formed formula (wff):

(1) If p is a propositional variable and t is an instant i.e. $t \in TIME$, then $T(t,p)$ is a wff.
(2) If $T(t,p)$ and $T(t,q)$ are wff's and t_1 and t_2 are instants, then $T(t,\sim p)$, $T(t,p \wedge q)$, $t_1 < t_2$ are also wff's.
(3) If X and Y are wff's and $t \in TIME$, then $\sim X$, $X \wedge Y$, $\exists t{:}X$, $\forall t{:}\ X$, are also wff's.
(4) Nothing else is a wff.

We shall use the same abbreviaitons and definitions as in tense logics.

B-logical tense-defintions:

(DF) $T(t,Fp) \equiv_{def} \exists t_1: (t<t_1 \wedge T(t_1,p))$
(DP) $T(t,Pp) \equiv_{def} \exists t_1: (t_1<t \wedge T(t_1,p))$
(DL) $T(t,Lp) \equiv_{def} \forall t_1: T(t_1,p)$

MINIMAL B-LOGIC, B_m

Axioms:

(T1) $T(t,p \wedge q) \equiv (T(t,p) \wedge T(t,q))$
(T2) $T(t,\sim p) \equiv \sim T(t,p)$

Some theorems in B_m:

(DG) $T(t,Gp) \equiv \forall t_1: (t<t_1 \supset T(t_1,p))$
(DH) $T(t,Hp) \equiv \forall t_1: (t_1<t \supset T(t_1,p))$

BRANCHING TIME LOGIC B_b

Add the following axioms to B_m:

(B1) $(t_1 < t_2 \wedge t_2 < t_3) \supset t_1 < t_3.$
(B2) $(t_1 < t_2 \wedge t_3 < t_2) \supset (t_1 < t_3 \vee t_1 = t_3 \vee t_3 < t_1)$

LINEAR TIME LOGIC B_l:

Add the following axioms to B_b:

(B3) $(t_2 < t_1 \wedge t_2 < t_3) \supset (t_1 < t_3 \vee t_1 = t_3 \vee t_3 < t_1)$
(B4) $\forall t_1 \exists t_2 : t_1 < t_2$
(B5) $\forall t_1 \exists t_2 : t_2 < t_1$
(B6) $\forall t_1 \forall t_2 \exists t_3 : t_1 < t_2 \supset (t_1 < t_3 \wedge t_3 < t_2)$

EXTENDED B-LOGIC:

Any of the systems B_m,B_b, and B_l can be extended by the axioms:

(TX1) $(\forall t: T(t,p)) \supset p$
(TX2) $(\forall t_1: T(t_1,p)) \supset T(t_2, \forall t_3: T(t_3,p))$
(TX3) $T(t_1,p) \supset T(t_2,T(t_1,p))$

and the rule:

(RT) If $\vdash p$, then $\vdash T(t,p)$ for any t

For the Leibnizian system we need the following B-logic:

Definition:
A Leibnizian structure is a quadruple $(TIME,<,\approx,T)$ where $TIME$ is a non-empty set with two relations $<$ and $\approx$ such that

(B1) $(t_1 < t_2 \wedge t_2 < t_3) \supset t_1 < t_3$

(B2) $(t_1 < t_2 \wedge t_3 < t_2) \supset (t_1 < t_3 \vee t_1 = t_3 \vee t_3 < t_1)$
(B3) $(t_2 < t_1 \wedge t_2 < t_3) \supset (t_1 < t_3 \vee t_1 = t_3 \vee t_3 < t_1)$
(B4) $\forall t_1 \exists t_2 : t_1 < t_2$
(B5) $\forall t_1 \exists t_2 : t_2 < t_1$
(B6) $\forall t_1 \forall t_2 \exists t_3 : t_1 < t_2 \supset (t_1 < t_3 \wedge t_3 < t_2)$
(B7) $t \approx t$
(B8) $t_1 \approx t_2 \supset t_2 \approx t_1$
(B9) $(t_1 \approx t_2 \wedge t_2 \approx t_3) \supset t_1 \approx t_3$
(B10) $(t_1 \approx t_2 \wedge t_3 < t_2) \supset \exists t_4: (t_3 \approx t_4 \wedge t_4 < t_1)$

and a truth operator T such that

(T1) $T(t,A \wedge B) \equiv (T(t,A) \wedge T(t,B))$
(T2) $T(t,\sim A) \equiv \sim T(t,A)$
(T3) $\forall x{:}T(t,A) \equiv T(t, \forall x{:}A)$ where x is foreign to t.
(T4) $T(t,FA) \equiv \exists t_1: (t < t_1 \wedge T(t_1,A))$
(T5) $T(t,PA) \equiv \exists t_1: (t_1 < t \wedge T(t_1,A))$
(T6) $T(t,NA) \equiv \forall t_1: (t_1 \approx t \supset T(t_1,A))$
(T7) $\forall t_1 \forall t_2: (t_1 \approx t_2 \wedge T(t_1,PA)) \supset T(t_2,PA)$
where A contains no occurrences of F.

2. METRICAL A-LOGIC (TENSE LOGICS)

The language of MT is based on a set of propositional variables (denoted p, q, r...) and a non-empty set of instant variables (denoted a, b, c ...). The set of well-formed formulae can be presented by the following definition:

(1) Propositional variables are wff.
(2) If α and β are wff, and x is a positive number, then $\sim\alpha$, $\alpha \supset \beta$, $\alpha \wedge \beta$, $\alpha \vee \beta$, $\forall x{:}\alpha$, $L\alpha$, $P(x)\alpha$, and $F(x)\alpha$ are all wff.
(3) If α is a wff, then $\forall a{:}\alpha$ is also a wff.
(4) There are no other wff.

We shall use the same quantifier symbols for quantification over numbers and instant variables and we shall use the same abbreviations/definitions as in the systems above with the addition of the following definitions:

(DGF) $G(x)\alpha \equiv_{def} \sim F(x) \sim \alpha$
(DHF) $H(x)\alpha \equiv_{def} \sim P(x) \sim \alpha.$
(DUF) $Fp \equiv_{def} \exists x{:}F(x)p$
(DUG) $Gp \equiv_{def} \forall x{:}G(x)p$
(DUP) $Pp \equiv_{def} \exists x{:}P(x)p$
(DUH) $Hp \equiv_{def} \forall x{:}H(x)p$

The axioms of the system MT are:

(MT1) $G(x)(p \supset q) \supset (G(x)p \supset G(x)q)$
(MT2) $F(x)H(x)p \supset p$
(MT3) $F(y+x)p \supset F(y)F(x)p$

The rules of the system MT are:

(RM) The 'mirror image' of any theorem (in which all occurrences of P are replaced by F and vice versa) is also a theorem.

(RMP) If $\vdash A$ and $\vdash A \supset B$, then $\vdash B$.
(RF) If $\vdash A$, then $\vdash G(x)A$.
(Π1) If $\vdash \phi(x) \supset \beta$ then $\vdash \forall x{:}\phi(x) \supset \beta$.
(Π2) If $\vdash \alpha \supset \phi(x)$ then $\vdash \alpha \supset \forall x{:}\phi(x)$, for x not free in α.

Some theorems in MT:

(MT4) $H(x)(p \supset q) \supset (P(x)p \supset P(x)q)$
(MT5) $p \supset G(x)P(x)p$
(MT6) $P(x)G(x)p \supset p$
(MT7) $\forall x{:}\ G(y)G(x)p \supset G(y)\forall x{:}\ G(x)p$
(MT8) $\forall x{:}\ G(y)H(x)p \supset G(y)\forall x{:}H(x)p$

LEIBNIZIAN METRIC TENSE LOGIC MLT

The axioms of the system MLT are:

(LT1) $G(x)(p \supset q) \supset (G(x)p \supset G(x)q)$
(LT2) $F(x)H(x)p \supset p$
(LT3) $F(y{+}x)p \supset F(y)F(x)p$
(LT4) $H(x)(p \supset q) \supset (H(x)p \supset H(x)q)$
(LT5) $P(x)G(x)p \supset p$
(LT6) $P(y{+}x)p \supset P(y)P(x)p$
(LT7) $F(x){\sim}p \equiv {\sim}F(x)p$
(LT8) $P(x){\sim}p \equiv {\sim}P(x)p$
(LT9) $NG(x)p \supset G(x)Np$
(LT10) $P(x)p \supset NP(x)p$,
where p contains no occurrences of F.

In addition we assume the S5-axioms hold for N.

3. THE SYSTEM OF PRIOR'S 3RD GRADE

Given a set of propositional variables (denoted p, q, r, ...) and a non-empty subset of this set i.e. the instant variables (denoted a, b, c ...). Then the language of non-metrical A-logics can be presented by the following formation rules for well-formed formula (wff):

(1) Propositional variables p, q, r ... are wff
(2) If p and q are wff, then $\sim p$, $p \wedge q$, Pp, Fp, Lp are also wff's.
(3) If p is a wff, then $\forall a{:}p$ is also a wff.
(4) Nothing else is a wff.

Abbreviations/definitions as in 1.1 and in addition:

(DE) $(\exists a{:}p) \equiv_{def} \sim(\forall a{:}\sim p)$
(DB) $a < b \equiv_{def} L(a \supset Fb)$
(DT) $T(a,p) \equiv_{def} L(a \supset p)$

Axioms for instant variables:

(I1) $\exists a{:}\ a$
(I2) $\sim L \sim a$
(I3) $L(a \supset p) \vee L(a \supset \sim p)$

Some theorems:

(DL) $\forall a{:}\ T(a,p) \equiv Lp$
(DG) $T(a,Gp) \equiv \forall b{:}(a<b \supset T(b,p))$
(DH) $T(a,Hp) \equiv \forall b{:}(b<a \supset T(b,p))$

3.1. INSTANT-LOGIC AND METRIC TENSE-LOGIC

SYSTEM MT*

Add to MT an modal operator for which the following axioms hold:

(BF1)	$\forall x{:}L\phi(x) \supset L(\forall x{:}\ \phi(x))$
(BF2)	$\exists x{:}L\phi(x) \supset L(\exists x{:}\ \phi(x))$
(LHX)	$Lq \supset H(x)q$
(LGX)	$Lq \supset G(x)q$

Definition:

(DB1) $before(a,b,x) \equiv_{def} L(a \supset F(x)b)$

Some theorems in MT*:

(L5)	$Lp \supset Gp$
(L6)	$Lp \supset Hp$
(T8)	$before(a,b,x) \equiv L(b \supset P(x)a)$
(T9)	$a<b \supset \exists x{:}\ before(a,b,x)$
(T10)	$\exists x{:}\ before(a,b,x) \supset a<b$
(T11)	$a<b \equiv \exists x{:}\ before(a,b,x)$
(T12)	$\exists b{:}\ (before(a,b,x) \wedge T(b,p)) \supset T(a,F(x)p)$
(T13)	$L(P(x)a \supset {\sim}p) \supset L(a \supset {\sim}F(x)p)$
(T14)	$T(a,F(x)p) \supset (\exists b{:}\ before(a,b,x) \wedge T(b,p))$
(T15)	$T(a,F(x)p) \equiv (\exists b{:}\ before(a,b,x) \wedge T(b,p))$
(T16)	$T(a,P(x)p) \equiv (\exists b{:}\ before(b,a,x) \wedge T(b,p))$
(T17)	$T(a,\exists x{:}q) \equiv \exists x{:}T(a,q)$
(T18)	$before(a,b,x+y) \supset \exists c{:}(before(a,c,x) \wedge before(c,b,y))$

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